

INTEGRALI FRATTI 5

$$\int \frac{1}{6x^3 - 7x^2 - 3x} dx$$

$$6x^3 - 7x^2 - 3x = x(6x^2 - 7x - 3)$$

calcoliamo $\Delta = 49 + 4 \cdot 3 \cdot 6 > 0$

$$\Delta = 49 + 72 = 121$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{7 \pm 11}{12} \quad \left\{ \begin{array}{l} x_1 = -\frac{4}{12} = -\frac{1}{3} \\ x_2 = \frac{18}{12} = \frac{3}{2} \end{array} \right.$$

$$6x^3 - 7x^2 - 3x = x \cdot 6 \left(x + \frac{1}{3}\right) \left(x - \frac{3}{2}\right) = x \cdot 6 \cdot \frac{1}{3} (3x+1) \cdot \frac{1}{2} (2x-3) =$$

$$= x(3x+1)(2x-3)$$

$$\frac{1}{6x^3 - 7x^2 - 3x} = \frac{A}{x} + \frac{B}{3x+1} + \frac{C}{2x-3} =$$

$$= \frac{A(3x+1)(2x-3) + Bx(2x-3) + Cx(3x+1)}{x(3x+1)(2x-3)} =$$

$$= \frac{A(6x^2 - 9x + 2x - 3) + B(2x^2 - 3x) + C(3x^2 + x)}{x(3x+1)(2x-3)} =$$

$$= \frac{A(6x^2 - 7x - 3) + B(2x^2 - 3x) + C(3x^2 + x)}{x(3x+1)(2x-3)} =$$



$$= \frac{(6A+2B+3C)x^2 - (7A+3B-C)x - 3A}{x(3x+1)(2x-3)} =$$

$$= \frac{1}{6x^3 - 7x^2 - 3x}$$

devo essere

$$\begin{cases} 6A + 2B + 3C = 0 \\ -(7A + 3B - C) = 0 \\ -3A = 1 \end{cases} \quad \begin{cases} -2 + 2B + 3C = 0 \\ -\frac{7}{3} + 3B - C = 0 \\ A = -\frac{1}{3} \end{cases}$$

$$\begin{cases} -2 + 2B + 9B - 7 = 0 \\ C = 3B - \frac{7}{3} \\ A = -\frac{1}{3} \end{cases} \quad \begin{cases} 11B = 9 \\ C = 3B - \frac{7}{3} \\ A = -\frac{1}{3} \end{cases} \quad \begin{cases} B = \frac{9}{11} \\ C = \frac{27}{11} - \frac{7}{3} = \frac{81-77}{33} \\ A = -\frac{1}{3} \end{cases}$$

$$\frac{1}{6x^3 - 7x^2 - 3x} = -\frac{1}{3x} + \frac{9}{11(3x+1)} + \frac{4}{33(2x-3)}$$



$$\int \frac{dx}{6x^3 - 7x^2 - 3x} = -\frac{1}{3} \int \frac{dx}{x} + \frac{9}{11} \int \frac{dx}{3x+1} + \frac{4}{33} \int \frac{dx}{2x-3} =$$

$$= -\frac{1}{3} \log|x| + \frac{9}{11} \int \frac{dx}{3x+1} + \frac{4}{33} \int \frac{dx}{2x-3}$$

$$\int \frac{dx}{3x+1} \quad t=3x+1 \quad dt=3dx \quad dx = \frac{dt}{3}$$

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$$\frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} \log|3x+1| + c$$

$$\int \frac{dx}{2x-3} \quad t=2x-3 \quad dt=2dx \quad dx = \frac{dt}{2}$$

↓

$$\frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \log|2x-3| + c$$

$$\int \frac{dx}{6x^3 - 7x^2 - 3x} = -\frac{1}{3} \log|x| + \frac{9}{11} \cdot \frac{1}{3} \log|3x+1| + \frac{4}{33} \cdot \frac{1}{2} \log|2x-3| + c$$

$$= -\log \sqrt[3]{x} + \log \sqrt[11]{|3x+1|^3} + \log \sqrt[33]{|2x-3|^2} + c$$

