

INTEGRALI FRATTI 7

$$\int \frac{x^2}{x^3+5x^2+8x+4} dx$$

Proviamo a scomporre il denominatore con la regola di Ruffini. Una radice, individuata per tentativi, è $x = -1$.

$$(-1)^3 + 5(-1)^2 + 8(-1) + 4 = -1 + 5 - 8 + 4 = -9 + 9 = 0$$

	1	5	8	4
-1		-1	-4	-4
	1	4	4	//

$$x^3 + 5x^2 + 8x + 4 = (x+1)(x^2 + 4x + 4) = (x+1)(x+2)^2$$

$$\frac{x^2}{x^3+5x^2+8x+4} = \frac{A}{x+1} + \frac{B_1}{x+2} + \frac{B_2}{(x+2)^2} =$$

$$= \frac{A(x+2)^2 + B_1(x+1)(x+2) + B_2(x+1)}{(x+1)(x+2)^2} =$$

$$= \frac{A(x^2+4x+4) + B_1(x^2+3x+2) + B_2(x+1)}{(x+1)(x+2)^2} =$$

$$= \frac{x^2(A+B_1) + x(4A+3B_1+B_2) + 4A+2B_1+B_2}{(x+1)(x+2)^2}$$



$$\begin{cases} A+B_1=1 \\ 4A+3B_1+B_2=0 \\ 4A+2B_1+B_2=0 \end{cases}$$

$$\begin{cases} A=1-B_1 \\ 4-4B_1+3B_1+B_2=0 \\ 4-4B_1+2B_1+B_2=0 \end{cases}$$

$$\begin{cases} A=1-B_1 \\ 4-B_1+B_2=0 \\ 4-2B_1+B_2=0 \end{cases}$$

$$\begin{cases} A=1-B_1 \\ B_2=B_1-4 \\ 4-2B_1+B_1-4=0 \end{cases}$$

$$\begin{cases} A=1 \\ B_2=-4 \\ -B_1=0 \end{cases}$$

$$\int \frac{x^2}{x^3+5x^2+8x+4} dx = \int \frac{dx}{x+1} + 4 \int \frac{dx}{(x+2)^2}$$

$$\int \frac{dx}{x+1} = \log|x+1| + C$$

$$\int \frac{dx}{(x+2)^2} = \int (x+2)^{-2} dx = -(x+2)^{-1} = -\frac{1}{x+2}$$

$$\int \frac{x^2}{x^3+5x^2+8x+4} dx = \log|x+1| + \frac{4}{x+2} + C$$

