INTEGRALI FRATTI 9

$$\int \frac{x^2}{(x+2)^2(x+4)^2}\,dx$$

Il denominatore ha le due radici doppie per cui la frazione si può scomporre nel modo seguente

$$\frac{x^2}{(x+2)^2(x+4)^2} = \frac{A_1}{x+2} + \frac{A_2}{(x+2)^2} + \frac{B_1}{x+4} + \frac{B_2}{(x+4)^2}$$

$$\frac{A_1(x+2)(x+4)^2 + A_2(x+4)^2 + B_1(x+2)^2(x+4) + B_2(x+2)^2}{(x+2)^2(x+4)^2}$$

$$A_1(x+2)(x^2+8x+16) + A_2(x^2+8x+16) + B_1(x^2+4x+4)(x+4) + B_2(x^2+4x+4)$$

$$A_1(x^3+8x^2+16x+2x^2+16x+32) + A_2(x^2+8x+16) + B_1(x^3+4x^2+4x+4x^2+16x+16) +$$
$$+ B_2(x^2+4x+4)$$

$$A_1(x^3+10x^2+32x+32) + A_2(x^2+8x+16) + B_1(x^3+8x^2+20x+16) + B_2(x^2+4x+4) =$$

$$= x^3(A_1+B_1) + x^2(10A_1+A_2+8B_1+B_2) + x(32A_1+8A_2+20B_1+4B_2) +$$
$$+ 32A_1+16A_2+16B_1+4B_2 = x^2$$

Per il principio di identità

$$\begin{cases} A_1+B_1=0 \\ 10A_1+A_2+8B_1+B_2=1 \\ 32A_1+8A_2+20B_1+4B_2=0 \\ 32A_1+16A_2+16B_1+4B_2=0 \end{cases}$$

$$\begin{cases} B_1 = -A_1 \\ 10A_1 + A_2 - 8A_1 + B_2 = 1 \\ 32A_1 + 8A_2 - 20A_1 + 4B_2 = 0 \\ 32A_1 + 16A_2 - 16A_1 + 4B_2 = 0 \end{cases} \qquad \begin{cases} B_1 = -A_1 \\ 2A_1 + A_2 + B_2 = 1 \\ 12A_1 + 8A_2 + 4B_2 = 0 \\ 16A_1 + 16A_2 + 4B_2 = 0 \end{cases}$$

$$\begin{cases} B_1 = -A_1 \\ B_2 = 1 - 2A_1 - A_2 \\ 12A_1 + 8A_2 + 4 - 8A_1 - 4A_2 = 0 \\ 16A_1 + 16A_2 + 4 - 8A_1 - 4A_2 = 0 \end{cases} \qquad \begin{cases} B_1 = -A_1 \\ B_2 = 1 - 2A_1 - A_2 \\ 4A_1 + 4A_2 + 4 = 0 \\ 8A_1 + 12A_2 + 4 = 0 \end{cases}$$

$$\begin{cases} B_1 = -A_1 \\ B_2 = 1 - 2A_1 - A_2 \\ 4A_1 + 4A_2 = 8A_1 + 12A_2 \\ 8A_1 + 12A_2 + 4 = 0 \end{cases} \qquad \begin{cases} B_1 = -A_1 \\ B_2 = 1 - 2A_1 - A_2 \\ 4A_1 + 8A_2 = 0 \\ 8A_1 + 12A_2 + 4 = 0 \end{cases}$$

$$\begin{cases} B_1 = -A_1 \\ B_2 = 1 - 2A_1 - A_2 \\ A_2 = -\dfrac{A_1}{2} \\ 8A_1 - 6A_2 + 4 = 0 \end{cases} \qquad \begin{cases} B_1 = -A_1 \\ B_2 = 1 - 2A_1 - A_2 \\ A_2 = -A_1/2 \\ 8A_1 + 3A_1 + 4 = 0 \end{cases}$$

$$\begin{cases} B_1 = -A_1 \\ B_2 = 1 - 2A_1 + \dfrac{x_1}{2} \\ A_2 = -A_1/2 \\ 8A_1 - \dfrac{3}{2}A_1 + 4 = 0 \end{cases} \qquad \begin{cases} B_1 = -A_1 \\ B_2 = 1 - 2A_1 + A_1/2 \\ A_2 = -A_1/2 \\ 13A_1 = -8 \end{cases}$$

$$\begin{cases} A_1 = -8/13 \\ A_2 = \left(-\frac{8}{13}\right)\cdot\frac{1}{2} = \frac{4}{13} \\ B_1 = \frac{8}{13} \\ B_2 = 1 + \frac{16}{13} - \frac{4}{13} = \frac{25}{13} \end{cases}$$

$$\frac{x^2}{(x+2)^2(x+4)^2} = -\frac{8}{13}\frac{1}{x+2} + \frac{4}{13}\frac{1}{(x+2)^2} + \frac{8}{13(x+4)} + \frac{25}{13(x+4)^2}$$

$$\int \frac{x^2}{(x+2)^2(x+4)^2}dx = -\frac{8}{13}\int\frac{dx}{x+2} + \frac{4}{13}\int\frac{dx}{(x+2)^2} + \frac{8}{13}\int\frac{dx}{x+4} + \frac{25}{13}\int\frac{dx}{(x+4)^2} =$$

$$= -\frac{8}{13}\log|x+2| - \frac{4}{13}\frac{1}{x+2} + \frac{8}{13}\log|x+4| - \frac{25}{13}\frac{1}{x+4} + c$$