

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{x^4 + x + 1} + 5x + \log(x^5 - 3x + 1)}{3x^2 + \log(2x^3 + 1) + \sqrt{x^2 + 5}}$$

$$= \lim_{x \rightarrow +\infty} \frac{x^2 \sqrt{1 + \frac{1}{x^3} + \frac{1}{x^4}} + 5x + \log(x^5 - 3x + 1)}{3x^2 + \log(2x^3 + 1) + x \sqrt{1 + \frac{5}{x^2}}}$$

$$= \lim_{x \rightarrow +\infty} \frac{x^2 \sqrt{1 + \frac{1}{x^3} + \frac{1}{x^4}} + 5x + 5 \log x + \log\left(1 - \frac{3}{x^4} + \frac{1}{x^5}\right)}{3x^2 + 3 \log x + \log\left(2 + \frac{1}{x^3}\right) + x \sqrt{1 + \frac{5}{x^2}}}$$

$$= \lim_{x \rightarrow +\infty} \frac{\sqrt{1 + \frac{1}{x^3} + \frac{1}{x^4}} + \frac{5}{x} + \frac{5 \log x}{x^2} + \frac{\log\left(1 - \frac{3}{x^4} + \frac{1}{x^5}\right)}{x^2}}{3 + \frac{3 \log x}{x^2} + \frac{\log\left(2 + \frac{1}{x^3}\right)}{x^2} + \frac{1}{x} \sqrt{1 + \frac{5}{x^2}}}$$

$$= \lim_{x \rightarrow +\infty} \frac{\sqrt{1 + \frac{1}{x^3} + \frac{1}{x^4}} + \frac{5}{x} + \frac{5 \log x}{x^2} + \frac{\log\left(1 - \frac{3}{x^4} + \frac{1}{x^5}\right)}{x^2}}{3 + \frac{3 \log x}{x^2} + \frac{\log\left(2 + \frac{1}{x^3}\right)}{x^2} + \frac{1}{x} \sqrt{1 + \frac{5}{x^2}}}$$

$$= \lim_{x \rightarrow +\infty} \frac{\sqrt{1 + \frac{1}{x^3} + \frac{1}{x^4}} + \frac{5}{x} + \frac{5 \log x}{x^2} + \frac{\log\left(1 - \frac{3}{x^4} + \frac{1}{x^5}\right)}{x^2}}{3 + \frac{3 \log x}{x^2} + \frac{\log\left(2 + \frac{1}{x^3}\right)}{x^2} + \frac{1}{x} \sqrt{1 + \frac{5}{x^2}}}$$

$$= \frac{1}{3}$$