

$$\begin{aligned}
 & \lim_{x \rightarrow +\infty} \frac{\log^2(x^4 + x + 1) - 3 \log(x^2 + 5x + 1) + 2}{\log^2(x^5 + 2x + 3) + 2 \log(x^3 + 2) + \cos^4 x} \\
 &= \frac{\left[4 \log x + \log \left(1 + \frac{1}{x^3} + \frac{1}{x^4} \right) \right]^2 - 6 \log x - 3 \log \left(1 + \frac{5}{x} + \frac{1}{x^2} \right) + 2}{\left[5 \log x + \log \left(1 + \frac{2}{x^4} + \frac{3}{x^5} \right) \right]^2 + 6 \log x + 2 \log \left(1 + \frac{2}{x^3} \right) + \cos^4 x} \\
 &= \frac{16 \log^2 x \left[1 + \frac{\log \left(1 + \frac{1}{x^3} + \frac{1}{x^4} \right)}{4 \log x} \right]^2 - 6 \log x - 3 \log \left(1 + \frac{5}{x} + \frac{1}{x^2} \right) + 2}{25 \log^2 x \left[1 + \frac{\log \left(1 + \frac{2}{x^4} + \frac{3}{x^5} \right)}{5 \log x} \right]^2 + 6 \log x + 2 \log \left(1 + \frac{2}{x^3} \right) + \cos^4 x}
 \end{aligned}$$

$$\frac{16}{25}$$