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$$\lim_{x \rightarrow -1} \frac{\sqrt{1 + \arctan^2 x} + 2 \arctan x + 1}{2 \arctan x + 3^x + 1}$$

$$y = \frac{e^x + e^{-x}}{e^x + e^{-x}} = y; \quad e^x + e^{-x} = y(e^x + e^{-x})$$
$$e^x(1-y) = 2e^{-x}$$

$$\arctan x = \frac{1}{2} \log \frac{1+x}{1-x}$$

$$x \rightarrow -1 \Rightarrow \frac{1}{2} \log \frac{1-1}{1+1} = -\infty$$

$$= \lim_{x \rightarrow -1} \frac{\left| \arctan x \right| \sqrt{\frac{1}{\arctan^2 x} + 1} + 2 \arctan x + 1}{\arctan x \left(2 + \frac{3^x}{\arctan x} + \frac{1}{\arctan x} \right)}$$

$$= \lim_{x \rightarrow -1} \frac{\sqrt{\frac{1}{\arctan^2 x} + 1} + 2 \frac{\arctan x}{\arctan x} + \frac{1}{\arctan x}}{2 + \frac{3^x}{\arctan x} + \frac{1}{\arctan x}}$$

$$= -\frac{1}{2}$$