$$\lim_{x\to+\infty} \frac{e^x+x+1}{\operatorname{arctg} x} \log \frac{e^x+2}{e^x-3} =$$

$$\begin{cases} \dfrac{e^x+2}{e^x-3} - 1 + 1 = \dfrac{e^x+2-e^x+3}{e^x-3} + 1 = \\ = \dfrac{5}{e^x-3} + 1 \end{cases}$$

$$\lim_{x\to+\infty} \frac{e^x+x+1}{\operatorname{arctg} x} \frac{5}{e^x-3} \cdot \frac{e^x-3}{5} \log\left(1+\frac{1}{\frac{e^x-3}{5}}\right) =$$

$$= \lim_{x\to+\infty} \frac{5}{\operatorname{arctg} x} \frac{e^x\left(1+\frac{x}{e^x}+\frac{1}{e^x}\right)}{e^x\left(1-\frac{3}{e^x}\right)} \lim_{y\to\infty} y \log\left(1+\frac{1}{y}\right) =$$

$$y = \frac{e^x-3}{5}$$

$$= \lim \frac{5}{\frac{\pi}{2}} \cdot 1 = \frac{10}{\pi}$$