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$$\lim_{x\to+\infty} \frac{\sqrt{x^2+1}+x-3}{3\sqrt{x}+1} \log \frac{2x+2\sqrt{x}-1}{2x+5}$$

$$\frac{2x+2\sqrt{x}-1}{2x+5} - 1 + 1 = \frac{2x+2\sqrt{x}-1-2x-5}{2x+5} + 1 =$$

$$= \frac{2\sqrt{x}-6}{2x+5} + 1$$

$$= \lim_{x\to+\infty} \frac{\sqrt{x^2+1}+x-3}{3\sqrt{x}+1} \log\left(1 + \frac{1}{\frac{2x+5}{2\sqrt{x}-6}}\right) =$$

$$= \lim_{x\to+\infty} \frac{\sqrt{x^2+1}+x-3}{3\sqrt{x}+1} \cdot \frac{2\sqrt{x}-6}{2x+5} \cdot \frac{2x+5}{2\sqrt{x}-6} \log\left(1 + \frac{1}{\frac{2x+5}{2\sqrt{x}-6}}\right)$$

$$= \lim_{x\to+\infty} \frac{2\sqrt{x^3+x} + 2\sqrt{x^3} - 6\sqrt{x} - 6\sqrt{x^2+1} + 6x + 18}{6\sqrt{x^3} + 2x + 15\sqrt{x} + 5} =$$

$$= \lim_{x\to+\infty} \frac{2x^{\frac{3}{2}}\sqrt{1+\frac{1}{x^3}} + 2x^{\frac{3}{2}} - 6x^{\frac{1}{2}} - 6x\sqrt{1+\frac{1}{x^2}} - 6x + 18}{6x^{\frac{3}{2}} + 2x + 15x^{\frac{1}{2}} + 5} =$$

$$= \lim_{x \to +\infty} \frac{x^{\frac{3}{2}} \left(2\sqrt{1+\frac{1}{x^3}} + 2 - \frac{6}{x} - \frac{6}{x^{\frac{1}{2}}}\sqrt{1+\frac{1}{x^2}} - \frac{6}{x^{\frac{1}{2}}} + \frac{18}{x^{\frac{3}{2}}} \right)}{x^{\frac{3}{2}} \left(6 + \frac{2}{x^{\frac{1}{2}}} + \frac{15}{x} + \frac{5}{x^{\frac{3}{2}}} \right)} =$$

$$= \frac{2+2}{6} = \frac{2}{3}$$