137

$$\lim_{x\to+\infty} \frac{x\log x+1}{2x+3} \log \frac{x\log x+x}{x\log x-3}$$

$$\frac{x\log x+x}{x\log x-3} - 1 + 1 = \frac{x\log x + x - x\log x + 3}{x\log x - 3} + 1 =$$

$$= \frac{x+3}{x\log x-3} + 1 = \frac{1+3/x}{\log x - \frac{3}{x}} + 1$$

$$\lim_{x\to+\infty} \frac{x\log x+1}{2x+3} \log\left(1 + \frac{1}{\frac{\log x - \frac{3}{x}}{1+\frac{3}{x}}}\right)$$

$$= \lim_{x\to+\infty} \frac{x\log x+1}{2x+3} \frac{1+\frac{3}{x}}{\log x - \frac{3}{x}} \cdot \frac{\log x - \frac{3}{x}}{1+\frac{3}{x}} \log\left(1 + \frac{1}{\frac{\log x - \frac{3}{x}}{1+\frac{3}{x}}}\right)$$

$$= \lim_{x\to+\infty} \frac{x\log x + 1 + 3\log x + \frac{3}{x}}{2x\log x - 3\log x + 3\log x - \frac{9}{x}} \cdot 1 =$$

$$= \lim_{x\to+\infty} \log x + \frac{1}{x} +$$

$$= \lim_{x \to +\infty} \frac{1 + \frac{1}{x \log x} + \frac{3}{x} + \frac{3}{x^2 \log x}}{2 - \frac{3}{x \log x} + \frac{3}{x} - \frac{9}{\log x}} =$$

$$= \frac{1}{2}$$