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$$\lim_{x \to \frac{1}{6}} \frac{3x+1}{1-\cos(6x-1)} \log^2(2 \operatorname{sen} \pi x) =$$

- $y = 6x - 1$; $x = \frac{y+1}{6}$; $x \to \frac{1}{6} \Rightarrow y \to 0$

$$= \lim_{y \to 0} \frac{3\frac{y+1}{6} + 1}{1 - \cos y}$$

$y = 6x - 1$; $x = \frac{y+1}{6}$; $x \to \frac{1}{6} \Rightarrow y \to 0$

$$\lim_{y \to 0} \frac{\frac{y+1}{2} + 1}{1 - \cos y} \log^2\left[2 \operatorname{sen}\left(\frac{\pi}{6} y + \frac{\pi}{6}\right)\right] =$$

$$= \lim_{y \to 0} \frac{1}{2} \frac{y+3}{1-\cos y} \log^2\left[2 \operatorname{sen}\frac{\pi}{6} y \cos\frac{\pi}{6} + 2\cos\frac{\pi}{6} y \operatorname{sen}\frac{\pi}{6}\right] =$$

$$= \lim_{y \to 0} \frac{1}{2} \frac{y+3}{1-\cos y} \log^2\left[\sqrt{3} \operatorname{sen}\frac{\pi}{6} y + \cos\frac{\pi}{6} y\right] =$$

$$= \lim_{y \to 0} \frac{1}{2} \frac{(y+3)}{y^2} \frac{y^2}{1-\cos y} \log^2\left[\sqrt{3} \operatorname{sen}\frac{\pi}{6} y\right.$$

1. QSB - Programmazione lineare 1 disciotto 1.44 MB MATEMATICA (DOS)
~~2 - 4/5 spot driving 2 dischetti: 5 1/4~~ ~~gibco (DOS)~~

$$= \lim_{x \to \frac{1}{6}} \frac{3x+1}{1-\cos(6x-1)} \log^2\left[1 + (2\sin \pi x - 1)\right] =$$

$$= \lim_{x \to \frac{1}{6}} \frac{3x+1}{(6x-1)^2} \cdot \frac{(6x-1)^2}{1-\cos(6x-1)} \cdot (2\sin \pi x - 1)^2 \frac{\log^2\left[1 + (2\sin \pi x - 1)\right]}{(2\sin \pi x - 1)^2}$$

$\to 1$

$y = 6x - 1 \quad ; \quad x = \frac{y+1}{6}$

$$= \lim_{y \to 0} \frac{\frac{y+1}{2} + 1}{y^2} \cdot \frac{y^2}{1-\cos y} \left[2\sin\left(\frac{\pi}{6}y + \frac{\pi}{6}\right) - 1\right]^2 \cdot \lim_{x \to \frac{1}{6}} (2\sin \pi x - 1)^2 =$$

$$= \lim_{y \to 0} \frac{1}{2} \frac{y+3}{y^2} \cdot \frac{y^2}{1-\cos y} \left[2\sin\frac{\pi}{6}y\cos\frac{\pi}{6} + 2\cos\frac{\pi}{6}y\sin\frac{\pi}{6} - 1\right]^2 =$$

$$= \lim_{y \to 0} \frac{1}{2} \frac{y+3}{y^2} \frac{y^2}{1-\cos y} \left[\sqrt{3}\sin\frac{\pi}{6}y + \cos\frac{\pi}{6}y - 1\right]^2 =$$

$\to \frac{1}{2}$

$$= \lim_{y\to 0} \frac{y+3}{y^2}\sqrt{3}\,\operatorname{sen}\frac{\pi}{6}y + \left(\cos\frac{\pi}{6}y - 1\right)\frac{y+3}{y^2}$$

$$= \lim_{y\to 0} \frac{\pi}{6}\frac{y^2+3y}{y^2}\sqrt{3}\,\frac{\operatorname{sen}\frac{\pi}{6}y}{\frac{\pi}{6}y} - \left(1-\cos\frac{\pi}{6}y\right)\frac{y+3}{y^2} =$$

$$= \lim_{y\to 0} \frac{\pi}{6}\frac{y^2+3y}{y^2}\sqrt{3}\,\frac{\operatorname{sen}\frac{\pi}{6}y}{\frac{\pi}{6}y} - 2\operatorname{sen}^2\frac{\pi}{12}y\,\frac{y+3}{y^2} =$$

$$= \lim_{y\to 0} \frac{\pi}{6}\frac{y^2+3y}{y^2}\sqrt{3}\,\frac{\operatorname{sen}\frac{\pi}{6}y}{\frac{\pi}{6}y} - 2\,\frac{\operatorname{sen}^2\frac{\pi}{12}y}{\frac{\pi^2}{12^2}y^2}\cdot$$

$$= \lim_{y\to 0} \frac{1}{2}\frac{y+3}{y^2}\frac{y^2}{1-\cos y}\Big[\lim_{x\to\frac{1}{6}}\left(4\operatorname{sen}^2\frac{\pi}{6}x + 1 - 4\operatorname{sen}\pi x\right) \cdot \sqrt{3}\operatorname{sen}\frac{\pi}{6}y\cdot$$

$$= \lim_{y\to 0}\frac{y+3}{y^2}\lim_{x\to\frac{1}{6}}\left(4\operatorname{sen}^2\pi x + 1 - 4\operatorname{sen}\pi x\right) =$$

$$= \lim_{y\to 0}\frac{y+3}{y^2}\left(4\operatorname{sen}^2\left(\frac{\pi}{6}y+\frac{\pi}{6}\right) + 1 - 4\operatorname{sen}\left(\frac{\pi}{6}y+\frac{\pi}{6}\right)\right)$$

$$= \lim_{y\to 0} \frac{y+3}{y^2}\left[4\left(\frac{\sqrt{3}}{2}\operatorname{sen}\frac{\pi}{6}y+\frac{1}{2}\cos\frac{\pi}{6}y\right)^2+1-4\left(\frac{\sqrt{3}}{2}\operatorname{sen}\frac{\pi}{6}y+\frac{1}{2}\cos\frac{\pi}{6}y\right)\right] =$$

$$= \lim_{y\to 0} \frac{y+3}{y^2}\left[3\operatorname{sen}^2\frac{\pi}{6}y+\cos^2\frac{\pi}{6}y+\sqrt{3}\operatorname{sen}\frac{\pi}{6}y\cos\frac{\pi}{6}y+1-2\sqrt{3}\operatorname{sen}\frac{\pi}{6}y-2\cos\frac{\pi}{6}y\right]$$

$$= \lim_{y\to 0} \frac{y+3}{y^2}\left[2\operatorname{sen}^2\frac{\pi}{6}y+2+\sqrt{3}\operatorname{sen}\frac{\pi}{6}y\cos\frac{\pi}{6}y-2\sqrt{3}\operatorname{sen}\frac{\pi}{6}y-2\cos\frac{\pi}{6}y\right]$$

$$= \lim_{y\to 0} \frac{y+3}{y^2}\left[2\operatorname{sen}^2\frac{\pi}{6}y+2\left(1-\cos\frac{\pi}{6}y\right)+\sqrt{3}\operatorname{sen}\frac{\pi}{6}y\cos\frac{\pi}{6}y-2\sqrt{3}\operatorname{sen}\frac{\pi}{6}y\right]$$

$$= \lim_{y\to 0} \frac{y+3}{y^2}\left[2\operatorname{sen}^2\frac{\pi}{6}y+4\operatorname{sen}^2\frac{\pi}{12}y+\sqrt{3}\operatorname{sen}\frac{\pi}{6}y\cos\frac{\pi}{6}y-2\sqrt{3}\operatorname{sen}\frac{\pi}{6}y\right]$$

$$= \lim_{y\to 0} \frac{y^2+3y}{y^2}\left[\frac{\frac{\pi}{6}\operatorname{sen}^2\frac{\pi}{6}y}{\frac{\pi}{6}y}\cdot\frac{\operatorname{sen}\frac{\pi}{6}y}{\frac{\pi}{6}y}+\frac{\pi\sqrt{3}}{6}\cdot\frac{\operatorname{sen}\frac{\pi}{6}y\cos\frac{\pi}{6}y}{\frac{\pi}{6}y}-\frac{\pi 2\sqrt{3}}{6}\cdot\frac{\operatorname{sen}\frac{\pi}{6}y}{\frac{\pi}{6}y}\right] =$$

$$= 1\left[0+\frac{\pi}{6}\sqrt{3}-\frac{\pi}{6}2\sqrt{3}\right] = -\frac{\pi}{6}\sqrt{3}$$