

$$\lim_{x \rightarrow +\infty} x^2 \left( \sqrt{\frac{x^2+1}{2x^2+3}} - \sqrt{\frac{x^3+5}{2x^3-1}} \right)$$

$$= \lim_{x \rightarrow +\infty} x^2 \sqrt{\frac{x^3+5}{2x^3-1}} \left( \sqrt{\frac{(x^2+1)(2x^3-1)}{(2x^2+3)(x^3+5)}} - 1 \right)$$

$$= \frac{1}{\sqrt{2}} \lim_{x \rightarrow +\infty} x^2 \left( \sqrt{\frac{2x^5 + 2x^3 - x^2 - 1}{2x^5 + 3x^3 + 10x^2 + 15}} - 1 \right)$$

$$= \frac{1}{\sqrt{2}} \lim_{x \rightarrow +\infty} x^2 \left( \sqrt{1 + \frac{-x^3 - 11x^2 - 16}{2x^5 + 3x^3 + 10x^2 + 15}} - 1 \right)$$

$$= \frac{1}{\sqrt{2}} \lim_{x \rightarrow +\infty} \frac{x^2(x^3 + 11x^2 + 16)}{2x^5 + 3x^3 + 10x^2 + 15} \left( \sqrt{1 - \frac{x^3 + 11x^2 + 16}{2x^5 + 3x^3 + 10x^2 + 15}} - 1 \right)$$

$$= -\frac{1}{\sqrt{2}} \cdot \frac{1}{2} \cdot \frac{1}{2} = -\frac{1}{4\sqrt{2}}$$