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$$\lim_{x \rightarrow \infty} \sqrt[4]{x^4 + 5x^3 + 2x + 1} - x$$

$$\lim_{y \rightarrow +\infty} \sqrt[4]{y^4 - 5y^3 - 2y + 1} - y$$

$$a^4 - b^4 = (a^2 + b^2)(a + b)(a - b)$$

$$\lim_{y \rightarrow +\infty} \sqrt[4]{y^4 - 5y^3 - 2y + 1} - y = \frac{\lim_{y \rightarrow +\infty} \sqrt[4]{y^4 - 5y^3 - 2y + 1} - y^4}{(\sqrt[4]{y^4 - 5y^3 - 2y + 1} + y^2)(\sqrt[4]{y^4 - 5y^3 - 2y + 1} + y)}$$

$$= \lim_{y \rightarrow +\infty} \frac{y^3 \left(-5 - \frac{2}{y^2} + \frac{1}{y^3} \right)}{y^2 \left(\sqrt{1 - \frac{5}{y} - \frac{2}{y^3} + \frac{1}{y^4}} + 1 \right) y \left(\sqrt[4]{1 - \frac{5}{y} - \frac{2}{y^3} + \frac{1}{y^4}} + 1 \right)}$$

$$= \frac{-5}{2 \cdot 2} = -\frac{5}{4}$$