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$$\lim_{x \rightarrow +\infty} \sqrt{x^2 + 3x + 5} - \sqrt{x^2 + x + 1} =$$

$$= \lim_{x \rightarrow +\infty} \frac{\cancel{x^2} + 3x + 5 - \cancel{x^2} - x - 1}{\sqrt{x^2 + 3x + 5} + \sqrt{x^2 + x + 1}} = \frac{2x + 4}{\sqrt{x^2 + 3x + 5} + \sqrt{x^2 + x + 1}} = 1$$

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$$\lim_{x \rightarrow +\infty} \sqrt{x^2 + x + 2} \log\left(1 + \cos \frac{1}{\sqrt{x}}\right) - x \log 2 =$$

$$= \lim_{x \rightarrow +\infty} \sqrt{x^2 + x + 2} \log\left(1 + \cos \frac{1}{\sqrt{x}}\right) - x \log 2 + \sqrt{x^2 + x + 2} \log 2 - \sqrt{x^2 + x + 2} \log 2 =$$

$$= \lim_{x \rightarrow +\infty} \log 2 \left(\sqrt{x^2 + x + 2} - x \right) + \sqrt{x^2 + x + 2} \log \frac{1 + \cos \frac{1}{\sqrt{x}}}{2} =$$

$$= \lim_{x \rightarrow +\infty} \log 2 \cdot \frac{x + 2}{\sqrt{x^2 + x + 2} + x} + \sqrt{x^2 + x + 2} \log \frac{1 + \cos \frac{1}{\sqrt{x}}}{2} =$$

$$= \frac{1}{2} \log 2 + \lim_{x \rightarrow +\infty} \sqrt{x^2 + x + 2} \log \left(1 - \sec^2 \frac{1}{2\sqrt{x}} \right)$$

$$= \frac{1}{2} \log 2 + \lim_{x \rightarrow +\infty} \sqrt{x^2 + x + 2} \left(-\sec^2 \frac{1}{2\sqrt{x}} \right) \log \left(1 - \sec^2 \frac{1}{2\sqrt{x}} \right)$$

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$$= \frac{1}{2} \log 2 + \lim_{x \rightarrow +\infty} \frac{\sec^2 \frac{1}{2\sqrt{x}} - \frac{1}{4x} \sqrt{x^2 + x + 2}}{\frac{1}{4x}} = \frac{1}{2} \log 2 + \frac{1}{2} = \frac{1}{2} \log 2 + \frac{1}{2}$$

$$= \frac{1}{2} \log 2 + \frac{1}{2}$$

