237

$$\lim_{x\to 0^-} \frac{3^{\frac{1}{x}}}{\operatorname{arctg}^3 x} = \frac{0}{0}$$

$$= \cancel{\lim_{x\to 0^-} (1+3^{\frac{1}{x}})-1}$$

$$= \lim_{x\to 0^-} \frac{x^3}{\operatorname{arctg}^3 x} \, \frac{3^{\frac{1}{x}}}{x^3} =$$

!

$$= \lim_{y\to -\infty} y^3 3^y = \lim_{y\to -\infty} \frac{y^3}{3^{-y}} =$$

$$= \lim_{t\to +\infty} -\frac{t^3}{3^t} = 0$$

$$\lim_{x\to \frac{2}{\pi}} \frac{\log|\ldots}{\log|\ldots}$$

$$y = x - \frac{2}{\pi}$$

$$\lim_{y\to 0} \frac{\log\ldots}{\log\ldots}$$

$$y = \pi - \frac{2}{x}$$

$$\lim_{y\to 0} \frac{\log\ldots}{\ldots}$$

$$= \lim_{y\to 0} \frac{\log|\ldots}{\ldots}$$

1- Albinoni: Adagio. Pachelbel. Canon. Bach. Aria - Vivaldi - La notte - Mozart - Serenata Notturna - Gluck: Dance of the blessed Spirits Herbert von Karajan - Berliner Philharmoniker audiocassette

2- Alice e le altre - Parole d'autore n. 1. L'Unità audiocassette.

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$$\lim_{x \to 0} \frac{1 - 3^{x^2}\cos x}{x^2} =$$

$$= \lim_{x \to 0} \frac{1 - 3^{x^2}\cos x + 3^{x^2} - 3^{x^2}}{x^2} =$$

$$= \lim_{x \to 0} 3^{x^2}\frac{(1-\cos x)}{x^2} - \frac{3^{x^2} - 1}{x^2} =$$

$$= 3^0 \cdot \frac{1}{2} - \log 3 = \frac{1}{2} - \log 3$$

188

$$\lim_{x\to+\infty} \sqrt{x^4+2x^2+x+3}\ \text{sen}\frac{\pi(x+1)}{2x+1} - x^2 =$$

$$= \lim_{x\to+\infty} \sqrt{x^4+2x^2+x+3}\cdot\left(1+\text{sen}\frac{\pi(x+1)}{2x+1}-1\right) -$$

$$= \lim_{x\to+\infty} \sqrt{x^4+2x^2+x+3}\ \frac{\left[1+\left(\text{sen}\frac{\pi(x+1)}{2x+1}-1\right)\right]-1}{\underbrace{\text{sen}\frac{\pi(x+1)}{2x+1}-1}_{\downarrow 1}}$$

$$= \lim_{x\to+\infty} \sqrt{x^4+2x^2+x+3}\left[\cos\left(\frac{\pi}{2}-\pi\frac{x+1}{2x+1}\right)-1\right]$$

$$= 1+\lim_{x\to+\infty} \sqrt{x^4+2x^2+x+3}\cdot 2\,\text{sen}^2\frac{\left(\frac{\pi}{2}-\pi\frac{x+1}{2x+1}\right)}{2}$$

$$= 1+\lim_{x\to+\infty} \sqrt{x^4+2x^2+x+3}\cdot 2\,\text{sen}^2\left(\frac{-\pi}{4x+2}\right)$$

$$\sqrt{x^4+2x^2+x+3} + \sqrt{x^4+2x^2+x+3} - x^2$$

$$\left(\text{sen}\frac{\pi(x+1)}{2x+1}-1\right) + \frac{\cancel{x^4}+2x^2+x+3-\cancel{x^4}}{\sqrt{x^4+2x^2+x+3}+x^2}$$

$$+1 =$$

$$= 1+\lim_{x\to+\infty} \sqrt{x^4+2x^2+x+3}\cdot 2\,\text{sen}^2\frac{2x\pi+\pi-2\pi x-2\pi}{4x+2}$$

$$= 1+\lim_{x\to+\infty} \frac{\pi^2\sqrt{x^4+2x^2+x+3}}{(4x+2)^2}\cdot 2\,\frac{\text{sen}^2\left(\frac{-\pi}{4x+2}\right)}{\frac{\pi^2}{(4x+2)^2}}$$

$$= 1 + 2\pi^2 \lim_{x \to +\infty} \sqrt{\frac{x^4 + 7x^2 + x + 3}{(16x^2 + 4 + 16x)^2}} =$$

$$= 1 + \frac{\pi}{8}^2$$

(189)

$$\lim_{x \to +\infty} \log\left(e^x + x^4 + 3x^2 + \operatorname{arctg} x\right) - x =$$

$$= \lim_{x \to +\infty} \log\left[e^x\left(1 + \frac{x^4}{e^x} + \frac{3x^2}{e^x} + \frac{\operatorname{arctg} x}{e^x}\right)\right] - x =$$

$$= \lim_{x \to +\infty} \cancel{x} + \log\left[1 + \frac{x^4}{e^x} + \frac{3x^2}{e^x} + \frac{\operatorname{arctg} x}{e^x}\right] - \cancel{x} =$$

$$= 0$$

190

$$\lim_{x \to +\infty} e^{\frac{3x^2+x+1}{x^3}} \sqrt[4]{x^4+x^3+5} - x =$$

$$= \lim_{x \to +\infty} e^{\frac{3x^2+x+1}{x^3}} \sqrt[4]{x^4+x^3+5} - \sqrt[4]{x^4+x^3+5} + \sqrt[4]{x^4+x^3+5}$$

$$= \lim_{x \to +\infty} \sqrt[4]{x^4+x^3+5}\left(e^{\frac{3x^2+x+1}{x^3}} - 1\right) + \sqrt[4]{x^4+x^3+5} - x$$

$$= \lim_{x \to +\infty} \sqrt[4]{x^4+x^3+5} \underbrace{\frac{e^{\frac{3x^2+x+1}{x^3}} - 1}{\frac{3x^2+x+1}{x^3}}}_{\to 1} \cdot \frac{3x^2+x+1}{x^3}$$

$$+ \lim_{x \to +\infty} \frac{\cancel{x^4}+x^3+5-\cancel{x^4}}{\left(\sqrt{x^4+x^3+5}+x^2\right)\left(\sqrt[4]{x^4+x^3+5}+x\right)} =$$

$$= \frac{1}{4} + \lim_{x \to +\infty} \sqrt[4]{\frac{(3x^2+x+1)^4}{x^{12}} \cdot (x^4+x^3+5)} =$$

$$= \frac{1}{4} + 3 = \frac{13}{4}$$

(181)

$$\lim_{x\to+\infty} \operatorname{senh} x - \log\left(\sqrt{x} + 3e^{\operatorname{senh} x} + 1\right) =$$

$$= \lim_{x\to+\infty} \frac{e^{x}+e^{-x}}{2} - \log\left[e^{\operatorname{senh} x}\left(3 + \frac{1}{\operatorname{senh} x} + \frac{\sqrt{x}}{\operatorname{senh} x}\right)\right] =$$

$$= \lim_{x\to+\infty} \operatorname{senh} x - \operatorname{senh} x - \log\left[3 + \frac{1}{\operatorname{senh} x} + \frac{\sqrt{3}}{\operatorname{senh} x}\right]$$

$$= -\log 3$$

$$\lim_{x\to 0} (\operatorname{sen}$$

$$= \lim_{x\to 0} e^{(1-}$$

$$= \lim_{x\to 0} e^{(1-c}$$

$$= \lim_{x\to 0} e^{(1-c}$$

$$= \lim_{x\to 0} e^{1}$$

$$= e^{\frac{1}{2}(c}$$

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$$\lim_{x\to 0} (\operatorname{sen} 5x)^{1-\cos x} =$$

$$= \lim_{x\to 0} e^{(1-\cos x)\log \operatorname{sen} 5x} =$$

$$= \lim_{x\to 0} e^{(1-\cos x)\log \frac{\operatorname{sen} 5x}{5x}\cdot 5x} =$$

$$= \lim_{x\to 0} e^{(1-\cos x)\left(\log \frac{\operatorname{sen} 5x}{5x} + \log 5x\right)}$$

$$= \lim_{x\to 0} e^{\frac{1-\cos x}{x^2}\left(x^2 \log \frac{\operatorname{sen} 5x}{5x} + x^2 \log 5x\right)} =$$

$$= e^{\frac{1}{2}(0+0)} = e^{0} = 1$$

$$\lim_{x\to 0}\left(\operatorname{tg}\sqrt[3]{x}\right)^{\operatorname{sen} 2x} =$$

$$= \lim_{x\to 0} e^{\operatorname{sen} 2x \log \operatorname{tg}\sqrt[3]{x}} =$$

$$= \lim_{x\to 0} e^{\operatorname{sen} 2x \cdot \operatorname{tg}\sqrt[3]{x} \frac{\log \operatorname{tg}\sqrt[3]{x}}{\operatorname{tg}\sqrt[3]{x}}} =$$

$$= \lim_{x\to 0} e^{\frac{\operatorname{sen} 2x}{\cancel{2x}} \cancel{2x} \operatorname{tg}\sqrt[3]{x} \log \frac{\operatorname{tg}\sqrt[3]{x}}{\operatorname{tg}\sqrt[3]{x}}} =$$

$$= e$$

$$= \lim_{x\to 0} e^{2\frac{\operatorname{sen} 2x}{2x} \cdot \frac{\sqrt[3]{x}}{\operatorname{tg}\sqrt[3]{x}} \cdot \frac{x}{\sqrt[3]{x}}\left(\operatorname{tg}\sqrt[3]{x} \log \operatorname{tg}\sqrt[3]{x}\right)}$$

$$= e^{2\cdot 1\cdot 1\cdot 0\cdot 0} = e^{0} = 1$$

1- Ciaikovski - Lo sch
bambini del teatro
/ Tartini - Concerto in
D 113 - Concerto in
archi e continuo.
de C. Scimone.
2- Carreras - Domingo -
3- Ciaikovski - Sinfo
filarmonica di Oslo -
4- Ciaikovski - Il
Orchestra - dir. Sei
8- Chopin - Sonat
Vladimir Ashke
10- Christmas in V
J. Carreras
12- Caro amico ti
13- Ciajkovskij. Sim
"Patetica". Chica
Claudio Abbado
14- Ciajkovsky - Sim
Oslo Philharmoni
a violoncello

15- Una città per cantare - Parole d'autore 3 - L'Unità -
16- Ciaikovski - Lo schiaccianoci - Il lago dei cigni - London Festival Orchestra - dir Alberto Lizzio - La bella addormentata New Philharmonic Orchestra - dir Alfred Scholz
17- Carboni L. - Inno nazionale - CD

194

$$\lim_{x\to 0}(1-\cos 3x)^{\operatorname{sen} x} =$$

$$= \lim_{x\to 0} e^{\operatorname{sen} x \log(1-\cos 3x)} =$$

$$= \lim_{x\to 0} e^{\operatorname{sen} x\,(1-\cos 3x)\frac{\log(1-\cos 3x)}{1-\cos 3x}} =$$

$$= \cancel{e^{?}} \lim_{x\to 0} e^{\frac{\operatorname{sen} x}{1-\cos 3x}\left((1-\cos 3x)\log(1-\cos 3x)\right)}$$

$$= \lim_{x\to 0} e^{\frac{\operatorname{sen} x}{x}\frac{3x}{1-\cos 3x}\frac{1}{3}\left((1-\cos 3x)\log(1-\cos 3x)\right)} = e^{1\cdot 0\cdot\frac{1}{3}\,0} = 1$$

195

$$\lim_{x\to 0}\left[\log(1+5x)\right]^{\operatorname{arctg} 3x} =$$

$$= \lim_{x\to 0} e^{\operatorname{arctg} 3x \,\log\log(1+5x)} =$$

$$= \lim_{x\to 0} e^{\operatorname{arctg} 3x \,\log(1+5x)\,\frac{\log\log(1+5x)}{\log(1+5x)}} =$$

$$\cancel{= e^{0\cdot 0\cdot 0} = 1}$$

$$= \lim_{x\to 0} e^{\frac{\operatorname{arctg} 3x}{3x}\,\frac{5x}{\log(1+5x)}\cdot\frac{5}{3}\,\log(1+5x)\log\log(1+5x)}$$

$$= e^{1\cdot 1\cdot\frac{5}{3}\cdot 0} = e^{0} = 1$$

e96

$$\lim_{x\to 0}\left(3^{5x}-1\right)^{\operatorname{senh} x} =$$

$$= \lim_{x\to 0} e^{\operatorname{senh} x \log\left(3^{5x}-1\right)} =$$

$$= \lim_{x\to 0} e^{\operatorname{senh} x \frac{\log\left(3^{5x}-1\right)}{3^{5x}-1}\left(3^{5x}-1\right)} =$$

$$= \cancel{e^{0\cdot 0\cdot 0} = 1}$$

$$\cancel{\operatorname{senh} x \cdot \frac{3^{5x}-1}{5}}$$

$$= \lim_{x\to 0} e^{\frac{1}{5}\frac{\operatorname{senh} x}{x}\frac{x}{3^{5x}-1}\left(3^{5x}-1\right)\log\left(3^{5x}-1\right)} =$$

$$= \lim_{x\to 0} e^{\frac{1}{5}\cdot 1\cdot \log 3\cdot 0} = e^{0} = 1$$

$$\lim_{x\to 0}\left(\operatorname{senh} tg \ldots\right.$$

$$= \lim_{x\to 0} e \ldots$$

$$= \lim_{x\to 0} e^{tg \ldots}$$

$$= \cancel{e \ldots = 1}$$

$$= \lim_{x\to 0} e^{tg \ldots}$$

$$= e^{5\cdot 1 \ldots}$$

107

$$\lim_{x\to 0} (\operatorname{sett\,tgh} x)^{tg\, 5x} =$$

$$= \lim_{x\to 0} e^{tg\, 5x \log \operatorname{sett\,tgh} x}$$

$$= \lim_{x\to 0} e^{tg\, 5x \cdot \operatorname{sett\,tgh} x \frac{\log \operatorname{sett\,tgh} x}{\operatorname{sett\,tgh} x}} =$$

$$= \lim_{x\to 0} e^{\frac{tg\, 5x}{5x} \cdot \frac{5x}{\operatorname{sett\,tgh} x} (\operatorname{sett\,tgh} x \log \operatorname{sett\,tgh} x)} =$$

$$= e^{1.1.0} = e^{0} = 1$$

$$\lim_{x\to+\infty}\left(\frac{x-1}{3x^2+5}\right)^{\frac{x^2+5x+4}{x^3+2}} =$$

$$= \lim_{x\to+\infty} e^{\frac{x^2+5x+4}{3x^3+2}\log\frac{x-1}{3x^2+5}} =$$

$$= \lim_{x\to+\infty} e^{\frac{x^2+5x+4}{3x^3+2}\cdot\frac{x-1}{3x^2+5}\,\frac{\log\frac{x-1}{3x^2+5}}{\frac{x-1}{3x^2+5}}} =$$

$$\cancel{= e^0 = 1}$$

$$= \lim_{x\to+\infty} e^{\frac{x^2+5x+4}{3x^3+2}\cdot\frac{3x^2+5}{x-1}\left(\frac{x-1}{3x^2+5}\log\frac{x-1}{3x^2+5}\right)} =$$

$$= e^{\frac{3}{3}\cdot 0} = e^0 = 1$$

1- Dvořák - Sinf

Smetana - La M

2- De Falla - El

Romande - Ernest

de España - Lo

Rafael Frühbec

populares españ

4 Dukas - L'Ap

Debussy, La

(1) Nouvel Orchestr

(2) Basler Sinf

6-

(199)

$$\lim_{x \to +\infty} \left(\operatorname{sen} \frac{2}{x} \right)^{\pi - 2 \operatorname{arctg} x} =$$

$$= \lim_{x \to +\infty} e^{(\pi - 2 \operatorname{arctg} x) \log \operatorname{sen} \frac{2}{x}} =$$

$$= \lim_{x \to +\infty} e^{(\pi - 2 \operatorname{arctg} x) \operatorname{sen} \frac{2}{x} \frac{\log \operatorname{sen} \frac{2}{x}}{\operatorname{sen} \frac{2}{x}}} =$$

$$= \lim_{x \to +\infty} e^{\frac{\pi - 2 \operatorname{arctg} x}{\operatorname{sen} \frac{2}{x}} \cdot \left(\operatorname{sen} \frac{2}{x} \log \operatorname{sen} \frac{2}{x} \right)} =$$

$$= \lim_{x \to +\infty} e^{\frac{(\pi - 2 \operatorname{arctg} x)}{\frac{2}{x}} \frac{\frac{2}{x}}{\operatorname{sen} \frac{2}{x}} \left(\operatorname{sen} \frac{2}{x} \log \operatorname{sen} \frac{2}{x} \right)}$$

$$= e^{-1 \cdot 1 \cdot 0} = e^{0} = 1$$

verificato Lazzeri

$$\frac{\pi - 2\,\mathrm{arctg}\,x}{\frac{2}{x}}$$

$$y = \pi - 2\,\mathrm{arctg}\,x$$

$$x = \mathrm{tg}\,\frac{\pi + y}{2} = \frac{\mathrm{sen}\left(\frac{\pi}{2} + \frac{y}{2}\right)}{\cos\left(\frac{y}{2} + \frac{\pi}{2}\right)} = -\frac{1}{\mathrm{tg}\,\frac{y}{2}}$$

$$= \frac{y}{-\frac{2}{\frac{1}{\mathrm{tg}\,\frac{y}{2}}}} = \frac{y}{-2\cdot\mathrm{tg}\,\frac{y}{2}} = \frac{1}{\frac{-\cancel{2}\,\mathrm{tg}\,\frac{y}{2}}{\cancel{2}\,\frac{y}{2}}}$$

$$\lim_{\cancel{x} \to +\infty} \frac{\pi - 2\,\mathrm{arctg}\,x}{\frac{2}{x}} = \lim_{y \to 0} \frac{1}{-\frac{\mathrm{tg}\,\frac{y}{2}}{\frac{y}{2}}} = -1$$

200

$$\lim_{x\to+\infty}\left(\operatorname{sen} e^{\frac{1-3x^3}{x^2+5}}\right)^{\frac{x-2}{4x^2+1}} =$$

$$= \lim_{x\to+\infty} e^{\frac{x-2}{4x^2+1}\log \operatorname{sen} e^{\frac{1-3x^3}{x^2+5}}} =$$

$$= \lim_{x\to+\infty} \cancel{e^{\frac{x-2}{4x^2+1}\cdot\frac{1}{\operatorname{sen} e^{\frac{1-3x^3}{x^2+5}}}\cdot \operatorname{sen} e^{\frac{1-3x^3}{x^2+5}} \log \operatorname{sen} e^{\frac{1-3x^3}{x^2+5}}}} =$$

$$= \lim_{x\to+\infty} e^{\frac{x-2}{4x^2+1}\log\sqrt{\frac{1-\cos 2e^{\frac{1-3x^3}{x^2+5}}}{2}}} = \lim_{x\to+\infty} e^{\frac{x-2}{4x^2+1}\cdot\frac{1}{2}\log\frac{1-\cos 2e^{\frac{1-3x^3}{x^2+5}}}{2}} =$$

$$= \lim_{x\to+\infty} e^{\frac{x-2}{4x^2+1}\cdot\frac{1}{2}\log\left(\frac{1-\cos 2e^{\frac{1-3x^3}{x^2+5}}}{2\cdot 4e^{\left(\frac{2-6x^3}{x^2+5}\right)}}\cdot 4e^{\left(\frac{2-6x^3}{x^2+5}\right)}\right)} = \lim_{x\to+\infty} e^{\frac{x-2}{4x^2+1}\cdot\frac{1}{2}\left[\log\frac{1-\cos 2e^{\frac{1-3x^3}{x^2+5}}}{2\cdot 4e^{\left(\frac{2-6x^3}{x^2+5}\right)}} + \frac{2-6x^3}{x^2+5} + \log\ldots\right]}$$

$$= \lim_{x\to+\infty} e^{\frac{x-2}{4x^2+1}\cdot\frac{1}{2}\log\frac{1-\cos 2e^{\frac{1-3x^3}{x^2+5}}}{2\cdot 4e^{\frac{2-6x^3}{x^2+5}}}}\; e^{\frac{x-2}{4x^2+1}\cdot\frac{1}{2}\cdot\frac{2(1-3x^3)}{x^2+5}}\; e^{\frac{x-2}{4x^2+1}\frac{1}{2}\log 4} =$$

$$= e^{0\cdot\frac{1}{2}\log\frac{1}{2}\cdot\frac{1}{2}}\; e^{-\frac{3}{4}}\; e^{0\frac{1}{2}\log 4} = e^{0}\, e^{-\frac{3}{4}}\, e^{0} = e^{-\frac{3}{4}}$$

200

$$\lim_{x\to+\infty}\left(\operatorname{sen} e^{\frac{1-3x^3}{x^2+5}}\right)^{\frac{x-2}{4x^2+1}} = \lim_{x\to+\infty} e^{\frac{x-2}{4x^2+1}\log \operatorname{sen} e^{\frac{1-3x^3}{x^2+5}}} =$$

$$\cancel{\lim_{x\to+\infty} e^{\frac{x-2}{4x^2+1}\cdot\frac{1}{\operatorname{sen} e^{\frac{1-3x^3}{x^2+5}}}\cdot \operatorname{sen} e^{\frac{1-3x^3}{x^2+5}} \log \operatorname{sen} e^{\frac{1-3x^3}{x^2+5}}}} =$$

$$\lim_{x\to+\infty} e^{\frac{x-2}{4x^2+1}\log\sqrt{\frac{1-\cos 2e^{\frac{1-3x^3}{x^2+5}}}{2}}} = \lim_{x\to+\infty} e^{\frac{x-2}{4x^2+1}\cdot\frac{1}{2}\log\frac{1-\cos 2e^{\frac{1-3x^3}{x^2+5}}}{2}} =$$

$$\lim_{x\to+\infty} e^{\frac{x-2}{4x^2+1}\cdot\frac{1}{2}\log\left(\frac{1-\cos 2e^{\frac{1-3x^3}{x^2+5}}}{2\cdot 4e^{2\left(\frac{2-6x^3}{x^2+5}\right)}}\cdot 4e^{2\left(\frac{2-6x^3}{x^2+5}\right)}\right)} = \lim_{x\to+\infty} e^{\frac{x-2}{4x^2+1}\cdot\frac{1}{2}\left[\log\frac{1-\cos 2e^{\frac{1-3x^3}{x^2+5}}}{2\cdot 4e^{2\left(\frac{2-6x^3}{x^2+5}\right)}} + \frac{2-6x^3}{x^2+5} + \log 4\right]} =$$

$$= \lim_{x\to+\infty} e^{\frac{x-2}{4x^2+1}\cdot\frac{1}{2}\log\frac{1-\cos 2e^{\frac{1-3x^3}{x^2+5}}}{2\cdot 4e^{\frac{2-6x^3}{x^2+5}}}}\, e^{\frac{x-2}{4x^2+1}\cdot\frac{1}{2}\cdot\frac{2(1-3x^3)}{x^2+5}}\, e^{\frac{x-2}{4x^2+1}\frac{1}{2}\log 4} =$$

$$= e^{0\cdot\frac{1}{2}\log\frac{1}{2}\cdot\frac{1}{2}}\, e^{-\frac{3}{4}}\, e^{0\cdot\frac{1}{2}\log 4} = e^{0}\, e^{-\frac{3}{4}}\, e^{0} = e^{-\frac{3}{4}}$$

$= e$

201

$$\lim_{x \to +\infty} \left(\operatorname{arctg} e^{-x} \right)^{\frac{x+3}{1-2x^2}} =$$

$$= \lim_{x \to +\infty} e^{\frac{x+3}{1-2x^2} \log \operatorname{arctg} e^{-x}} =$$

$$= \lim_{x \to +\infty} e^{\frac{x+3}{1-2x^2} \log \frac{\operatorname{arctg} e^{-x}}{e^{-x}} \cdot e^{-x}} =$$

$$= \lim_{x \to +\infty} e^{\frac{x+3}{1-2x^2} \left(\log \frac{\operatorname{arctg} e^{-x}}{e^{-x}} - x \right)} =$$

$$= \lim_{x \to +\infty} e^{\frac{x+3}{1-2x^2} \log \frac{\operatorname{arctg} e^{-x}}{e^{-x}}} \cdot e^{\frac{x^2+3x}{2x^2-1}} =$$

$$= e^{0 \log 1} \, e^{\frac{1}{2}} = e^{\frac{1}{2}} = \sqrt{e}$$

$$\lim_{x \to -\infty} \left(\sqrt[5]{1+e \ldots} \right.$$

$$= \lim_{x \to -\infty} e^{\frac{3x+\ldots}{x^2+\ldots}}$$

$$= \lim_{x \to -\infty} e^{\frac{3x+1}{x^2+5} \ldots}$$

$$= \lim_{x \to -\infty} e^{\frac{3x+1}{x^2+5} \log \ldots}$$

$$= e^{\infty} \cdot e^{3} = \ldots$$

E02

$$\lim_{x\to-\infty}\left(\sqrt[5]{1+e^x}-1\right)^{\frac{3x+1}{x^2+5}} =$$

$$= \lim_{x\to-\infty} e^{\frac{3x+1}{x^2+5}\log\left(\frac{\sqrt[5]{1+e^x}-1}{e^x}\, e^x\right)} =$$

$$= \lim_{x\to-\infty} e^{\frac{3x+1}{x^2+5}\left(\log\frac{\sqrt[5]{1+e^x}-1}{e^x}+x\right)} =$$

$$= \lim_{x\to-\infty} e^{\frac{3x+1}{x^2+5}\log\frac{\sqrt[5]{1+e^x}-1}{e^x}} \cdot e^{\frac{3x^2+x}{x^2+5}} =$$

$$= e^{\infty}\cdot e^{3} = e^{3}$$

203

$$\lim_{x\to-\infty}\left[\log(1+3^x)\right]^{\operatorname{arctg}\frac{x^2+1}{x^3}} =$$

$$= \lim_{x\to-\infty} e^{\operatorname{arctg}\frac{x^2+1}{x^3}\,\log\log(1+3^x)} =$$

$$= \lim_{x\to-\infty} e^{\operatorname{arctg}\frac{x^2+1}{x^3}\,\log\left(3^x\cdot\frac{\log(1+3^x)}{3^x}\right)} =$$

$$= \lim_{x\to-\infty} e^{\operatorname{arctg}\frac{x^2+1}{x^3}\left[\log 3^x + \log\frac{\log(1+3^x)}{3^x}\right]} =$$

$$= \lim_{x\to-\infty} e^{\left(\operatorname{arctg}\frac{x^2+1}{x^3}\right)\log 3^x}\ \lim_{x\to-\infty} e^{\operatorname{arctg}\frac{x^2+1}{x^3}\log\frac{\log(1+3^x)}{3^x}}$$

$$\left\{\begin{array}{l} x\to-\infty \Rightarrow 3^x\to 0 \\ \lim_{x\to-\infty}\frac{\log(1+3^x)}{3^x} = \lim_{y\to 0}\frac{\log(1+y)}{y} = 1 \end{array}\right\}$$

$$= \lim_{x\to\infty} e^{\frac{\operatorname{arctg}\frac{x^2+1}{x^3}}{\frac{x^2+1}{x^3}}\cdot\frac{x^2+1}{x^3}\cdot\log_3 3^x\log 3}\cdot e^0$$

$$= \lim_{x\to+\infty} e^{\frac{\operatorname{arctg}\frac{x^2+1}{x^3}}{\frac{x^2+1}{x^3}}\cdot\frac{x^2+1}{x^3}\times\log 3} =$$

$$= e^{1\cdot 1\cdot\log 3} = e^{\log 3} = 3$$

204

$$\lim_{x \to +\infty} x^{\frac{x+1}{x^2+5}} =$$

$$= \lim_{x \to +\infty} e^{\frac{x+1}{x^2+5} \log x} =$$

$$= \lim_{x \to +\infty} e^{\frac{x+1}{x^2+5} \times \frac{\log x}{x}} =$$

$$= e^{1 \cdot 0} = 1$$

$$\lim_{x \to +\infty} \left(e^x + x^3 + \dots\right.$$

$$= \lim_{x \to +\infty} e^{\frac{x^2 + x\sqrt{\dots}}{2x^3 - \dots}}$$

$$= \lim_{x \to +\infty} e^{\frac{x^2 + x\sqrt{\dots}}{2x^3 - \dots}}$$

$$= \lim_{x \to +\infty} e^{\frac{x^3 + x^2 \dots}{2x^3 - x \dots}}$$

$$= e^{\frac{1}{2}} e^{0} \dots$$

205

$$\lim_{x \to +\infty} \left(e^x + x^3 + \cos x + 1 \right)^{\frac{x^2 + x\sqrt{x} + 3}{2x^3 - x^2 + 1}} =$$

$$= \lim_{x \to +\infty} e^{\frac{x^2 + x\sqrt{x} + 3}{2x^3 - x^2 + 1} \log e^x \left(1 + \frac{x^3}{e^x} + \frac{\cos x}{e^x} + \frac{1}{e^x} \right)}$$

$$= \lim_{x \to +\infty} e^{\frac{x^2 + x\sqrt{x} + 3}{2x^3 - x^2 + 1} \cdot \log e^x} \; e^{\frac{x^2 + x\sqrt{x} + 3}{2x^3 - x^2 + 1} \log \left(1 + \frac{x^3}{e^x} + \frac{\cos x}{e^x} \right)}$$

$$= \lim_{x \to +\infty} e^{\frac{x^3 + x^2\sqrt{x} + 3x}{2x^3 - x^2 + 1}} \; e^{\frac{x^2 + x\sqrt{x} + 3}{2x^3 - x^2 + 1} \log \left(1 + \frac{x^3}{e^x} + \frac{\cos x}{e^x} + \frac{1}{e^x} \right)}$$

$$= e^{\frac{1}{2}} \, e^{0} = \sqrt{e}$$

206

$$\lim_{x\to+\infty}\left(x^4+3^x+\operatorname{sen}^4x+2\right)^{\frac{\sqrt{x^2+3x+4}}{5x^2+2x+3}} =$$

$$= \lim_{x\to+\infty} e^{\frac{\sqrt{x^2+3x+4}}{5x^2+2x+3}\log\left(3^x\left[\frac{x^4}{3^x}+1+\frac{\operatorname{sen}^4x}{3^x}+\frac{2}{3^x}\right]\right)} =$$

$$= \lim_{x\to+\infty} e^{\frac{x\sqrt{1+\frac{3}{x}+\frac{4}{x^2}}}{5x^2+2x+3}\log_3 3^x\log 3}\cdot e^{\frac{\sqrt{x^2+3x+4}}{5x^2+2x+3}\log\left[1+\frac{x^4}{3^x}+\frac{\operatorname{sen}^4x}{3^x}+\frac{2}{3^x}\right]} =$$

$$= e^{\frac{1}{5}\log 3}\, e^{0} = e^{\log\sqrt[5]{3}} = \sqrt[5]{3}$$

$$\lim_{x\to-\infty}\left(e^{-x}+\operatorname{arctg}\right.$$

$$\frac{2x^3-5x^2+}{x^4+2\cos x}$$

$$= \lim_{x\to-\infty} e$$

$$\frac{2x^4-}{x^4+}$$

$$= \lim_{x\to-\infty} e$$

$$= e^{-2}\cdot e^{0}$$

$$\lim_{x\to-\infty}\left(e^{-x}+\operatorname{arctg}x+2\right)^{\frac{2x^3-5x^2+3}{x^4+2\cos x+1}} =$$

$$=\lim_{x\to-\infty} e^{\frac{2x^3-5x^2+3}{x^4+2\cos x+1}\log e^{-x}\left(1+\frac{\operatorname{arctg}x}{e^{-x}}+\frac{2}{e^{-x}}\right)}$$

$$=\lim_{x\to-\infty} e^{-\frac{2x^4-5x^3+3x}{x^4+2\cos x+1}}\; e^{\frac{2x^3-5x^2+3}{x^4+2\cos x+1}\log\left(1+\frac{\operatorname{arctg}x}{e^{-x}}+\frac{2}{e^{-x}}\right)}$$

$$= e^{-2}\cdot e^{0} = \frac{1}{e^2}$$

208

$$\lim_{x\to+\infty}\left(\frac{x^2+2}{x+3\,\text{sen}\,x}\right)^{\frac{x^3+2x^2+\text{arctg}\,x}{4x^5-3x^4+\log x}} =$$

$$= \lim_{x\to+\infty} e^{\frac{x^3+2x^2+\text{arctg}\,x}{4x^5-3x^4+\log x}\log\frac{x^2+2}{x+3\,\text{sen}\,x}} =$$

$$= \lim_{x\to+\infty} e^{\frac{x^3+2x^2+\text{arctg}\,x}{4x^5-3x^4+\log x}\log\left(\frac{x^2}{x}\,\frac{\left(1+\frac{2}{x^2}\right)}{1+3\frac{\text{sen}\,x}{x}}\right)}$$

$$= \lim_{x\to+\infty} e^{\frac{(x^3+2x^2+\text{arctg}\,x)x^2}{4x^5-3x^4+\log x}\cdot\frac{\log x}{x^2}} \cdot e^{\frac{x^3+2x^2+\text{arctg}\,x}{4x^5-3x^4+\log x}\log\left(\frac{1+\frac{2}{x^2}}{1+3\frac{\text{sen}\,x}{x}}\right)} =$$

$$= e^{\frac{1}{4}\cdot 0}\, e^{0\cdot 0} = 1$$

1) Liszt / Rapsodia...

2- Linee di ...

$\lim_{x\to+\infty} (\text{senh}\,x)$ ...

$= \lim_{x\to+\infty} e^{\frac{3x^2+5x \dots}{2x^3-x \dots}}$

$= \lim_{x\to+\infty} e^{\frac{3x^2+5x \dots}{2x^3-x \dots}}$

$= \lim_{x\to+\infty} e^{\frac{3x^2+5x \dots}{2x^3-x \dots}}$

$= \lim_{x\to+\infty} e^{\frac{3x^2+5x \dots}{2x^3-x \dots}}$

$= e^{\frac{3}{2}}\, e^{0}$ ...

$\frac{x}{x}$

$=$

$\log \frac{x^2+2}{x+3\,\mathrm{sen}\,x}$

$=$

$\log\left(\frac{x^2}{x} \frac{\left(1+\frac{2}{x^2}\right)}{1+3\frac{\mathrm{sen}\,x}{x}}\right)$

$\frac{3x}{x^2}$ ·

$\log\left(\frac{1+\frac{2}{x^2}}{1+3\frac{\mathrm{sen}\,x}{x}}\right)$

$=$

~~1) List - Rapsodia ungherese n/2 16/12/15.~~ LP

~~2- Linea di confine - Ampliocentrità~~

209

$$\lim_{x\to+\infty} (\mathrm{senh}\,x)^{\frac{3x^2+5x+1}{2x^3-x+5}} =$$

$$= \lim_{x\to+\infty} e^{\frac{3x^2+5x+1}{2x^3-x+5}\cdot \log \mathrm{senh}\,x} =$$

$$= \lim_{x\to+\infty} e^{\frac{3x^2+5x+1}{2x^3-x+5} \log \frac{e^x - e^{-x}}{2}} =$$

$$= \lim_{x\to+\infty} e^{\frac{3x^2+5x+1}{2x^3-x+5} \log e^x \left(\frac{1-e^{-2x}}{2}\right)} =$$

$$= \lim_{x\to+\infty} e^{\frac{3x^2+5x+1}{2x^3-x+5}\cdot x}\, e^{\frac{3x^2+5x+1}{2x^3-x+5} \log\left(\frac{1-e^{-2x}}{2}\right)}$$

$$= e^{\frac{3}{2}} e^{0} = e^{\frac{3}{2}}$$

210

$$\lim_{x\to+\infty}\left(\frac{x^2+x+3}{x^2+3x+1}\right)^{\frac{x^2+1}{ax+1}} =$$

$$\lim_{x\to+\infty} e^{\frac{x^2+1}{ax+1}\log\frac{x^2+x+3}{x^2+3x+1}} =$$

$$= \lim_{x\to+\infty} e^{\frac{x^2+1}{ax+1}\log\left[1+\left(\frac{x^2+x+3}{x^2+3x+1}-1\right)\right]} =$$

$$= \lim_{x\to+\infty} e^{\frac{x^2+1}{ax+1}\log\left(1+\frac{\cancel{x^2}+x+3-\cancel{x^2}-3x-1}{x^2+3x+1}\right)} =$$

$$= \lim_{x\to+\infty} e^{\frac{x^2+1}{ax+1}\log\left(1+\frac{-2x+2}{x^2+3x+1}\right)} =$$

$$= \lim_{x\to+\infty} e^{\frac{\frac{x^2+1}{ax+1}\left(-\frac{2x-2}{x^2+3x+1}\right)\log\left(1-\frac{2x-2}{x^2+3x+1}\right)}{-\frac{2x-2}{x^2+3x+1}}} =$$

$$= e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

211

$$\lim_{x\to+\infty}\left(\frac{x\log x+x+3}{x\log x+2}\right)^{\frac{x\log x+1}{2x+5}} =$$

$$= \lim_{x\to+\infty} e^{\frac{x\log x+1}{2x+5}\log\frac{x\log x+x+3}{x\log x+2}} =$$

$$= \lim_{x\to+\infty} e^{\frac{x\log x+1}{2x+5}\log\left[1+\left(\frac{x\log x+x+3}{x\log x+2}-1\right)\right]} =$$

$$= \lim_{x \to +\infty} e^{\frac{\frac{x^2+1}{4x+1}\left(-\frac{2x-2}{x^2+3x+1}\right)\log\left(1-\frac{2x-2}{x^2+3x+1}\right)}{-\frac{2x-2}{x^2+3x+1}}} =$$

$$= e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

211

$$\lim_{x \to +\infty} \left(\frac{x\log x + x + 3}{x \log x + 2}\right)^{\frac{x \log x + 1}{2x+5}} =$$

$$= \lim_{x \to +\infty} e^{\frac{x\log x+1}{2x+5}\log\frac{x\log x + x + 3}{x\log x + 2}} =$$

$$= \lim_{x \to +\infty} e^{\frac{x\log x+1}{2x+5}\log\left[1+\left(\frac{x\log x + x + 3}{x\log x + 2}-1\right)\right]} =$$

$$\lim_{x\to+\infty} e^{\frac{x\log x+1}{2x+5}\log\left[1+\frac{x\cancel{\log x}+x+3-x\cancel{\log x}-2}{x\log x+2}\right]} =$$

$$\lim_{x\to+\infty} e^{\frac{x\log x+1}{2x+5}\log\left[1+\frac{x-1}{x\log x+2}\right]} =$$

$$= \lim_{x\to+\infty} e^{\frac{x\log x+1}{2x+5}\cdot\frac{x-1}{x\log x+2}\cdot\frac{\log\left(1+\frac{x-1}{x\log x+2}\right)}{\frac{x-1}{x\log x+2}}} =$$

$$= \lim_{x\to+\infty} e^{\frac{x^2\log x\left(1+\frac{1}{x\log x}\right)\left(1-\frac{1}{x}\right)}{x^2\log x\left(2+\frac{5}{x}\right)\left(1+\frac{2}{x\log x}\right)}\cdot\frac{\log\left(1+\frac{x-1}{x\log x+2}\right)}{\frac{x-1}{x\log x+2}}} =$$

$$= e^{\frac{1}{2}}$$

(212)

$$\lim_{x\to+\infty}\left(\frac{e^x+2}{e^x-3}\right)^{\frac{e^x+x+3}{1+\operatorname{arctg}x}} = \lim_{x\to+\infty} e^{\frac{e^x+x+3}{1+\operatorname{arctg}x}\log\frac{e^x+2}{e^x-3}} =$$

$$= \lim_{x\to+\infty} e^{\frac{e^x\left(1+\frac{x}{e^x}+\frac{3}{e^x}\right)}{1+\operatorname{arctg}x}\cdot\log\left[1+\left(\frac{e^x+2}{e^x-3}-1\right)\right]} =$$

$$= \lim_{x\to+\infty} e^{\frac{e^x\left(1+\frac{x}{e^x}+\frac{3}{e^x}\right)}{1+\operatorname{arctg}x}\log\left[1+\frac{e^x+2-e^x+3}{e^x-3}\right]} =$$

$$= \lim_{x\to+\infty} e^{\frac{e^x\left(1+\frac{x}{e^x}+\frac{3}{e^x}\right)}{1+\operatorname{arctg}x}\log\left(1+\frac{5}{e^x-3}\right)} =$$

$$= \lim_{x\to+\infty} e^{\frac{e^x\left(1+\frac{x}{e^x}+\frac{3}{e^x}\right)}{1+\operatorname{arctg}x}\left(\frac{5}{e^x-3}\right)\frac{\log\left(1+\frac{5}{e^x-3}\right)}{\frac{5}{e^x-3}}} =$$

$$= \lim_{x\to+\infty} e^{\frac{1+\frac{x}{e^x}+\frac{3}{e^x}}{1+\operatorname{arctg}x}\cdot\frac{5}{1-\frac{3}{e^x}}\cdot\frac{\log\left(1+\frac{5}{e^x-3}\right)}{\frac{5}{e^x-3}}} =$$

$$= e^{\frac{1+0+0}{1+\frac{\pi}{2}}\cdot\frac{5}{1+0}\cdot 1} = e^{\frac{5}{1+\frac{\pi}{2}}} = e^{\frac{10}{\pi+2}}$$

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213

$$\lim_{x\to+\infty}\left(2e^{\sqrt{\frac{x^2+x+1}{x^2-x+3}}-1}-1\right)^{\frac{x^2+3}{3x+2}} =$$

$$\lim_{x\to+\infty} e^{\frac{x^2+3}{3x+2}\log\left(2e^{\sqrt{\frac{x^2+x+1}{x^2-x+3}}-1}-1\right)} =$$

$$= \cancel{\lim_{x\to+\infty} e^{\frac{x^2+3}{3x+2}\log\left(e^{\sqrt{\frac{x^2+x+1}{x^2-x+3}}-1}+e^{\sqrt{\frac{x^2+x+1}{x^2-x+3}}-1}\right.}}$$

$$= \lim_{x\to+\infty} e^{\frac{x^2+3}{3x+2}\log\left[1+\left(2e^{\sqrt{\frac{x^2+x+1}{x^2-x+3}}-1}-2\right)\right]} =$$

$$= \lim_{x\to+\infty} e^{\frac{x^2+3}{3x+2}\cdot\left(2e^{\sqrt{\frac{x^2+x+1}{x^2-x+3}}-1}-2\right)\frac{\log\left[1+\left(2e^{\sqrt{\frac{x^2+x+1}{x^2-x+3}}-1}-2\right)\right]}{2e^{\sqrt{\frac{x^2+x+1}{x^2-x+3}}-1}-2}} =$$

$$= \lim_{x\to+\infty} e^{\frac{2x^2+6}{3x+2}\left(\sqrt{\frac{x^2+x+1}{x^2-x+3}}-1\right)\frac{e^{\sqrt{\frac{x^2+x+1}{x^2-x+3}}-1}-1}{\sqrt{\frac{x^2+x+1}{x^2-x+3}}-1}\cdot\frac{\log\left[1+\left(2e^{\sqrt{\frac{x^2+x+1}{x^2-x-3}}-1}-2\right)\right]}{2e^{\sqrt{\frac{x^2+x+1}{x^2+x+3}}}-2}}$$

$$= \lim_{x\to+\infty} \frac{2x^2+6}{3x+2}\cdot\frac{\frac{x^2+x+1}{x^2-x+3}-1}{\sqrt{\frac{x^2+x+1}{x^2-x+3}}+1}\cdot\frac{e^{\sqrt{\frac{x^2+x+1}{x^2-x+3}}}-1}{\sqrt{\frac{x^2+x+1}{x^2-x+3}}-1}\cdot$$

$$\cdot\frac{\log\left[1+\left(2e^{\sqrt{\frac{x^2+x+1}{x^2-x+3}}-1}-2\right)\right]}{2e^{\sqrt{\frac{x^2+x+1}{x^2-x+3}}-1}-2} =$$

$$= \lim_{x\to+\infty} e^{\frac{\frac{2x-2}{x^2-x+3}}{\sqrt{\frac{x^2+x+1}{x^2-x+3}}+1}\cdot\frac{2x^2+6}{3x+2}\cdot 1} =$$

$$= \emptyset\cdot e^{\frac{\frac{4}{3}}{2}\cdot 1} = \emptyset\cdot e^{\frac{2}{3}}$$

214

$$\lim_{x\to 0}\left(\frac{2^x+5^x}{3^x+4^x}\right)^{\frac{1}{x}} =$$

$$= \lim_{x\to 0} e^{\frac{1}{x}\log\frac{2^x+5^x}{3^x+4^x}} =$$

$$= \lim_{x\to 0} e^{\frac{1}{x}\log\left[1+\left(\frac{2^x+5^x}{3^x+4^x}-1\right)\right]} =$$

$$= \lim_{x\to 0} e^{\frac{1}{x}\left(\frac{2^x+5^x}{3^x+4^x}-1\right)\frac{\log\left[1+\left(\frac{2^x+5^x}{3^x+4^x}-1\right)\right]}{\left(\frac{2^x+5^x}{3^x+4^x}-1\right)}} =$$

$$= \lim_{x\to 0} e^{\frac{1}{x}\frac{2^x+5^x-3^x-4^x}{3^x+4^x}\frac{\log\left[1+\left(\frac{2^x+5^x}{3^x+4^x}-1\right)\right]}{\left(\frac{2^x+5^x}{3^x+4^x}-1\right)}} =$$

$$= \lim_{x\to 0} e^{\frac{1}{3^x+4^x}\left(\frac{2^x-1}{x}+\frac{5^x-1}{x}-\frac{3^x-1}{x}-\frac{4^x-1}{x}\right)\cdot\frac{\log\left[1+\left(\frac{2^x+5^x}{3^x+4^x}-1\right)\right]}{\left(\frac{2^x+5^x}{3^x+4^x}-1\right)}}$$

$$= e^{\frac{1}{2}(\log 2+\log 5-\log 3-\log 4)\cdot 1} =$$

$$= e^{\frac{1}{2}\left(\log\frac{10}{12}\right)} = e^{\log\sqrt{\frac{5}{6}}} =$$

$$= \sqrt{\frac{5}{6}}$$

215

$$\lim_{x\to 0}\left(\frac{3^x+6^x}{2^x+3^x}\right)^{\frac{1}{\operatorname{arctg} x}} = \lim_{x\to 0} e^{\frac{1}{\operatorname{arctg} x}\log\frac{3^x+6^x}{2^x+3^x}} =$$

$$= \lim_{x\to 0} e^{\frac{1}{\operatorname{arctg} x}\log\left[1+\left(\frac{3^x+6^x}{2^x+3^x}-1\right)\right]} =$$

$$= \lim_{x\to 0} e^{\frac{1}{\operatorname{arctg} x}\left(\frac{3^x+6^x}{2^x+3^x}-1\right)\frac{\log\left[1+\left(\frac{3^x+6^x}{2^x+3^x}-1\right)\right]}{\left(\frac{3^x+6^x}{2^x+3^x}-1\right)}} =$$

$$= \lim_{x\to 0} e^{\frac{x}{\operatorname{arctg} x}\cdot\frac{1}{2^x+3^x}\left(3^x+6^x-2^x-3^x\right)\cdot\frac{\log\left[1+\left(\frac{3^x+6^x}{2^x+3^x}-1\right)\right]}{\left(\frac{3^x+6^x}{2^x+3^x}-1\right)}}$$

$$= \lim_{x\to 0} e^{\frac{x}{\operatorname{arctg} x}\frac{1}{2^x+3^x}\left(\frac{3^x-1}{x}+\frac{6^x-1}{x}-\frac{2^x-1}{x}-\frac{3^x-1}{x}\right)\frac{\log\left[1+\left(\frac{3^x+6^x}{2^x+3^x}-1\right)\right]}{\left(\frac{3^x+6^x}{2^x+3^x}-1\right)}}$$

$$= \lim_{x\to 0} e^{1\cdot\frac{1}{2}\left(\log\frac{3\cdot\cancel{6}^{\,3}}{2\cdot\cancel{3}}\right)\cdot 1} = e^{\log\sqrt{3}} = \sqrt{3}$$

~~1. Puccini. Turandot / Tosca - 3 cassette~~

~~4 Prokofiev - Pierino e il lupo. The Chamber Orchestra of Europe - Claudio Abbado CD + audiocassette~~

~~6. Prokofiev. Romeo & Giulietta (estratti). Berliner Philharmoniker. Esa Pekka Salonen~~

216

$$\lim_{x\to 0}\left(\frac{5^x+1}{x^2+2}\right)^{\frac{x+1}{x}} = \lim_{x\to 0} e^{\frac{x+1}{x}\log\frac{5^x+1}{x^2+2}} =$$

$$\lim_{x\to 0} e^{\frac{x+1}{x}\log\left[1+\left(\frac{5^x+1}{x^2+2}-1\right)\right]} =$$

$$= \lim_{x\to 0} e^{\frac{x+1}{x}\left(\frac{5^x+1}{x^2+2}-1\right)\frac{\log\left[1+\left(\frac{5^x+1}{x^2+2}-1\right)\right]}{\left(\frac{5^x+1}{x^2+2}-1\right)}} =$$

$$= \lim_{x\to 0} e^{\frac{x+1}{x^2+2}\left(\frac{5^x+1-x^2-2}{x}\right)\frac{\log\left[1+\left(\frac{5^x+1}{x^2+2}-1\right)\right]}{\left(\frac{5^x+1}{x^2+2}-1\right)}} =$$

$$= \lim_{x\to 0} e^{\frac{x+1}{x^2+2}\left(\frac{5^x-1}{x}-x\right)\frac{\log\left[1+\left(\frac{5^x+1}{x^2+2}-1\right)\right]}{\left(\frac{5^x+1}{x^2+2}-1\right)}} =$$

$$= e^{\frac{1}{2}(\log 5 - 1)} = e^{\log\sqrt{5}-\frac{1}{2}} =$$

$$= \sqrt{5}\sqrt{e^{-1}} = \sqrt{\frac{5}{e}}$$

217

$$\lim_{x\to+\infty}\left(3e^{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}-2\right)^x=$$

$$=\lim_{x\to+\infty} e^{x\log\left(3e^{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}-2\right)}=$$

$$=\lim_{x\to+\infty} e^{x\log\left[1+\left(3e^{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}-3\right)\right]}=$$

$$=\lim_{x\to+\infty} e^{x\left(3e^{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}-3\right)\frac{\log\left[1+\left(3e^{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}-3\right)\right]}{3e^{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}-3}}=$$

$$=\lim_{x\to+\infty} e^{3x\cdot\frac{\left(e^{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}-1\right)}{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}\cdot\left(\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}\right)\frac{\log\left[1+\left(3e^{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}-3\right)\right]}{3e^{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}-3}}=$$

vedi foglio allegato

$$=\lim_{x\to+\infty} e^{3\left(-\frac{1}{2}\right)}$$

$$= e^{-\frac{3}{2}}$$

$$\lim_{x\to+\infty} x\left(\operatorname{arctg} x - \frac{\pi x^2+1}{2x^2+5}\right)$$

$$\lim_{x\to+\infty} x \frac{2x^2\operatorname{arctg} x + 5\operatorname{arctg} x - \pi x^2 - 1}{2x^2+5} =$$

$$= \lim_{x\to+\infty} x \frac{\cancel{x^2}\left[2\operatorname{arctg} x - \pi + \frac{5\operatorname{arctg} x}{x^2} - \frac{1}{x^2}\right]}{\cancel{x^2}\left(2+\frac{5}{x}\right)} =$$

$$= \lim_{x\to+\infty} \frac{x\left(2\operatorname{arctg} x - \pi\right) + \frac{5\operatorname{arctg} x}{x} \nearrow 0 - \frac{1}{x} \nearrow 0}{\left(2+\frac{5}{x}\right) \searrow 2} =$$

$$= \frac{1}{2}\lim_{x\to+\infty} x\left(2\operatorname{arctg} x - \pi\right)$$

$$y = 2\operatorname{arctg} x - \pi \qquad x = \operatorname{tg}\frac{y+\pi}{2}$$

$$= \frac{1}{2}\lim_{y\to 0} \operatorname{tg}\left(\frac{y}{2}+\frac{\pi}{2}\right)\cdot y = -\frac{1}{2}\lim \frac{y}{\operatorname{tg} y} = -\frac{1}{2}$$

218

$$\lim_{x\to 0}\left(\frac{\operatorname{sen}x+2}{\cos x+1}\right)^{\frac{x^2+\operatorname{sen}^3x}{x^3+2x^4}} =$$

$$= \lim_{x\to 0} e^{\frac{x^2+\operatorname{sen}^3x}{x^3+2x^4}\log\frac{\operatorname{sen}x+2}{\cos x+1}} =$$

$$= \lim_{x\to 0} e^{\frac{x^2+\operatorname{sen}^3x}{x^3+2x^4}\log\left[1+\left(\frac{\operatorname{sen}x+2}{\cos x+1}-1\right)\right]} =$$

$$= \lim_{x\to 0} e^{\frac{x^2+\operatorname{sen}^3x}{x^3+2x^4}\left(\frac{\operatorname{sen}x+2}{\cos x+1}-1\right)\frac{\log\left[1+\left(\frac{\operatorname{sen}x+2}{\cos x+1}-1\right)\right]}{\left(\frac{\operatorname{sen}x+2}{\cos x+1}-1\right)}} =$$

$$= \lim_{x\to 0} e^{\frac{x^2+\operatorname{sen}^3x}{x^3+2x^4}\frac{\operatorname{sen}x-\cos x+1}{1+\cos x}\frac{\log\left[1+\left(\frac{\operatorname{sen}x+2}{\cos x+1}-1\right)\right]}{\left(\frac{\operatorname{sen}x+2}{\cos x+1}-1\right)}} =$$

$$= \lim_{x\to 0} e^{\frac{x^2+\operatorname{sen}^3x}{x^3+2x^4}\frac{\operatorname{sen}x+1-\cos x}{1+\cos x}\frac{\log\left[1+\frac{\operatorname{sen}x+2}{\cos x+1}-1\right]}{\frac{\operatorname{sen}x+2}{\cos x+1}-1}} =$$

$$= \lim_{x\to 0} e^{\frac{x^2\left(1+\frac{\operatorname{sen}^3x}{x^2}\right)}{x^3(1+2x)}\frac{\operatorname{sen}x+1-\cos x}{1+\cos x}\frac{\log\left[1+\frac{\operatorname{sen}x+2}{\cos x+1}-1\right]}{\frac{\operatorname{sen}x+2}{\cos x+1}-1}}$$

$$= \lim_{x\to 0} e^{\frac{1+\frac{\operatorname{sen}^3x}{x^2}}{1+2x}\frac{\frac{\operatorname{sen}x}{x}+\frac{1-\cos x}{x}}{1+\cos x}\frac{\log\left[1+\frac{\operatorname{sen}x+2}{\cos x+1}-1\right]}{\frac{\operatorname{sen}x+2}{\cos x+1}-1}}$$

$$= e^{1\cdot\frac{1}{2}\cdot 1} = e^{\frac{1}{2}}$$

219

$$\lim_{x\to 0}\left(\frac{\operatorname{sen}x+\cos x}{x^2-3x+1}\right)^{\frac{x\log 3+\operatorname{sen}^2x}{4x^2}} =$$

$$\lim_{x\to 0} e^{\frac{x\log 3+\operatorname{sen}^2x}{4x^2}\log\left[1+\left(\frac{\operatorname{sen}x+\cos x}{x^3-3x+1}-1\right)\right]} =$$

$$\lim_{x\to 0} e^{\frac{x\log 3+\operatorname{sen}^2x}{4x^2}\cdot\left(\frac{\operatorname{sen}x+\cos x}{x^3-3x+1}-1\right)\frac{\log\left[1+\left(\frac{\operatorname{sen}x+\cos x}{x^3-3x+1}-1\right)\right]}{\frac{\operatorname{sen}x+\cos x}{x^3-3x+1}-1}} =$$

$$=\lim_{x\to 0} e^{\frac{x\left(\log 3+\frac{\operatorname{sen}^2x}{x}\right)}{4x^2}\cdot\frac{\operatorname{sen}x+\cos x-x^3+3x-1}{x^3-3x+1}\frac{\log\left[1+\frac{\operatorname{sen}x+\cos x}{x^3-3x+1}-1\right]}{\frac{\operatorname{sen}x+\cos x}{x^3-3x+1}-1}}$$

$$=\lim_{x\to 0} e^{\frac{\log 3+\frac{\operatorname{sen}^2x}{x}}{4}\frac{\frac{\operatorname{sen}x}{x}+\frac{\cos x-1}{x}-x^2+3}{x^3-3x+1}\frac{\log\left[1+\frac{\operatorname{sen}x+\cos x}{x^3-3x+1}-1\right]}{\frac{\operatorname{sen}x+\cos x}{x^3-3x+1}-1}} =$$

$$=\lim_{x\to 0} e^{\frac{\log 3+0}{4}\cdot\frac{4}{1}\cdot 1} = e^{\log 3} = 3$$

220

$$\lim_{x\to 1}\left(\frac{2x+5}{x-1+\operatorname{sen}^2(x-1)}\right)^{\pi-4\operatorname{arctg}x} =$$

$$=\lim_{x\to 1} e^{(\pi-4\operatorname{arctg}x)\log\frac{2x+5}{x-1+\operatorname{sen}^2(x-1)}} =$$

$$=\lim_{x\to 1} e^{\frac{\pi-4\operatorname{arctg}x}{\frac{2x+5}{x-1+\operatorname{sen}^2(x-1)}}\frac{2x+5}{x-1+\operatorname{sen}^2(x-1)}\log\frac{2x+5}{x-1+\operatorname{sen}^2(x-1)}} =$$

$$=\lim_{x\to 1} e^{\frac{\pi-4\operatorname{arctg}x}{2x+5}\cdot(x-1)+\operatorname{sen}^2(x-1))\frac{2x+5}{x-1+\operatorname{sen}^2(x-1)}\log\frac{2x+5}{(x-1)+\operatorname{sen}^2(x-1)}} =$$

$$= e^{\frac{0}{7}\cdot 0\cdot 0} = e^0 = 1$$

R S Soc T U V W X Y Z

$$\operatorname{sen}^2 x$$

$$=$$

$$\left[\frac{\operatorname{sen}x+\cos x}{x^3-3x+1}-1\right)\Big]$$

$$=$$

$$-1\Big)\log\left[1+\left(\frac{\operatorname{sen}x+\cos x}{x^3-3x+1}-1\right)\right] \Big/ \left(\frac{\operatorname{sen}x+\cos x}{x^3-3x+1}-1\right)$$

$$=$$

$$\frac{\cos x-x^3+3x-1}{x^3-3x+1}\;\frac{\log\left[1+\frac{\operatorname{sen}x+\cos x}{x^3-3x+1}-1\right]}{\frac{\operatorname{sen}x+\cos x}{x^3-3x+1}-1}$$

$$\frac{\frac{2x-1}{x}-x^2+3}{3x+1}\;\frac{\log\left[1+\frac{\operatorname{sen}x+\cos x}{x^3-3x+1}-1\right]}{\frac{\operatorname{sen}x+\cos x}{x^3-3x+1}-1} =$$

$$= \lim_{x\to 0} e^{\frac{\log 3+0}{4}\cdot\frac{4}{1}\cdot 1} = e^{\log 3} = 3$$

220

$$\lim_{x\to 1}\left(\frac{2x+5}{x-1+\operatorname{sen}^2(x-1)}\right)^{\pi-4\operatorname{arctg}x} =$$

$$= \lim_{x\to 1} e^{(\pi-4\operatorname{arctg}x)\log\frac{2x+5}{x-1+\operatorname{sen}^2(x-1)}} =$$

$$= \lim_{x\to 1} e^{\frac{\pi-4\operatorname{arctg}x}{\frac{2x+5}{x-1+\operatorname{sen}^2(x-1)}}\cdot\frac{2x+5}{x-1+\operatorname{sen}^2(x-1)}\log\frac{2x+5}{x-1+\operatorname{sen}^2(x-1)}} =$$

$$= \lim_{x\to 1} e^{\frac{\pi-4\operatorname{arctg}x}{2x+5}\cdot((x-1)+\operatorname{sen}^2(x-1))\frac{2x+5}{x-1+\operatorname{sen}^2(x-1)}\log\frac{2x+5}{(x-1)+\operatorname{sen}^2(x-1)}} =$$

$$= e^{\frac{0}{7}\cdot 0\cdot 0} = e^{0} = 1$$

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221

$$\lim_{x\to+\infty}\left(\frac{x^4-3x^3+x^2-1}{x^2-2x\,\mathrm{arctg}\,x}\right)^{\frac{1}{x}} =$$

$$\lim_{x\to+\infty} e^{\frac{1}{x}\log\frac{x^4-3x^3+x^2-1}{x^2-2x\,\mathrm{arctg}\,x}} =$$

$$\lim_{x\to+\infty} e^{\frac{1}{x}\log\frac{x^4\left(1-\frac{3}{x}+\frac{1}{x^2}-\frac{1}{x^4}\right)}{x^2\left(1-\frac{2\,\mathrm{arctg}\,x}{x}\right)}} =$$

$$= \lim_{x\to+\infty} e^{\frac{1}{x}\left[\log x^2+\log\left(\frac{1-\frac{3}{x}+\frac{1}{x^2}-\frac{1}{x^4}}{1-\frac{2\,\mathrm{arctg}\,x}{x}}\right)\right]} =$$

$$= \lim_{x\to+\infty} e^{\frac{2\log x}{x}}\; e^{\frac{1}{x}\log\left(\frac{1-\frac{3}{x}+\frac{1}{x^2}-\frac{1}{x^4}}{1-\frac{2\,\mathrm{arctg}\,x}{x}}\right)}$$

$$= \lim_{x\to+\infty} \underbrace{e^{\frac{2\log x}{x}}}_{\to e^0}\; e^{\frac{1}{x}\log\left[1+\left(\frac{1-\frac{3}{x}+\frac{1}{x^2}-\frac{1}{x^4}}{1-\frac{2\,\mathrm{arctg}\,x}{x}}-1\right)\right]} =$$

$$= \lim_{x\to+\infty} e^{\frac{1}{x}\left(\frac{1-\frac{3}{x}+\frac{1}{x^2}-\frac{1}{x^4}}{1-\frac{2\,\mathrm{arctg}\,x}{x}}-1\right)\frac{\log\left[1+\left(\frac{1-\frac{3}{x}+\frac{1}{x^2}-\frac{1}{x^4}}{1-\frac{2\,\mathrm{arctg}\,x}{x}}-1\right)\right]}{\frac{1-\frac{3}{x}+\frac{1}{x^2}-\frac{1}{x^4}}{1-\frac{2\,\mathrm{arctg}\,x}{x}}-1}} =$$

$$= e^{0\cdot0\cdot0} = 1$$

222

$$\lim_{x\to -1}\left(\frac{x^2+2x+3}{x+3}\right)^{\frac{1}{\operatorname{sen}(x+1)}} =$$

$$= \lim_{x\to -1} e^{\frac{1}{\operatorname{sen}(x+1)}\log\frac{x^2+2x+3}{x+3}} =$$

$$= \lim_{x\to -1} e^{\frac{1}{\operatorname{sen}(x+1)}\log\left[1+\left(\frac{x^2+2x+3}{x+3}-1\right)\right]} =$$

$$= \lim_{x\to -1} e^{\frac{1}{\operatorname{sen}(x+1)}\left(\frac{x^2+2x+3}{x+3}-1\right)\frac{\log\left[1+\left(\frac{x^2+2x+3}{x+3}-1\right)\right]}{\left(\frac{x^2+2x+3}{x+3}-1\right)}} =$$

$$= \lim_{x\to -1} e^{\frac{1}{\operatorname{sen}(x+1)}\frac{x^2+2x+3-x-3}{x+3}\frac{\log\left[1+\left(\frac{x^2+2x+3}{x+3}-1\right)\right]}{\left(\frac{x^2+2x+3}{x+3}-1\right)}} =$$

$$= \lim_{x\to -1} e^{\frac{1}{\operatorname{sen}(x+1)}\frac{x^2+x}{x+3}\frac{\log\left[1+\left(\frac{x^2+2x+3}{x+3}-1\right)\right]}{\frac{x^2+2x+3}{x+3}-1}} =$$

1- Rimsky-Korsakow -
Berliner Philharmon
2- Rodrigo - Concierto
gentilhombre · Orc
dir. Odón Alonso ·
3- Ravel: Bolero / Be
Rimski · Korsakov
Enesco: Rapsod
sinfonica di B
4- Rostropovich -
cello concert in B
Cello concert.
5- Rachmaninov ·
Vladimir Ashken
Orchestra - Anche
7- Ravel · Bolero ·
in Sol maggiore
Orchestra sinfo
Lubiana · Anton
8- Rossini · Stabat
& Corus · Carlo M

$$= \lim_{x\to -1} e^{\frac{x+1}{\text{sen}(x+1)}\cdot\frac{x}{x+3}\cdot\frac{\log\left[1+\left(\frac{x^2+2x+3}{x+3}-1\right)\right]}{\frac{x^2+2x+3}{x+3}-1}} =$$

$$= e^{1\cdot\frac{-1}{2}\cdot 1} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

223

$$\lim_{x\to 2}\left(\frac{x^3+x+1}{x^2+7x-7}\right)^{\frac{11e^2}{e^x-e^2}} =$$

$$= \lim_{x\to 2} e^{\frac{11e^2}{e^x-e^2}\log\frac{x^3+x+1}{x^2+7x-7}} =$$

$$= \lim_{x\to 2} e^{\frac{11e^2}{e^x-e^2}\log\left[1+\left(\frac{x^3+x+1}{x^2+7x-7}-1\right)\right]} =$$

$$= \lim_{x\to 2} e^{\frac{11e^2}{e^x-e^2}\cdot\left(\frac{x^3+x+1}{x^2+7x-7}-1\right)\frac{\log\left[1+\left(\frac{x^3+x+1}{x^2+7x-7}-1\right)\right]}{\left(\frac{x^3+x+1}{x^2+7x-7}-1\right)}} =$$

$$= \lim_{x\to 2} e^{\frac{11e^2}{e^x-e^2}\,\frac{x^3 \dots}{x^2 \dots}}$$

$$= \lim_{x\to 2} e^{\frac{11(x-2)}{e^{x-2}-1} \dots}$$

$$= \lim_{x\to 2} e$$

$x^3-x^2-6 \dots$
x^3-2x^2
$// \; x^2-6 \dots$
$x^2-2 \dots$
$// \; -4 \dots$
$-4 \dots$

$$= e^{11\cdot 1\cdot\frac{2}{11}\cdot \dots}$$

$$\lim_{x\to -1} e^{\frac{x+1}{\sin(x+1)}\,\frac{x}{x+3}\,\frac{\log\left[1+\left(\frac{x^2+2x+3}{x+3}-1\right)\right]}{\frac{x^2+2x+3}{x+3}-1}} =$$

$$e^{1\cdot\frac{-1}{2}\cdot 1} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

223

$$\lim_{x\to 2}\left(\frac{x^3+x+1}{x^2+7x-7}\right)^{\frac{11e^2}{e^x-e^2}} =$$

$$\lim_{x\to 2} e^{\frac{11e^2}{e^x-e^2}\log\frac{x^3+x+1}{x^2+7x-7}} =$$

$$\lim_{x\to 2} e^{\frac{11e^2}{e^x-e^2}\log\left[1+\left(\frac{x^3+x+1}{x^2+7x-7}-1\right)\right]} =$$

$$=\lim_{x\to 2} e^{\frac{11e^2}{e^x-e^2}\cdot\left(\frac{x^3+x+1}{x^2+7x-7}-1\right)\frac{\log\left[1+\left(\frac{x^3+x+1}{x^2+7x-7}-1\right)\right]}{\left(\frac{x^3+x+1}{x^2+7x-7}-1\right)}} =$$

$$=\lim_{x\to 2} e^{\frac{11e^2}{e^x-e^2}\,\frac{x^3-x^2-6x+8}{x^2+7x-7}\,\frac{\log\left[1+\left(\frac{x^3+x+1}{x^2+7x-7}-1\right)\right]}{\left(\frac{x^3+x+1}{x^2+7x-7}-1\right)}} =$$

$$=\lim_{x\to 2} e^{\frac{11(x-2)}{e^{x-2}-1}\,\frac{x^2+x-4}{x^2+7x-7}\,\frac{\log\left[1+\left(\frac{x^3+x+1}{x^2+7x-7}-1\right)\right]}{\left(\frac{x^3+x+1}{x^2+7x-7}-1\right)}} =$$

$$\left\{\begin{array}{l|l}
x^3-x^2-6x+8 & x-2 \\ \hline
x^3-2x^2 & x^2+x-4 \\ \hline
// \quad x^2-6x+8 & \\
x^2-2x & \\ \hline
// \quad -4x+8 & \\
-4x+8 & \\ \hline
// \quad // &
\end{array}\right\}$$

$$= e^{\cancel{11}\cdot 1\cdot\frac{2}{\cancel{11}}\cdot 1} = e^2$$

S Soc. T U V W X Y Z

224

$$\lim_{x\to1}\left(\frac{\operatorname{sen}\pi x+3x}{x^2+x+1}\right)^{\frac{1}{tg(x-1)}} =$$

$$\lim_{x\to1} e^{\frac{1}{tg(x-1)}\log\frac{\operatorname{sen}\pi x+3x}{x^2+x+1}} =$$

$$= \lim_{x\to1} e^{\frac{1}{tg(x-1)}\log\left[1+\left(\frac{\operatorname{sen}\pi x+3x}{x^2+x+1}-1\right)\right]} =$$

$$= \lim_{x\to1} e^{\frac{1}{tg(x-1)}\cdot\frac{\operatorname{sen}\pi x-x^2+2x-1}{x^2+x+1}\cdot\frac{\log\left[1+\left(\frac{\operatorname{sen}\pi x+3x}{x^2+x+1}-1\right)\right]}{\frac{\operatorname{sen}\pi x+3x}{x^2+x+1}-1}} =$$

$$= \lim_{x\to1} e^{-\frac{\pi}{3}}$$

$$\left\{\begin{array}{l} x-1=y\ ;\quad x=y+1\ ,\ x\to1 \Rightarrow y\to0 \\ \lim_{y\to0} \frac{1}{tg\,y}\,\frac{\operatorname{sen}\pi(y+1)-(y+1)^2+2(y+1)-1}{(y+1)^2+y+1+1} = \end{array}\right.$$

$$= \lim_{y\to0}\frac{1}{tg\,y}\,\frac{-\operatorname{sen}\pi y-y^2\cancel{-1}\cancel{-2y}\cancel{+2y}\cancel{+2}\cancel{-1}}{y^2+1+2y+y+2} =$$

$$= \lim_{y\to0}\frac{1}{tg\,y}\,\frac{-\operatorname{sen}\pi y-y^2}{y^2+3y+3} =$$

$$= \lim_{y\to0}\frac{y}{tg\,y}\,\frac{\left(-\frac{\pi\operatorname{sen}\pi y}{\pi y}-y\right)}{y^2+3y+3} = 1\,\frac{-\pi-0}{3}$$

$$= -\frac{\pi}{3}$$

225

$$\lim_{x\to 0}\left(\frac{x^2+x+3}{tg^2x+2\sqrt{x}+sen x}\right)^{\sqrt{arcsen x}} =$$

$$= \lim_{x\to 0} e^{\sqrt{arcsen x}\,\log\frac{x^2+x+3}{tg^2x+2\sqrt{x}+sen x}}$$

$$\cancel{= \lim_{x\to 0} e^{\sqrt{arcsen x}\cdot tg^2x+2\sqrt{x}}}$$

$$\lim_{x\to 0} e^{\sqrt{arcsen x}\cdot\frac{x^2+x+3}{tg^2x+2\sqrt{x}+sen x}\cdot\frac{\log\frac{x^2+x+3}{tg^2x+2\sqrt{x}+sen x}}{\frac{x^2+x+3}{tg^2x+2\sqrt{x}+sen x}}} =$$

$$= \lim_{x\to 0} e^{\sqrt{\frac{arcsen x}{x}}\cdot\frac{x^2+x+3}{\frac{tg^2x}{x^2}+\frac{2}{x\sqrt{x}}+\frac{sen x}{x^2}}\cdot\frac{\log\frac{x^2+x+3}{tg^2x+2\sqrt{x}+sen x}}{\frac{x^2+x+3}{tg^2x+2\sqrt{x}+sen x}}} =$$

$$= e^{1\cdot\frac{3}{1+0+0}\cdot 0} = 1$$

1- Sibelius - Pell
Philharmoniker
2- Schubert - I
op 142 D 935.
4- Strauss Rich
Also sprach Z
Verklärung
Vladimir A
6- Stivell Alan
Audiocassett
7- Stivell Alan
8- Stravinsky
Detroit Sym
- Audiocassetto

226

$$\lim_{x\to 0}\left(\frac{1+\operatorname{sen} x}{5^x+4^x-3^x-2^x}\right)^{\sqrt{1+\operatorname{arctg} x}\,-1} =$$

$$= \lim_{x\to 0} e^{\left(\sqrt{1+\operatorname{arctg} x}-1\right)\log\frac{1+\operatorname{sen} x}{5^x+4^x-3^x-2^x}} =$$

$$= \lim_{x\to 0} e^{\left(\sqrt{1+\operatorname{arctg} x}-1\right)\frac{1+\operatorname{sen} x}{5^x+4^x-3^x-2^x}\cdot\frac{\log\frac{1+\operatorname{sen} x}{5^x+4^x-3^x-2^x}}{\frac{1+\operatorname{sen} x}{5^x+4^x-3^x-2^x}}} =$$

$$= \lim_{x\to 0} e^{\frac{\sqrt{1+\operatorname{arctg} x}-1}{\operatorname{arctg} x}\cdot\frac{\operatorname{arctg} x}{x}\cdot\frac{1+\operatorname{sen} x}{\frac{5^x-1}{x}+\frac{4^x-1}{x}-\frac{3^x-1}{x}-\frac{2^x-1}{x}}\cdot\frac{\log\frac{1+\operatorname{sen} x}{5^x+4^x-3^x-2^x}}{\frac{1+\operatorname{sen} x}{5^x+4^x-3^x-2^x}}} =$$

$$= e^{\frac{1}{2}\cdot 1\cdot\frac{1+0}{\log 5+\log 4-\log 3-\log 2}\cdot 0} = 1$$

227 esercizi d'esame

$$\lim_{x \to +\infty} \frac{\left(\arcsin \frac{1}{x}\right)^7}{16 \operatorname{arctg}^4 x - \pi^4} =$$

$$= \lim_{x \to +\infty} \frac{\left(\arcsin \frac{1}{x}\right)^7}{\frac{1}{x^7}} \cdot \frac{\frac{1}{x^7}}{16 \operatorname{arctg}^4 x - \pi^4}$$

$$= 1 \cdot \lim_{x \to +\infty} \frac{\frac{1}{x^7}}{16 \operatorname{arctg}^4 x - \pi^4} =$$

$$= \lim_{x \to +\infty} \frac{\frac{1}{x^7}}{(4 \operatorname{arctg}^2 x - \pi^2)(4 \operatorname{arctg}^2 x + \pi^2)} =$$

$$= \lim_{x \to +\infty} \frac{\frac{1}{x^7}}{(2 \operatorname{arctg} x - \pi)(2 \operatorname{arctg} x + \pi)(4 \operatorname{arctg}^2 x + \pi^2)}$$

$$\left\{ \lim_{x \to +\infty} \frac{\frac{1}{x}}{2 \operatorname{arctg} x - \pi} \right.$$

$$2 \operatorname{arctg} x - \pi = y$$

$$x = \operatorname{tg} \frac{\pi + y}{2}$$

$$x \to +\infty \Rightarrow y \to 0$$

$$= \lim_{y \to 0} \frac{\frac{1}{\operatorname{tg} \frac{\pi + y}{2}}}{y} =$$

$$= \lim_{y \to 0} \operatorname{tg}\left(\frac{\pi + y}{2}\right) = \frac{\operatorname{sen}\left(\frac{\pi}{2} + \frac{y}{2}\right)}{\cos\left(\frac{\pi}{2} + \frac{y}{2}\right)} =$$

$$= \frac{\cos \frac{y}{2}}{-\operatorname{sen} \frac{y}{2}} = \frac{-1}{\operatorname{tg} \frac{y}{2}}$$

$$= \lim_{y \to 0} -\frac{1}{2} \frac{\operatorname{tg} \frac{y}{2}}{y/2} = -\frac{1}{2}$$

$$= \lim_{x \to +\infty} \frac{\frac{1}{x^6}}{(2 \operatorname{arctg} x + \pi)(4 \operatorname{arctg} x^2 + \pi^2)} =$$

$$= \frac{0}{\left(2 \frac{\pi}{2} + \pi\right)\left(4 \frac{\pi^2}{4} + \pi^2\right)} = 0$$

1. Tezquolje

228

$$\lim_{x \to 0} \left(\frac{1}{x^2} \right.$$

$$= \lim_{x \to 0}$$

$$= \lim_{x \to 0} \left(\frac{1}{x} \right.$$

$$= \lim_{x \to 0} \left(\frac{1}{x} \right.$$

$$= (\infty$$

~~1. Tezeuole - Iu populit - Audiotentie~~

228 E. E.

$$\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{tg^2 x} \right) =$$

$$= \cancel{\lim_{x \to 0} \frac{tg^2 x - x^2}{x^2 tg^2 x}}$$

$$= \lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{tg x} \right) \left(\frac{1}{x} + \frac{1}{tg x} \right)$$

$$= \lim_{x \to 0} \left(\frac{1}{x} + tg(\pi + x) \right) \left(\frac{1}{x} + \frac{1}{tg x} \right) =$$

$$= (\infty + 0)(\infty + \infty) = \infty$$

229 E. B.

$$\lim_{x\to+\infty}\left[\sin e^{\frac{\cos x-2x^5-3x+5}{2\sin x+5x^3+3x+7}}\right]^{\frac{2x^2+10x+3}{3x^4-6x^3+7}} =$$

$$= \lim_{x\to+\infty} e^{\frac{2x^2-10x+3}{3x^4-6x^3+7}\log \sin e^{\frac{\cos x}{2\sin x}-\frac{2x^5}{+}\frac{-3x+5}{5x^3+3x+7}}}$$

$$= \lim_{x\to+\infty} e^{\frac{2x^2-10x+3}{3x^4-6x^3+7}\cdot\frac{1}{\sin e^{\frac{\cos x-2x^5-3x+5}{2\sin x+5x^3+3x+7}}}}$$

$$= \lim_{x\to+\infty} e^{\frac{2x^2-10x+3}{3x^4-6x^3+7}\frac{e^{\frac{\cos x-2x^5-3x+5}{2\sin x+5x^3+3x+7}}}{\sin e^{\frac{\cos x-2x^5-3x+5}{2\sin x+5x^3+3x+7}}}\cdot\frac{1}{e^{\frac{\cos x-2x^5-3x+5}{2\sin x+5x^3+3x+7}}}}$$

$$\sin e^{\frac{\cos x-2x^5-3x+5}{2\sin x+5x^3+3x+7}} \quad \log\sin e^{\frac{\cos x-2x^5-3x+5}{2\sin x+5x^3+3x+}} \to 0$$

$$= \lim_{x\to+\infty} e^{\frac{2x^2-10x+3}{3x^4-6x^3+7}\log\frac{\sin e^{\frac{\cos x}{2\sin x}-\frac{2x^5}{+5x^3}-\frac{3x+5}{+3x+7}}}{e^{\frac{\cos x-2x^5-3x+5}{2\sin x+5x^3+3x+7}}}\, e^{\frac{\cos x-2x^5-3x+5}{2\sin x+5x^3+3x+7}}} =$$

$$\lim_{x\to+\infty} e^{\frac{2x^2-10x+3}{3x^4-6x^3+7}\log \sin \frac{e^{\frac{\cos x-2x^5-3x+5}{2\sin x+5x^3+3x+7}}}{e^{\frac{\cos x-2x^5-3x+5}{2\sin x+5x^3+3x+7}}}} + \frac{2x^2-10x+3}{3x^4+6x^3+7}\cdot\frac{\cos x-2x^5-3x+5}{2\sin x+5x^3+3x+7}$$

$$0\cdot 0$$

$$= e^{\lim_{x\to+\infty} e^{\frac{2x^4-10x+3}{3x^4+6x^3+7}\cdot\frac{\frac{\cos x}{x^5}-2-\frac{3}{x^4}+\frac{5}{x^5}}{\frac{2\sin x}{x^3}+5+\frac{3}{x^2}+\frac{7}{x^7}}}} =$$

$$= \lim_{x\to+} e^{\frac{2}{3}\cdot\left(-\frac{2}{5}\right)} = e^{-\frac{4}{15}}$$

230   B2B.

$$\lim_{x\to+\infty} \frac{x^7+\sin x+\cos x}{x^6+2\sqrt[3]{x}+\cos x}\left(\sqrt[3]{\frac{x+1}{x+4}}-\sqrt[3]{\frac{x-1}{x+6}}\right)=$$

$$\lim_{x\to+\infty} \frac{1+\frac{\sin x}{x^7}+\frac{\cos x}{x^7}}{1+2\frac{\sqrt[3]{x}}{x^6}+\frac{\cos x}{x^6}}\cdot x\ \frac{\frac{x+1}{x+4}-\frac{x-1}{x+6}}{\sqrt[3]{\left(\frac{x+1}{x+4}\right)^2}+\sqrt[3]{\frac{(x+1)(x-1)}{(x+4)(x+6)}}+\sqrt[3]{\left(\frac{x-1}{x+6}\right)^2}}=$$

$$\left\{ a^3-b^3=(a-b)\left(a^2+ab+b^2\right)\right\}$$

$$=1\cdot\lim_{x\to+\infty}\frac{x\ \frac{(x+1)(x+6)-(x-1)(x+4)}{(x+4)(x+6)}}{1+1+1}=$$

$$\frac{1}{3}\lim_{x\to+\infty} x \frac{\cancel{x^2}+7x+6-\cancel{x^2}-3x+4}{x^2+10x+24}$$

$$=\frac{1}{3}\lim_{x\to+\infty}\frac{4x^2+10x}{x^2+10x+24}=$$

$$=\frac{1}{3}\cdot 4=\frac{4}{3}$$

1 - Verdi - Aida - Vienna State Opera Chorus & Vienna Philharmonic Orchestra - Herbert Von Karajan - 3 cassette

4 - Vivaldi - Gloria RV 588 - Regensburger Domspatzen Capella Academica Wien - Hanns-Martin Schneidt direttore / Bach - Concerti per organo BWV 593, 5[illegible] K. Richter organista audiocassette + C[illegible]

6 - Vivaldi - "Le quattro stagioni" (Orchestra da Camera di Praga - L. Hlavacek) - La Tempesta di mare (Concerto per flauto op 10 n 2 "La notte" - Musici di Praga - J. Jelinek flauto - dir G. Munclinger - Audiocassette

7 - Vivaldi - Le quattro stagioni - La Tempesta di mare - La caccia - Orchestra da camera italiana - Salvatore Accardo - Audiocassette

8 - Verdi - Otello - Orchestra e coro della Radio di Tokyo - dir - Nino Verchi 2 Audiocassette

231 E.B.

$$\lim_{x\to 0} \frac{\operatorname{arcsen}\left(2-e^{-tgx}-e^{\operatorname{sen}x}\right)}{x\left[(1+tgx)^5-1\right]}=$$

$$=\lim_{x\to 0} \frac{\operatorname{arcsen}\left(2-e^{-tgx}-e^{\operatorname{sen}x}\right)}{\underbrace{2-e^{-tgx}-e^{\operatorname{sen}x}}_{\to 1}} \cdot \frac{2-e^{-tgx}-e^{\operatorname{sen}x}}{x} \cdot \frac{tgx}{\underbrace{(1+tgx)^5-1}_{\to 5}} \cdot \frac{x}{tgx}$$

$$=\frac{1}{5}\cdot\lim_{x\to 0}\left[-\frac{e^{-tgx}-1}{x}-\frac{e^{\operatorname{sen}x}-1}{x}\right]=-\frac{2}{5}$$

232

$$\lim_{x\to 0}\left(1-2x^2\log x\right)^{\frac{1}{2\,\mathrm{arcsen}^2 x}} = \lim_{x\to 0} e^{\frac{1}{2\,\mathrm{arcsen}^2 x}\log\left(1-2x^2\log x\right)}$$

$$= \lim_{x\to 0} e^{\frac{1}{2\,\mathrm{arcsen}^2 x}\frac{\log\left[1-2x^2\log x\right]}{-2x^2\log x}\left(-2x^2\log x\right)}$$

$$= \lim_{x\to 0} e^{\frac{-2x^2}{2\,\mathrm{arcsen}^2 x}\log x\,\frac{\log\left[1-2x^2\log x\right]}{-2x^2\log x}} =$$

$$= \lim_{x\to 0} e^{-1\cdot(-\infty)\cdot 1} = e^{\infty} = \infty$$

1 - West Side Story - Kiri Te Kanawa - José Carreras
Leonard Bernstein CD+ audiocassette
3. Weber

233

$$\lim_{x \to 0^+} \left(\left| x + \log_7 \operatorname{sen} x \right| - \operatorname{cotg} x^2 \right)$$

$$\lim_{x \to 0^+} \operatorname{cotg} x^2 \left(\frac{\left| x + \log_7 \operatorname{sen} x \right|}{\operatorname{cotg} x} - 1 \right)$$

$$= \lim_{x \to 0^+} \operatorname{cotg} x^2 \left(\left| \frac{x}{\operatorname{cotg} x^2} + \frac{\log_7 \operatorname{sen} x}{\operatorname{cotg} x^2} \right| - 1 \right)$$

$$= \lim_{x \to 0^+} \operatorname{cotg} x^2 \left(\left| \underbrace{\frac{x \operatorname{sen} x^2}{\cos x^2}}_{0} + \frac{\operatorname{sen} x^2 \log_7 \operatorname{sen} x}{\cos x^2} \right| - 1 \right)$$

$$= \lim_{x \to 0^+} \operatorname{cotg} x^2 \left(\left| 0 \right| - 1 \right) = \infty$$

$$\ldots \operatorname{tg} x^2)$$

$$\ldots \Big| - 1)$$

$$\ldots \frac{\log_7 \sin x}{\operatorname{cotg} x^2} \Big| - 1)$$

$$\ldots \frac{\log_7 \sin x}{\cos x^2} \Big| - 1)$$

$$\infty$$

234

$$\lim_{x \to +\infty} \left(\cos \left(\operatorname{arctg} \frac{1}{x} \right) \right)^{x} =$$

$$= \lim_{x \to +\infty} e^{x \log \cos \left(\operatorname{arctg} \frac{1}{x} \right)} =$$

$$= \lim_{x \to +\infty} e^{x \log \left[1 + \left(\cos \left(\operatorname{arctg} \frac{1}{x} \right) - 1 \right) \right]}$$

$$= \lim_{x \to +\infty} e^{x \left[\cos \left(\operatorname{arctg} \frac{1}{x} \right) - 1 \right] \frac{\log \left[1 + \left(\cos \operatorname{arctg} \frac{1}{x} - 1 \right) \right]}{\cos \operatorname{arctg} \frac{1}{x} - 1}} =$$

$$= \lim_{x \to +\infty} e^{-x \frac{1 - \cos \left(\operatorname{arctg} \frac{1}{x} \right)}{\operatorname{arctg} \frac{1}{x}} \cdot \operatorname{arctg} \frac{1}{x} \frac{\log \left[1 + \left(\cos \operatorname{arctg} \frac{1}{x} - 1 \right) \right]}{\cos \operatorname{arctg} \frac{1}{x} - 1}}$$

$$= e^{-1 \cdot 0 \cdot 1} = 1$$

235

$$\lim_{x\to+\infty} \left(x - \log_2 (2^x - 4)\right) =$$

$$= \lim_{x\to+\infty} \left(x - \log_2 2^x\left(1 - \frac{4}{2^x}\right)\right) =$$

$$= \lim_{x\to+\infty} \cancel{x} - \cancel{x} - \log_2\left(1 - \frac{4}{2^x}\right) =$$

$$= 0$$

$$\lim_{x\to 0} \frac{e^x -}{tg\, x}$$

$$= \lim_{x\to 0}$$

$$= \lim_{x\to 0} e$$

$$= \lim_{x\to 0} e^{\operatorname{sen} x}$$

$$= e^0 \log e \lim_{x\to 0}$$

236

$$\lim_{x\to 0} \frac{e^x - e^{\operatorname{sen} x}}{\operatorname{tg} x - \operatorname{sen} x} =$$

$$= \lim_{x\to 0} e^{\operatorname{sen} x} \frac{\left(e^{x-\operatorname{sen} x} - 1\right)}{x - \operatorname{sen} x} \frac{x - \operatorname{sen} x}{\operatorname{sen} x \left(\frac{1-\cos x}{\cos x}\right)} =$$

$$= \lim_{x\to 0} e^{\operatorname{sen} x} \frac{e^{x-\operatorname{sen} x} - 1}{x - \operatorname{sen} x} \frac{x - \operatorname{sen} x}{\operatorname{sen} x \, \frac{1-\cos x}{\cos x}} =$$

$$= \lim_{x\to 0} e^{\operatorname{sen} x} \frac{e^{x-\operatorname{sen} x} - 1}{x - \operatorname{sen} x} \cos x \frac{(x - \operatorname{sen} x)}{\frac{\operatorname{sen}^3 x}{x^3}} \frac{1}{\frac{1-\cos x}{x^2}} =$$

$$= \lim_{x\to 0} e^{\operatorname{sen} x} \frac{e^{x-\operatorname{sen} x} - 1}{x - \operatorname{sen} x} \frac{\cos x}{\frac{\operatorname{sen}^2 x}{x^2}} \frac{x - \operatorname{sen} x}{x^3} \frac{x^2}{1-\cos x}$$

$$= e^0 \log e \cdot 1 \cdot \frac{1}{6} \cdot 2 = \frac{1}{3}$$

$$\cancel{= \lim_{x\to 0} e^{\operatorname{sen} x} \frac{e^{x-\operatorname{sen} x} - 1}{x - \operatorname{sen} x} \frac{\cos x}{\frac{\operatorname{sen}^3 x}{x^3}} \frac{x^3 - \operatorname{sen}^3 x}{x^3 (x^2 + x \operatorname{sen} x + \ldots}}$$

$$\cancel{= \lim_{x\to 0} e^{\operatorname{sen} x} \frac{e^{x-\operatorname{sen} x} - 1}{x - \operatorname{sen} x} \frac{\cos x}{\frac{\operatorname{sen}^3 x}{x^3}} \frac{1 - \frac{\operatorname{sen}^3 x}{x^3}}{x^2\left(1 + \frac{\operatorname{sen} x}{x} + \frac{\operatorname{sen}^2 x}{x^2}\right)}}$$

$$\cancel{= \lim_{x\to 0} e^{\operatorname{sen} x} \frac{e^{x-\operatorname{sen} x} - 1}{x - \operatorname{sen} x} \frac{\cos x}{\frac{\operatorname{sen}^3 x}{x^3}} \frac{x\left(1 - \frac{\operatorname{sen} x}{x}\right)}{x^3}} =$$

$$\cancel{= \lim_{x\to 0} e^{\operatorname{sen} x} \frac{e^{x-\operatorname{sen} x} - 1}{x - \operatorname{sen} x} \frac{\cos x}{\frac{\operatorname{sen}^3 x}{x^3}} \frac{1}{x^2} \frac{1 - \frac{\operatorname{sen}^3 x}{x^3}}{(1+}}$$

con l'Hopital

$$\lim_{x\to 0} \frac{x - \operatorname{sen} x}{x^3} = \lim_{x\to 0} \frac{1-\cos x}{3x^2} = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$$

238

$\frac{y^3}{3^{-y}} =$

$$\lim_{x \to \frac{2}{\pi}} \frac{\log\left|\operatorname{tg}\frac{1}{x}\right|}{\log\left|\pi - \frac{2}{x}\right|} =$$

$y = x - \frac{2}{\pi}$ $\quad x = y + \frac{2}{\pi}$ $\quad x \to \frac{2}{\pi} \Rightarrow y \to 0$

$$\cancel{\lim_{y \to 0} \frac{\log\left|\operatorname{tg}\frac{1}{y+\frac{2}{\pi}}\right|}{\log\left|\pi - \frac{2}{y+}\right.}}$$

$y = \pi - \frac{2}{x}$ $\quad \frac{y+\pi}{2} = \frac{1}{x}$; $\quad x \to \frac{2}{\pi} \Rightarrow y \to 0$

$$\lim_{y \to 0} \frac{\log\left|\operatorname{tg}\left(\frac{y}{2} + \frac{\pi}{2}\right)\right|}{\log y} =$$

0

$$= \lim_{y \to 0} \frac{\log\left|\frac{\cos\frac{y}{2}}{\operatorname{sen}\frac{y}{2}}\right|}{\log y} = \lim_{y \to 0} \frac{\log\cos\frac{y}{2} - \log\operatorname{sen}\frac{y}{2}}{\log y}$$

$$= \lim_{y\to 0} \frac{\log \cos \frac{y}{2}}{\log y} - \frac{\log\left(\frac{\operatorname{sen}\frac{y}{2}}{\frac{y}{2}} \frac{y}{2}\right)}{\log y} =$$

$$= \lim_{y\to 0} \frac{\log \cos \frac{y}{2}}{\log y} - \frac{\log\left(\frac{\operatorname{sen}\frac{y}{2}}{\frac{y}{2}}\right)}{\log y} - \frac{\log y}{\log y} + \frac{\log 2}{\log y} =$$

$$= 0 - 0 - 1 + 0 = -1$$