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$$\lim_{x\to+\infty} \sqrt{x^4+2x^2+x+3}\ \operatorname{sen}\frac{\pi(x+1)}{2x+1} - x^2 =$$

$$= \lim_{x\to+\infty} \sqrt{x^4+2x^2+x+3}\cdot\left(1+\frac{\operatorname{sen}\pi(x+1)}{2x+1}-1\right) -$$

$$= \lim_{x\to+\infty} \sqrt{x^4+2x^2+x+3}\ \frac{\left[1+\left(\operatorname{sen}\frac{\pi(x+1)}{2x+1}-1\right)\right]-1}{\underbrace{\operatorname{sen}\frac{\pi(x+1)}{2x+1}-1}_{\downarrow\ 1}}$$

$$= \lim_{x\to+\infty} \sqrt{x^4+2x^2+x+3}\left[\cos\left(\frac{\pi}{2}-\pi\frac{x+1}{2x+1}\right)-1\right]$$

$$= 1+\lim_{x\to+\infty} \sqrt{x^4+2x^2+x+3}\cdot 2\operatorname{sen}^2\frac{\left(\frac{\pi}{2}-\pi\frac{x+1}{2x+1}\right)}{2}$$

$$= 1+\lim_{x\to+\infty} \sqrt{x^4+2x^2+x+3}\cdot 2\operatorname{sen}^2\left(\frac{-\pi}{4x+2}\right)$$

$$\sqrt{x^4+2x^2+x+3} + \sqrt{x^4+2x^2+x+3} - x^2$$

$$\left(\operatorname{sen}\frac{\pi(x+1)}{2x+1}-1\right) + \frac{\cancel{x^4}+2x^2+x+3-\cancel{x^4}}{\sqrt{x^4+2x^2+x+3}+x^2}$$

$$+1 =$$

$$= 1+\lim_{x\to+\infty} \sqrt{x^4+2x^2+x+3}\cdot 2\operatorname{sen}^2\frac{2x\cancel{\pi}+\pi-2\cancel{\pi}x-2\pi}{4x+2}$$

$$= 1+\lim_{x\to+\infty} \pi^2\frac{\sqrt{x^4+2x^2+x+3}}{(4x+2)^2}\cdot 2\,\frac{\operatorname{sen}^2\left(\frac{-\pi}{4x+2}\right)}{\frac{\pi^2}{(4x+2)^2}}$$

$$= 1 + 2\pi^2 \lim_{x \to +\infty} \sqrt{\frac{x^4 + 2x^2 + x + 3}{(16x^2 + 4 + 16x)^2}} =$$

$$= 1 + \frac{\pi^2}{8}$$