(203)

$$\lim_{x\to-\infty}\left[\log(1+3^x)\right]^{\operatorname{arctg}\frac{x^2+1}{x^3}} =$$

$$= \lim_{x\to-\infty} e^{\operatorname{arctg}\frac{x^2+1}{x^3}\,\log\log(1+3^x)} =$$

$$= \lim_{x\to-\infty} e^{\operatorname{arctg}\frac{x^2+1}{x^3}\,\log\left(3^x\cdot\frac{\log(1+3^x)}{3^x}\right)} =$$

$$= \lim_{x\to-\infty} e^{\operatorname{arctg}\frac{x^2+1}{x^3}\left[\log 3^x + \log\frac{\log(1+3^x)}{3^x}\right]} =$$

$$= \lim_{x\to-\infty} e^{\left(\operatorname{arctg}\frac{x^2+1}{x^3}\right)\log 3^x} \quad \lim_{x\to-\infty} e^{\operatorname{arctg}\frac{x^2+1}{x^3}\log\frac{\log(1+3^x)}{3^x}}$$

$$\begin{cases} x\to-\infty \Rightarrow 3^x\to 0 \\ \lim\limits_{x\to-\infty}\frac{\log(1+3^x)}{3^x} = \lim\limits_{y\to 0}\frac{\log(1+y)}{y} = 1 \end{cases}$$

$$= \lim_{x\to\infty} e^{\frac{\operatorname{arctg}\frac{x^2+1}{x^3}}{\frac{x^2+1}{x^3}}\cdot\frac{x^2+1}{x^3}\cdot\log_3 3^x \log 3} \cdot e^0$$

$$= \lim_{x\to+\infty} e^{\frac{\operatorname{arctg}\frac{x^2+1}{x^3}}{\frac{x^2+1}{x^3}}\cdot\frac{x^2+1}{x^3}\times\log 3} =$$

$$= e^{1\cdot 1\cdot\log 3} = e^{\log 3} = 3$$