

Q10

$$\lim_{x \rightarrow +\infty} \left(\frac{x^2+x+3}{x^2+3x+1} \right)^{\frac{x^2+1}{x+1}}$$

$$\lim_{x \rightarrow +\infty} \frac{x^2+1}{x+1} \log \frac{x^2+x+3}{x^2+3x+1}$$

$$= \lim_{x \rightarrow +\infty} \frac{x^2+1}{x+1} \log \left[1 + \left(\frac{x^2+x+3}{x^2+3x+1} - 1 \right) \right]$$

$$= \lim_{x \rightarrow +\infty} \frac{x^2+1}{x+1} \log \left(1 + \frac{x^2+x+3 - x^2 - 3x - 1}{x^2+3x+1} \right)$$

$$= \lim_{x \rightarrow +\infty} \frac{x^2+1}{x+1} \log \left(1 + \frac{-2x+2}{x^2+3x+1} \right)$$

$$= \lim_{x \rightarrow +\infty} \frac{x^2+1}{x+1} \log \left[1 + \left(\frac{-2x+2}{x^2+3x+1} - 1 \right) \right]$$

$$= \lim_{x \rightarrow +\infty} \frac{x^2+1}{x+1} \left(-\frac{2x-2}{x^2+3x+1} \right) \log \left(1 - \frac{2x-2}{x^2+3x+1} \right)$$

$$= e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

2.11

$$\lim_{x \rightarrow +\infty} \left(\frac{x \log x + x + 3}{x \log x + 2} \right)^{\frac{x \log x + 1}{2x + 5}}$$

$$= \lim_{x \rightarrow +\infty} e^{\frac{x \log x + 1}{2x + 5} \log \frac{x \log x + x + 3}{x \log x + 2}}$$

$$= \lim_{x \rightarrow +\infty} e^{\frac{x \log x + 1}{2x + 5} \log \left[1 + \left(\frac{x \log x + x + 3}{x \log x + 2} - 1 \right) \right]}$$

M N O P