217

$$\lim_{x\to+\infty}\left(3e^{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}-2\right)^x=$$

$$=\lim_{x\to+\infty} e^{x\log\left(3e^{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}-2\right)}=$$

$$=\lim_{x\to+\infty} e^{x\log\left[1+\left(3e^{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}-3\right)\right]}=$$

$$=\lim_{x\to+\infty} e^{x\left(3e^{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}-3\right)\frac{\log\left[1+\left(3e^{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}-3\right)\right]}{3e^{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}-3}}=$$

$$=\lim_{x\to+\infty} e^{3x\cdot\frac{\left(e^{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}-1\right)}{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}\cdot\left(\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}\right)\frac{\log\left[1+\left(3e^{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}-3\right)\right]}{3e^{\operatorname{arctg}x-\frac{\pi x^2+1}{2x^2+5}}-3}}$$

vedi foglio allegato

$$=\lim_{x\to+\infty} e^{3\left(-\frac{1}{2}\right)}$$

$$= e^{3\left(-\frac{1}{2}\right)}=e^{-\frac{3}{2}}$$

$$\lim_{x\to+\infty} x\left(\operatorname{arctg} x - \frac{\pi x^2+1}{2x^2+5}\right)$$

$$\lim_{x\to+\infty} x\,\frac{2x^2\operatorname{arctg}x + 5\operatorname{arctg}x - \pi x^2 - 1}{2x^2+5} =$$

$$= \lim_{x\to+\infty} x\,\frac{\cancel{x^2}\left[2\operatorname{arctg}x - \pi + \frac{5\operatorname{arctg}x}{x^2} - \frac{1}{x^2}\right]}{\cancel{x^2}\left(2+\frac{5}{x}\right)} =$$

$$= \lim_{x\to+\infty} \frac{x\left(2\operatorname{arctg}x - \pi\right) + \overbrace{\frac{5\operatorname{arctg}x}{x}}^{\to 0} - \overbrace{\frac{1}{x}}^{\to 0}}{\underbrace{\left(2+\frac{5}{x}\right)}_{\to 2}} =$$

$$= \frac{1}{2}\lim_{x\to+\infty} x\left(2\operatorname{arctg}x - \pi\right)$$

$$y = 2\operatorname{arctg}x - \pi \qquad x = \operatorname{tg}\frac{y+\pi}{2}$$

$$= \frac{1}{2}\lim_{y\to 0} \operatorname{tg}\left(y + \frac{\pi}{2}\right)\cdot y \;=\; -\frac{1}{2}\lim \frac{y}{\operatorname{tg} y} = -\frac{1}{2}$$