227 esercizi d'esame

$$\lim_{x\to+\infty} \frac{\left(\arcsin \frac{1}{x}\right)^7}{16\,\mathrm{arctg}^4 x - \pi^4} =$$

$$= \lim_{x\to+\infty} \frac{\left(\arcsin \frac{1}{x}\right)^7}{\frac{1}{x^7}} \cdot \frac{\frac{1}{x^7}}{16\,\mathrm{arctg}^4 x - \pi^4}$$

$$= 1 \cdot \lim_{x\to+\infty} \frac{\frac{1}{x^7}}{16\,\mathrm{arctg}^4 x - \pi^4} =$$

$$= \lim_{x\to+\infty} \frac{\frac{1}{x^7}}{(4\,\mathrm{arctg}^2 x - \pi^2)(4\,\mathrm{arctg}^2 x + \pi^2)} =$$

$$= \lim_{x\to+\infty} \frac{\frac{1}{x^7}}{(2\,\mathrm{arctg}\, x - \pi)(2\,\mathrm{arctg}\, x + \pi)(4\,\mathrm{arctg}^2 x + \pi^2)}$$

$$\left\{ \lim_{x\to+\infty} \frac{\frac{1}{x}}{2\,\mathrm{arctg}\, x - \pi} \right.$$

$2\,\mathrm{arctg}\, x - \pi = y$

$x = \mathrm{tg}\,\frac{\pi + y}{2}$

$x \to +\infty \Rightarrow y \to 0$

$$= \lim_{y\to 0} \frac{\frac{1}{\mathrm{tg}\frac{\pi+y}{2}}}{y} =$$

$$= \lim_{y\to 0} \mathrm{tg}\left(\frac{\pi+y}{2}\right) = \frac{\mathrm{sen}\left(\frac{\pi}{2} + \frac{y}{2}\right)}{\cos\left(\frac{\pi}{2} + \frac{y}{2}\right)} =$$

$$= \frac{\cos \frac{y}{2}}{-\mathrm{sen}\,\frac{y}{2}} = \frac{-1}{\mathrm{tg}\,\frac{y}{2}}$$

$$= \lim_{y \to 0} -\frac{1}{2} \frac{\operatorname{tg} \frac{y}{2}}{\frac{y}{2}} = -\frac{1}{2}$$

$$= \lim_{x \to +\infty} \frac{\frac{1}{x^6}}{(2 \operatorname{arctg} x + \pi)(4 \operatorname{arctg} x^2 + \pi^2)} =$$

$$= \frac{0}{\left(2 \frac{\pi}{2} + \pi\right)\left(4 \frac{\pi^2}{4} + \pi^2\right)} = 0$$

1. [illegible]

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$$\lim_{x \to 0} \left(\frac{1}{x^2} \right.$$

$$= \lim_{x \to 0}$$

$$= \lim_{x \to 0} \left(\frac{1}{x} \right.$$

$$= \lim_{x \to 0} \left(\frac{1}{x} \right.$$

$$= (\infty$$