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$$\lim_{x\to 0} \frac{\log\left[1+\operatorname{senh}\left(tg^2x-sen^2x\right)\right]}{\left(\sqrt{1+x}-1\right)\operatorname{arctg}^3x} =$$

$$= \lim_{x\to 0} \frac{\log\left[1+\operatorname{senh}\left(tg^2x-sen^2x\right)\right]}{\operatorname{senh}\left(tg^2x-sen^2x\right)} \cdot \frac{\operatorname{senh}\left(tg^2x-sen^2x\right)}{tg^2x-sen^2x} \cdot \frac{tg^2x-sen^2x}{\left(\sqrt{1+x}-1\right)\operatorname{arctg}^3x} =$$

$$\left\{ \lim_{x\to 0} tg^2x - sen^2x = 0 \rightarrow \lim_{y\to 0} \frac{\operatorname{senh} y}{y} = 1 \right\}$$

$$= 1\cdot 1\cdot \lim_{x\to 0} \frac{tg^2x-sen^2x}{\left(\sqrt{1+x}-1\right)\operatorname{arctg}^3x} = \lim_{x\to 0} \frac{1}{x^4}\,\frac{tg^2x-sin^2x}{\frac{\sqrt{1+x}-1}{x}\;\frac{\operatorname{arctg}^3x}{x^3}} =$$

$$= \lim_{x \to 0} \frac{\dfrac{tg^2 x - \sin^2 x}{x^4}}{\frac{1}{2} \cdot 1} =$$

$$= 2 \lim_{x \to 0} \frac{1}{x^4}\left(\frac{\sin^2 x}{\cos^2 x} - \sin^2 x\right) =$$

$$= 2 \lim_{x \to 0} \frac{\sin^2 x}{x^4}\left(\frac{1 - \cos^2 x}{\cos^2 x}\right) =$$

$$= 2 \lim_{x \to 0} \frac{\sin^2 x}{x^2} \cdot \frac{\sin^2 x}{x^2} \frac{1}{\cos^2 x} = 2$$

$$\lim_{x \to +\infty} \left[\frac{x^2+x+3}{x-1} \right.$$

$$= \lim_{x \to +\infty} \frac{x^2+x+3}{x-1}$$

$$= \lim_{x \to +\infty} 5x^2$$

$$= \lim_{x \to +\infty} \frac{(x^2+x+3}{(}$$

$$= \lim_{x \to +\infty} \frac{5x^2}{5}$$