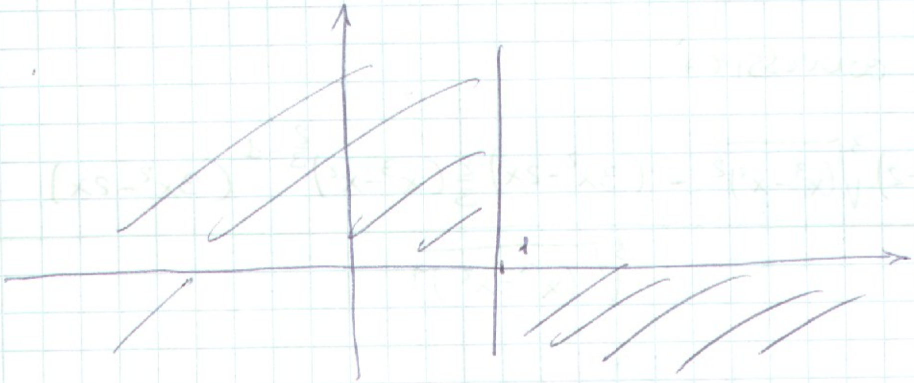
$f(x) = \sqrt[3]{x^3 - x^2}$

CE: $\mathbb{R}$

POSITIVITÀ

$f(x) > 0 \qquad x^3 - x^2 > 0 \; ; \; x^2(x-1) > 0 \; ; \; x > 1$



INTERSEZIONI CON GLI ASSI

asse x $\Rightarrow$ $y = 0 \qquad x = 0 \; , \; x = 1$

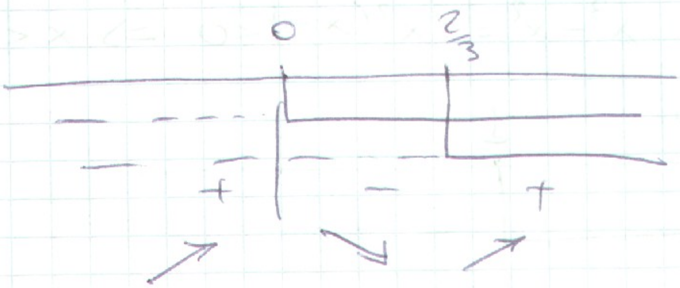
asse y - $x = 0 \qquad y = 0$

MAX E MIN

$f'(x) = \frac{1}{3}(x^3 - x^2)^{\frac{1}{3} - 1} (3x^2 - 2x) =$

$= \frac{1}{3} \frac{3x^2 - 2x}{\sqrt[3]{(x^3 - x^2)^2}}$

$f'(x) = 0 \qquad x(3x - 2) = 0 \qquad x = 0 \; ; \; x = \frac{2}{3}$



$f(0) = 0$

$x = \frac{2}{3}$ min $\qquad f\left(\frac{2}{3}\right) = \sqrt[3]{\left(\frac{2}{3}\right)^3 - \left(\frac{2}{3}\right)^2} = \sqrt[3]{\frac{8}{27} - \frac{4}{9}} =$

$$= \sqrt[3]{\frac{8-12}{27}} = \sqrt[3]{-\frac{4}{27}} = -\frac{\sqrt[3]{4}}{3}$$

in $x=0$ e $x=1$

la derivata è infinita

$\lim_{x\to 0^-} f'(x) = +\infty \qquad \lim_{x \to 0^+} f'(x) = -\infty$

CONCAVITÀ E CONVESSITÀ

$$f''(x) = \frac{1}{3} \frac{(6x-2)\sqrt[3]{(x^3-x^2)^2} - (3x^2-2x)\frac{2}{3}(x^3-x^2)^{\frac{2}{3}-1}(3x^2-2x)}{\sqrt[3]{(x^3-x^2)^4}} =$$

$$= \frac{1}{3} \frac{(6x-2)\sqrt[3]{(x^3-x^2)^2} - x^2(9x^2+4-12x)\frac{2}{3}\frac{1}{\sqrt{x^3-x^2}}}{\sqrt[3]{(x^3-x^2)^4}} =$$

$$= \frac{1}{3} \frac{(6x-2)(x^3-x^2) - 6x^4 - \frac{8}{3}x^2 + 8x^3}{\sqrt[3]{x^3-x^2}\,\sqrt[3]{(x^3-x^2)^4}}$$

$$= \frac{1}{3} \frac{6x^4-6x^3-2x^3+2x^2-6x^4-\frac{8}{3}x^2+8x^3}{\sqrt[3]{x^3-x^2}\,\sqrt[3]{(x^3-x^2)^4}} = \frac{1}{3} \frac{-\frac{8}{3}x^2}{\sqrt[3]{x^3-x^2}\,\sqrt[3]{(x^3-x^2)^4}}$$

$f'(x) > 0 \Rightarrow \quad x^3 - x^2 = x^2(x-1) < 0 \Rightarrow x < 1$

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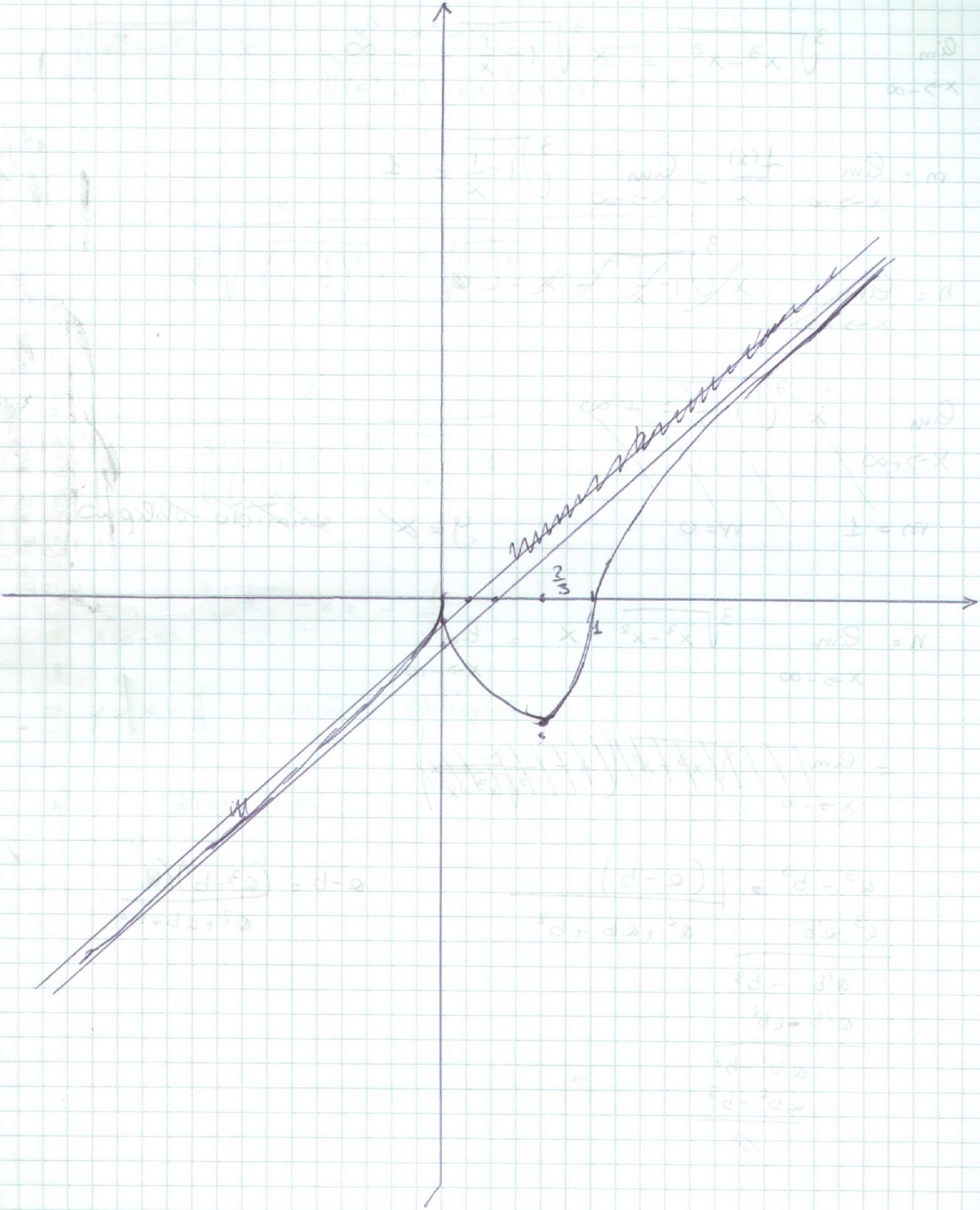

2/3
1

$$\lim_{x\to-\infty} \sqrt[3]{x^3-x^2} = x\sqrt[3]{1-\frac{1}{x}} = -\infty$$

$$m = \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \sqrt[3]{1-\frac{1}{x}} = 1$$

$$\cancel{n = \lim_{x\to-\infty} x\sqrt[3]{1-\frac{1}{x}} - x = }$$

$$\cancel{\lim_{x\to+\infty} x\sqrt[3]{1-\frac{1}{x}} = +\infty}$$

$m = 1$ $\qquad n = 0$ $\qquad$ $y = \cancel{x}$ ~~asíntota oblicua~~

$$n = \lim_{x\to-\infty} \sqrt[3]{x^3-x^2} - x = \lim_{x\to-\infty}$$

$$= \lim_{x\to-\infty} \cancel{\sqrt[3]{x^3-x^2}\left(x - \frac{x}{\sqrt[3]{x^3-x^2}}\right)}$$

$$\begin{array}{l|l} a^3 - b^3 & (a-b) \\ \hline a^3 - a^2b & a^2+ab+b^2 \\ \hline \quad a^2b - b^3 & \\ \quad a^2b + ab^2 & \\ \hline \qquad ab^2 - b^3 & \\ \qquad ab^2 - b^3 & \\ \hline \qquad // & \end{array}$$

$$a - b = \frac{(a^3-b^3)(a}{a^2+ab+b^2}$$

$$\sqrt[3]{x^3-x^2} - x = \frac{x^3-x^2-x^3}{\sqrt[3]{(x^3-x^2)^2} + x\sqrt[3]{x^3-x^2} + x^2} =$$

$$= -\frac{x^2}{x^2\sqrt[3]{\left(1-\frac{1}{x}\right)^2} + x^2\sqrt[3]{1-\frac{1}{x}} + x^2} =$$

$$= -\frac{1}{\sqrt[3]{\left(1-\frac{1}{x}\right)^2} + \sqrt[3]{1-\frac{1}{x}} + 1}$$

$$n = \lim_{x \to -\infty} \sqrt[3]{x^3-x^2} - x = -\frac{1}{3}$$

$y = x - \frac{1}{3}$ asintoto obliquo

$$\lim_{x \to +\infty} f(x) = +\infty$$

$m = 1$ $\qquad n = -\frac{1}{3}$