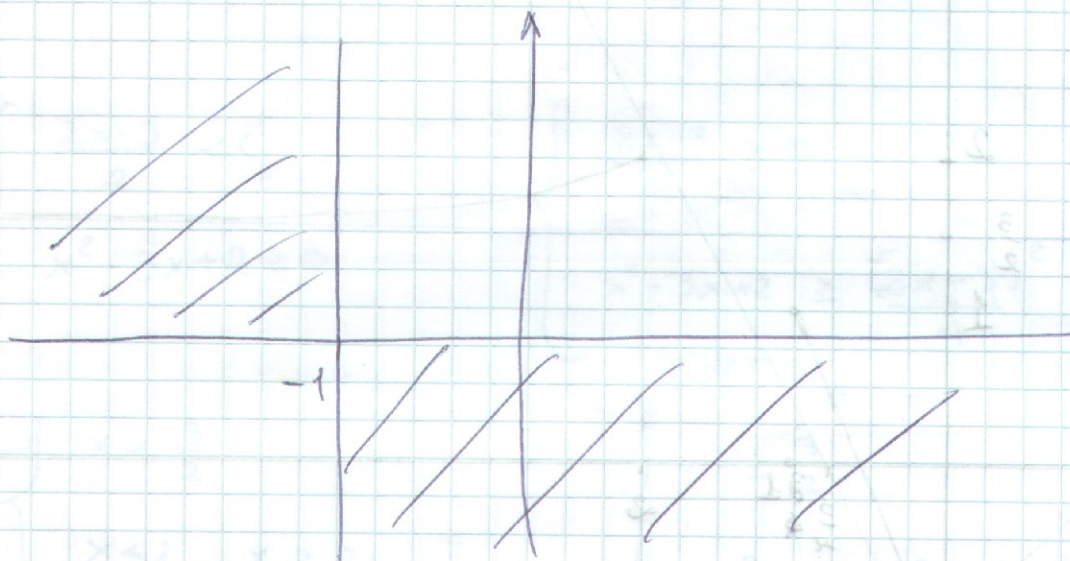


$$f(x) = \sqrt[3]{(x-3)^2(x+1)}$$

$$ce: \mathbb{R}$$

POSITIVITÀ:

$$f(x) \geq 0 \quad \left\{ \begin{array}{l} (x-3)^2 \geq 0 \\ x+1 > 0 \end{array} \right. \quad \left\{ \begin{array}{l} \forall x \neq 3 \\ x > -1 \end{array} \right.$$



INTERSEZIONE CON GLI ASSI

$$\text{asse } y \quad x=0 \Rightarrow y = \sqrt[3]{9}$$

$$\text{asse } x \quad y=0 \Rightarrow x=3, x=-1$$

$$\lim_{x \rightarrow -\infty} f(x) = -\infty$$

$$x \rightarrow -\infty$$

$$m = \lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \lim_{x \rightarrow -\infty} \sqrt[3]{\frac{(x-3)^2(x+1)}{x^3}} = 1$$

$$n = \lim_{x \rightarrow -\infty} \sqrt[3]{(x-3)^2(x+1)} - x =$$

$$= \lim_{y \rightarrow \infty} \sqrt[3]{(-y-3)^2(-y+1)} + y =$$

$$= \lim_{y \rightarrow \infty} -\sqrt[3]{(y+3)^2(y-1)} + y =$$

$$= \lim_{y \rightarrow \infty} y - \sqrt[3]{(y+3)^2(y-1)} =$$

$$= \lim_{y \rightarrow \infty} \frac{y^3 - (y+3)^2(y-1)}{y^2 + y \sqrt[3]{(y+3)^2(y-1)} + \sqrt[3]{(y+3)^4(y-1)^2}} =$$

$$= \lim_{y \rightarrow \infty} \frac{y^3 - (y^2 + 9 + 6y)(y-1)}{y^2 + y^2 \sqrt[3]{\left(1 + \frac{3}{y}\right)^2 \left(1 - \frac{1}{y}\right)} + y^2 \sqrt[3]{\left(1 + \frac{3}{y}\right)^2 \left(1 - \frac{1}{y}\right)^2}} =$$

$$= \lim_{y \rightarrow \infty} \frac{\cancel{y^3} - \cancel{y^3} - 9y - 6y^2 + y^2 + 9 + 6y}{y^2 \left[ 1 + \sqrt[3]{\left(1 + \frac{3}{y}\right)^2 \left(1 - \frac{1}{y}\right)} + \sqrt[3]{\left(1 + \frac{3}{y}\right)^2 \left(1 - \frac{1}{y}\right)^2} \right]} =$$

$$= \lim_{y \rightarrow \infty} \frac{-5 - \frac{3}{y} + \frac{9}{y^2}}{1 + \sqrt[3]{\left(1 + \frac{3}{y}\right)^2 \left(1 - \frac{1}{y}\right)} + \sqrt[3]{\left(1 + \frac{3}{y}\right)^2 \left(1 - \frac{1}{y}\right)^2}} = -\frac{5}{3}$$

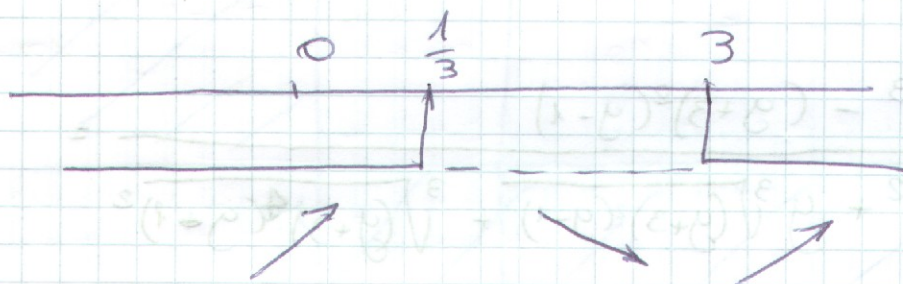
esimio obliquo :  $y = x - \frac{5}{3}$

MAX E MIN.

$$f'(x) = \frac{2(x-3)(x+1) + (x-3)^2}{3 \sqrt[3]{(x-3)^4 (x+1)^2}} = \frac{2x^2 + 2x - 6x - 6 + x^2 + 9 - 6x}{3 \sqrt[3]{(x-3)^4 (x+1)^2}}$$

$$= \frac{3x^2 - 10x + 3}{3 \sqrt[3]{(x-3)^4 (x+1)^2}} \rightarrow x = \frac{5 \pm \sqrt{25-9}}{3} = \frac{5 \pm 4}{3} \begin{cases} \frac{1}{3} \\ \frac{9}{3} = 3 \end{cases}$$

$f'(x) > 0$        $x < \frac{1}{3}, x > 3$



~~$x = \frac{1}{3}$~~   $x = \frac{1}{3}$  punto de MAX

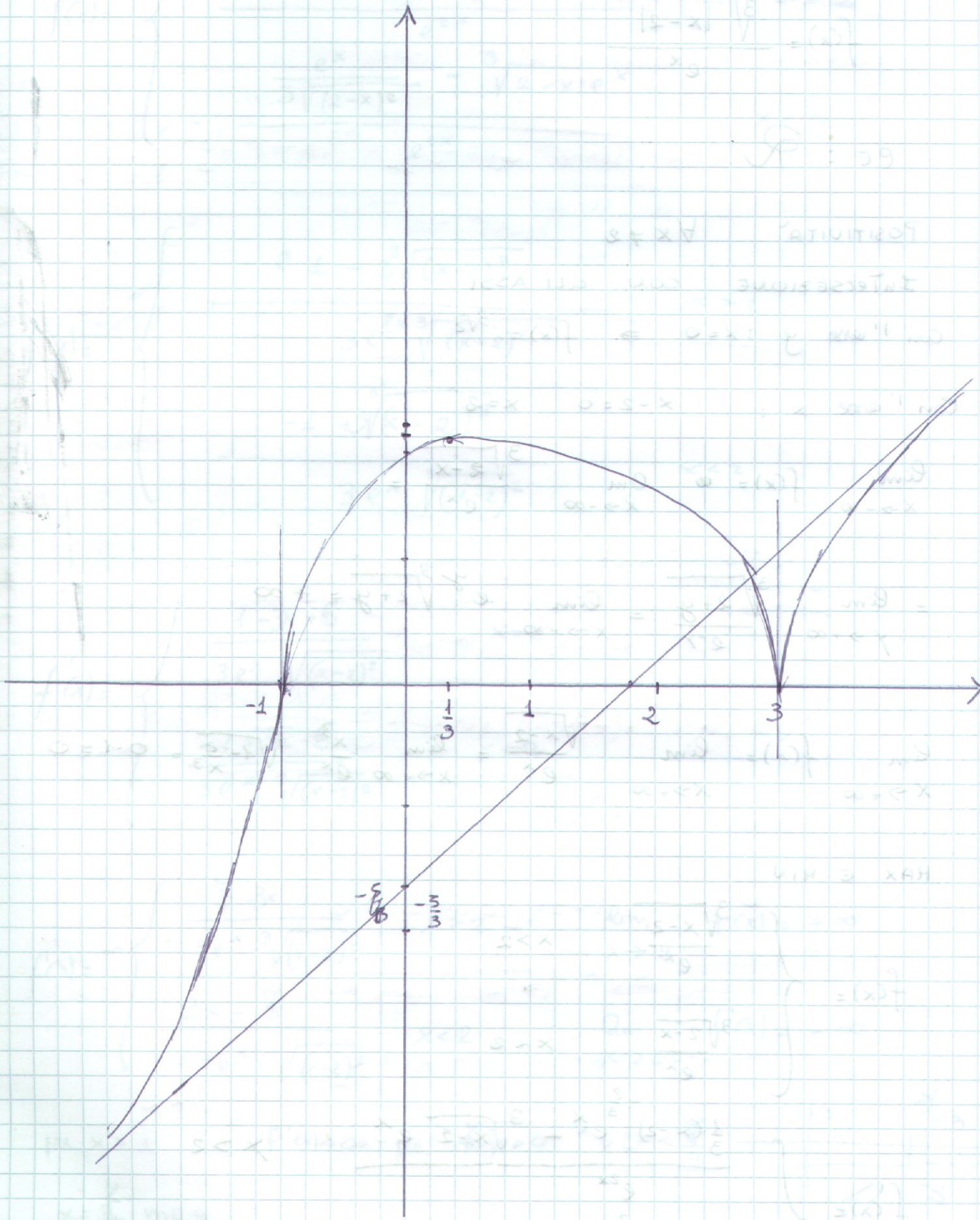
in  $x=1$  e  $x=3$  la  $f'(x) \rightarrow$  non è definita  $\rightarrow \pm \infty / \pm \infty$

$$\lim_{x \rightarrow 1} f'(x) = -\infty$$

$$\lim_{x \rightarrow 3^-} f'(x) = -\infty$$

$$\lim_{x \rightarrow 3^+} f'(x) = +\infty$$

$$f\left(\frac{1}{3}\right) = \sqrt[3]{\left(\frac{1}{3} - 3\right)^2 \left(\frac{1}{3} + 1\right)} = \sqrt[3]{\frac{64}{9} \cdot \frac{4}{3}} = \frac{\sqrt[3]{256}}{3}$$



$f(x) = x^3 - 3x^2 + 2x$   
 $f'(x) = 3x^2 - 6x + 2$   
 $f''(x) = 6x - 6$

(positiv)  $\Delta x \neq 0$   
 INTERSEKTIONEN  
 $f(x) = 0 \Rightarrow x^3 - 3x^2 + 2x = 0$   
 $x(x^2 - 3x + 2) = 0$   
 $x(x-1)(x-2) = 0$   
 $x = 0, 1, 2$

$f'(x) = 0 \Rightarrow 3x^2 - 6x + 2 = 0$   
 $x = \frac{6 \pm \sqrt{36 - 24}}{6} = \frac{6 \pm \sqrt{12}}{6} = \frac{6 \pm 2\sqrt{3}}{6} = 1 \pm \frac{\sqrt{3}}{3}$   
 $x \approx 1.577, 0.423$

$f''(x) > 0 \Rightarrow 6x - 6 > 0 \Rightarrow x > 1$   
 $f''(x) < 0 \Rightarrow 6x - 6 < 0 \Rightarrow x < 1$

$f(1) = 1 - 3 + 2 = 0$   
 $f(2) = 8 - 12 + 4 = 0$   
 $f(0) = 0$

$f(x) = x^3 - 3x^2 + 2x$   
 $f'(x) = 3x^2 - 6x + 2$   
 $f''(x) = 6x - 6$