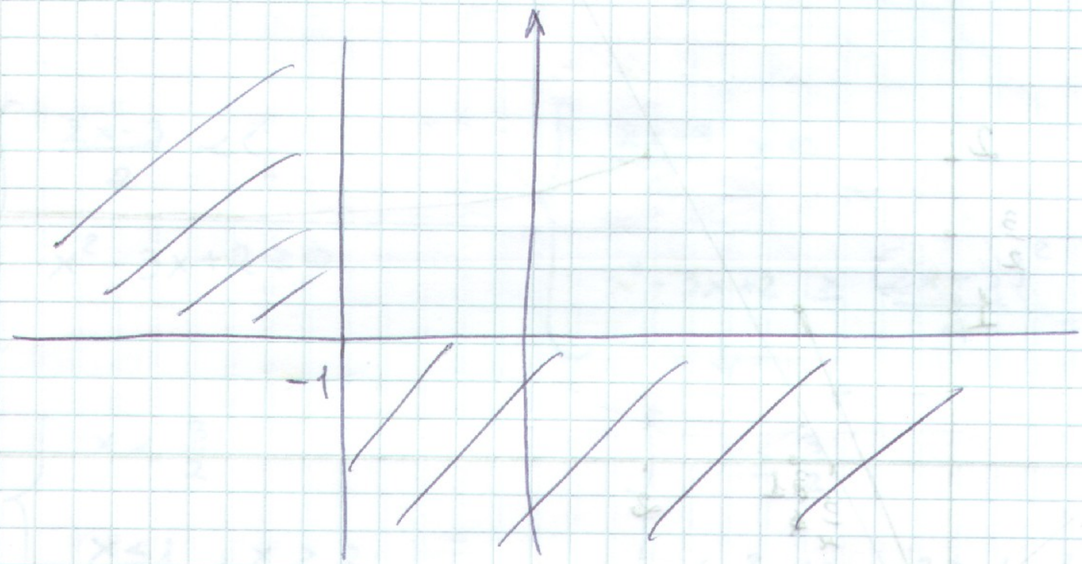
$$f(x) = \sqrt[3]{(x-3)^2(x+1)}$$

CE : $\mathbb{R}$

POSITIVITÀ :

$$f(x) \geq 0 \quad \begin{cases} (x-3)^2 \geq 0 \\ x+1 > 0 \end{cases} \quad \begin{cases} \forall x \neq 3 \\ x > -1 \end{cases}$$



INTERSEZIONE CON GLI ASSI

asse y $\quad x = 0 \Rightarrow \quad y = \sqrt[3]{9}$

asse x $\quad y = 0 \Rightarrow \quad x = 3 \; , \; x = -1$

$$\lim_{x \to -\infty} f(x) = -\infty$$

$$m = \lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \sqrt[3]{\frac{(x-3)^2(x+1)}{x^3}} = 1$$

$$n = \lim_{x \to -\infty} \sqrt[3]{(x-3)^2(x+1)} - x =$$

$$= \lim_{y \to \infty} \sqrt[3]{(-y-3)^2(-y+1)} + y =$$

$$= \lim_{y \to +\infty} -\sqrt[3]{(y+3)^2(y-1)} + y =$$

$$= \lim_{y \to +\infty} y - \sqrt[3]{(y+3)^2(y-1)} =$$

$$= \lim_{y \to +\infty} \frac{y^3 - (y+3)^2(y-1)}{y^2 + y\sqrt[3]{(y+3)^2(y-1)} + \sqrt[3]{(y+3)^4(y-1)^2}} =$$

$$= \lim_{y \to +\infty} \frac{y^3 - (y^2+9+6y)(y-1)}{y^2 + y^2\sqrt[3]{\left(1+\frac{3}{y}\right)^2\left(1-\frac{1}{y}\right)} + y^2\sqrt[3]{\left(1+\frac{3}{y}\right)^2\left(1-\frac{1}{y}\right)}^2} =$$

$$= \lim_{y \to +\infty} \frac{\cancel{y^3} - \cancel{y^3} - 9y - 6y^2 + y^2 + 9 + 6y}{y^2\left[1 + \sqrt[3]{\left(1+\frac{3}{y}\right)^2\left(1-\frac{1}{y}\right)} + \sqrt[3]{\left(1+\frac{3}{y}\right)^2\left(1-\frac{1}{y}\right)}^2\right]} =$$

$$= \lim_{y \to +\infty} \frac{-5 - \frac{3}{y} + \frac{9}{y^2}}{1 + \sqrt[3]{\left(1+\frac{3}{y}\right)^2\left(1-\frac{1}{y}\right)} + \sqrt[3]{\left(1+\frac{3}{y}\right)^2\left(1-\frac{1}{y}\right)^2}} = -\frac{5}{3}$$

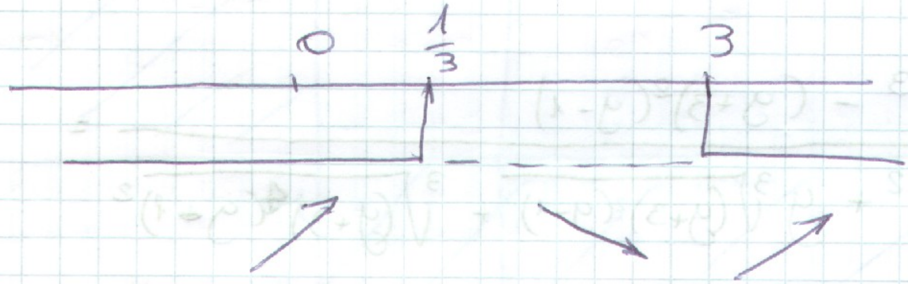
asintoto obliquo : $y = x - \frac{5}{3}$

MAX E MIN.

$$f'(x) = \frac{2(x-3)(x+1) + (x-3)^2}{3\sqrt[3]{(x-3)^4(x+1)^2}} = \frac{2x^2+2x-6x-6+x^2+9-6x}{3\sqrt[3]{(x-3)^4(x+1)^2}} =$$

$$= \frac{3x^2-10x+3}{3\sqrt[3]{(x-3)^4(x+1)^2}} \longrightarrow x = \frac{5 \pm \sqrt{25-9}}{3} = \frac{5 \pm 4}{3} \begin{cases} \frac{1}{3} \\ \frac{9}{3} = 3 \end{cases}$$

$f'(x) > 0 \qquad x < \frac{1}{3}, \ x > 3$



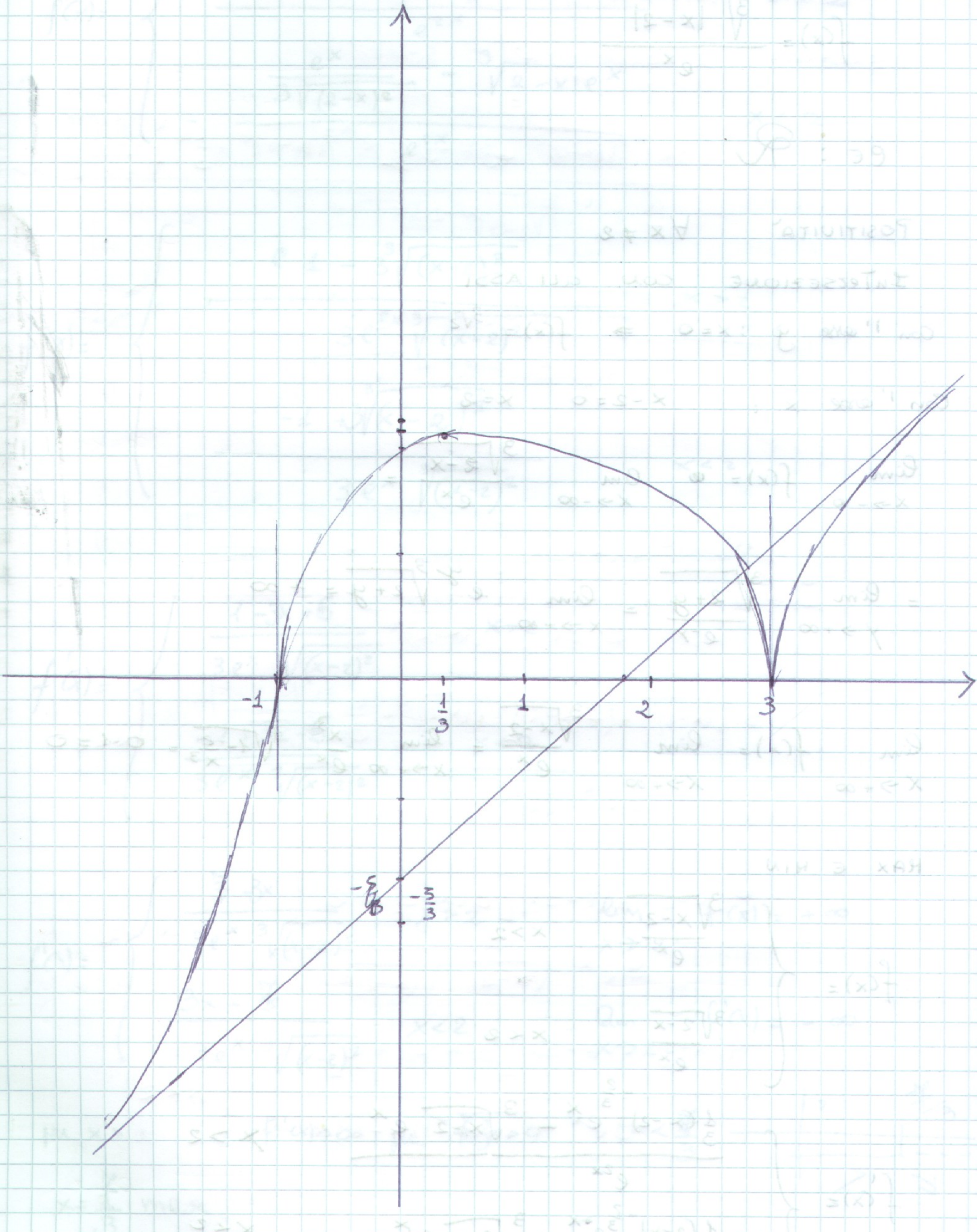
$\cancel{f(\frac{1}{3})}$ $\quad x = \frac{1}{3}$ punto di MAX

in $x = 1$ e $x = 3$ la $f'(x) \rightarrow$ non è definita ~~e → ∞~~

$$\lim_{x \to 1} f'(x) = -\infty \qquad \lim_{x \to 3^-} f'(x) = -\infty$$

$$\lim_{x \to 3^+} f'(x) = +\infty$$

$$f\left(\frac{1}{3}\right) = \sqrt[3]{\left(\frac{1}{3} - 3\right)^2 \left(\frac{1}{3} + 1\right)} = \sqrt[3]{\frac{64}{9} \cdot \frac{4}{3}} = \frac{\sqrt[3]{256}}{3}$$



-1
$\frac{1}{3}$
1
2
3
$-\frac{5}{3}$