$$f(x) = \frac{\sqrt[3]{|x-2|}}{e^x}$$

CE : $\mathbb{R}$

POSITIVITÀ $\forall x \neq 2$

INTERSEZIONE CON GLI ASSI

Con l'asse y : $x = 0 \Rightarrow f(x) = \sqrt[3]{2}$

Con l'asse x : $x - 2 = 0 \quad x = 2$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{\sqrt[3]{2-x}}{e^x} =$$

$$= \lim_{y \to +\infty} \frac{\sqrt[3]{2+y}}{e^{-y}} = \lim_{x \to +\infty} e^y \sqrt[3]{2+y} = +\infty$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{\sqrt[3]{x-2}}{e^x} = \lim_{x \to +\infty} \frac{x^3}{e^x} \sqrt[3]{1 - \frac{2}{x^3}} = 0 \cdot 1 = 0$$

MAX E MIN

$$f(x) = \begin{cases} \dfrac{\sqrt[3]{x-2}}{e^x} & x > 2 \\[2ex] \dfrac{\sqrt[3]{2-x}}{e^x} & x < 2 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{\frac{1}{3}(x-2)^{-\frac{2}{3}} e^x - \sqrt[3]{x-2}\, e^x}{e^{2x}} & x > 2 \\[2ex] -\frac{1}{3}(2-x)^{-\frac{2}{3}} e^{x} - \sqrt[3]{2-x}\, e^x & x < 2 \end{cases}$$

$$f(x) = \frac{\quad}{e^x}$$

$C.E.: \mathbb{R}$

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$$f'(x) = \begin{cases} \dfrac{\frac{1}{3}(x-2)^{-\frac{2}{3}} e^x - \sqrt[3]{x-2}\, e^x}{e^{2x}} & x > 2 \\[2ex] \dfrac{-\frac{1}{3}(2-x)^{-\frac{2}{3}} e^{x} - \sqrt[3]{2-x}\, e^x}{e^{2x}} & x < 2 \end{cases}$$

$$f'(x)=\begin{cases} \dfrac{\dfrac{1}{3}\dfrac{e^x}{\sqrt[3]{(x-2)^2}} - e^x\sqrt[3]{x-2}}{e^{2x}} & x>2 \\[2ex] -\dfrac{\dfrac{e^x}{3\sqrt[3]{(2-x)^2}} - \sqrt[3]{2-x}\,e^x}{e^{2x}} & x<2 \end{cases}$$

$$f'(x)=\begin{cases} \dfrac{\cancel{e}\;1-3\sqrt[3]{(x-2)^3}}{3e^{\cancel{2}x}\sqrt[3]{(x-2)^2}} & x>2 \\[2ex] \dfrac{-1+3\sqrt[3]{(x-2)^3}}{3e^x\sqrt[3]{(x-2)^2}} & x<2 \end{cases}$$

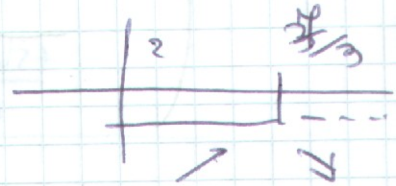
$$f'(x)=\begin{cases} \dfrac{1-3x+6}{3e^x\sqrt[3]{(x-2)^2}} & x>2 \\[2ex] \dfrac{-1+3x-6}{3e^x\sqrt[3]{(x-2)^2}} & x<2 \end{cases}$$

$$f'(x)=\begin{cases} \dfrac{7-3x}{3e^x\sqrt[3]{(x-2)^2}} & x>2 \\[2ex] \dfrac{3x-7}{3e^x\sqrt[3]{(x-2)^2}} & x<2 \end{cases}$$

$$\lim_{x\to 2^+} f'(x) = +\infty$$

$$\lim_{x\to 2^-} f'(x) = -\infty$$

per $x>2$ $\quad f'(x)>0$ per $7-3x>0$ $\quad x<\dfrac{7}{3}$



$x=\dfrac{7}{3}$ max

$f'(x)>0$ per $3x-7>0$ $\quad x>\dfrac{7}{3}$

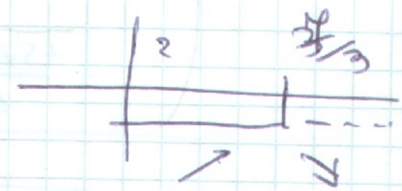
$$\begin{cases} - \dfrac{\dfrac{e^x}{3\sqrt[3]{(2-x)^2}} - \sqrt[3]{2-x}\, e^x}{e^{2x}} & x < 2 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{\cancel{e}\ 1 - 3\sqrt[3]{(x-2)^3}}{3e^{\cancel{2}x}\sqrt[3]{(x-2)^2}} & x > 2 \\[2ex] \dfrac{-1 + 3\sqrt[3]{(x-2)^3}}{3e^x\sqrt[3]{(x-2)^2}} & x < 2 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{1 - 3x + 6}{3e^x\sqrt[3]{(x-2)^2}} & x > 2 \\[2ex] \dfrac{-1 + 3x - 6}{3e^x\sqrt[3]{(x-2)^2}} & x < 2 \end{cases}$$

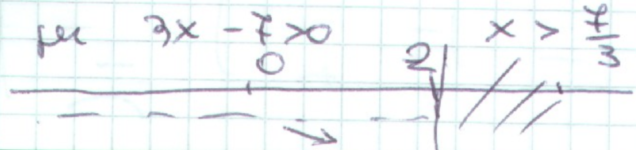
$$f'(x) = \begin{cases} \dfrac{7 - 3x}{3e^x\sqrt[3]{(x-2)^2}} & x > 2 \qquad \lim_{x \to 2^+} f'(x) = +\infty \\[2ex] \dfrac{3x - 7}{3e^x\sqrt[3]{(x-2)^2}} & x < 2 \qquad \lim_{x \to 2^-} f'(x) = -\infty \end{cases}$$

per $x > 2$ $\quad f'(x) > 0$ per $7 - 3x > 0$ $\quad x < \frac{7}{3}$



$x = \frac{7}{3}$ max

per $x < 2$ $\quad f'(x) > 0$ per $3x - 7 > 0$ $\quad x > \frac{7}{3}$



quindi la $f(x)$ è decrescente per $x \in]-\infty, 2[\cup]\frac{7}{3}, +\infty[$

e crescente per $x \in]2, \frac{7}{3}[$

CONCAVITÀ E CONVESSITÀ

$$f''(x) = \begin{cases} \dfrac{-3\left(3e^x \sqrt[3]{(x-2)^2}\right) + 3(7-3x)\left[e^x \sqrt[3]{(x-2)^2} + \frac{2}{3} e^x \frac{1}{\sqrt[3]{x-2}}\right]}{9 e^{2x} \sqrt[3]{(x-2)^4}} & x>2 \\[2ex] \dfrac{3\left(3e^x \sqrt[3]{(x-2)^2}\right) + 3(3x-7)\left[e^x \sqrt[3]{(x-2)^2} + \frac{2}{3} e^x \frac{1}{\sqrt[3]{x-2}}\right]}{9 e^{2x} \sqrt[3]{(x-2)^4}} & x<2 \end{cases}$$

$$f''(x) = \begin{cases} -\dfrac{3\sqrt[3]{(x-2)^2} + (7-3x)\left[\sqrt[3]{(x-2)^2} + \frac{2}{3}\frac{1}{\sqrt[3]{x-2}}\right]}{\cancel{9}\, 3e^x \sqrt[3]{(x-2)^4}} & x>2 \\[2ex] \dfrac{3\sqrt[3]{(x-2)^2} + (7-3x)\left[\sqrt[3]{(x-2)^2} + \frac{2}{3}\frac{1}{\sqrt[3]{x-2}}\right]}{3e^x \sqrt[3]{(x-2)^4}} & x<2 \end{cases}$$

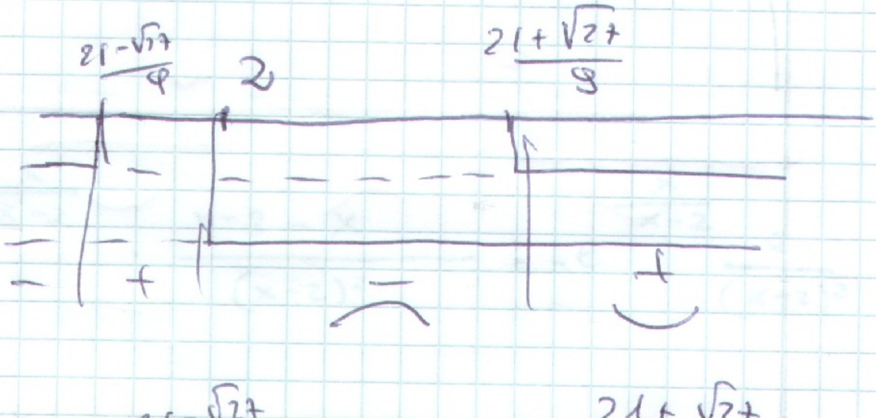
$$f''(x) = \begin{cases} -\dfrac{9(x-2) + (7-3x)\left[3(x-2) + 2\right]}{9 e^x \sqrt[3]{x-2}\, \sqrt[3]{(x-2)^4}} & x>2 \\[2ex] \dfrac{9(x-2) + (7-3x)\left[3(x-2) + 2\right]}{9 e^x \sqrt[3]{x-2}\, \sqrt[3]{(x-2)^4}} & x<2 \end{cases}$$

$$f''(x)=\begin{cases} -\dfrac{9x-18+(7-3x)(3x-4)}{9e^{x}\sqrt[3]{x-2}\,\sqrt[3]{(x-2)^4}} & x>2 \\[2ex] \dfrac{9x-18+21x-28-9x^2+12x}{9e^{x}\sqrt[3]{x-2}\,\sqrt[3]{(x-2)^4}} & x<2 \end{cases}$$

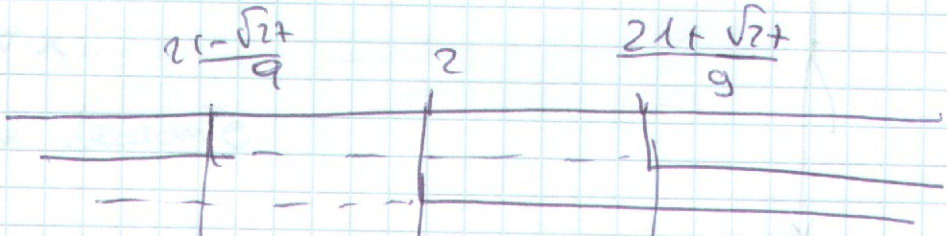
$$f''(x)=\begin{cases} +\dfrac{+9x^2-42x+46}{9e^{x}\sqrt[3]{x-2}\,\sqrt[3]{(x-2)^4}} & x>2 \\[2ex] -\dfrac{9x^2-42x+46}{9e^{x}\sqrt[3]{x-2}\,\sqrt[3]{(x-2)^4}} & x<2 \end{cases}$$

$$x=\frac{21\pm\sqrt{441-414}}{\cancel{18}\,9}=\frac{21\pm\sqrt{27}}{\cancel{18}\,9}$$

$f''(x)$ per $x>2$



per $x<2$



$$f''(x)=\begin{cases} -\dfrac{9x-18+(7-3x)(3x-4)}{9e^{x}\sqrt[3]{x-2}\,\sqrt[3]{(x-2)^4}} & x>2 \\[2ex] \dfrac{9x-18+21x-28-9x^2+12x}{9e^{x}\sqrt[3]{x-2}\,\sqrt[3]{(x-2)^4}} & x<2 \end{cases}$$

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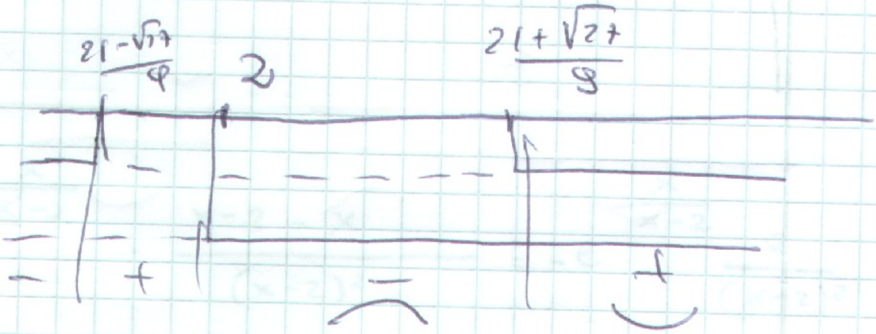
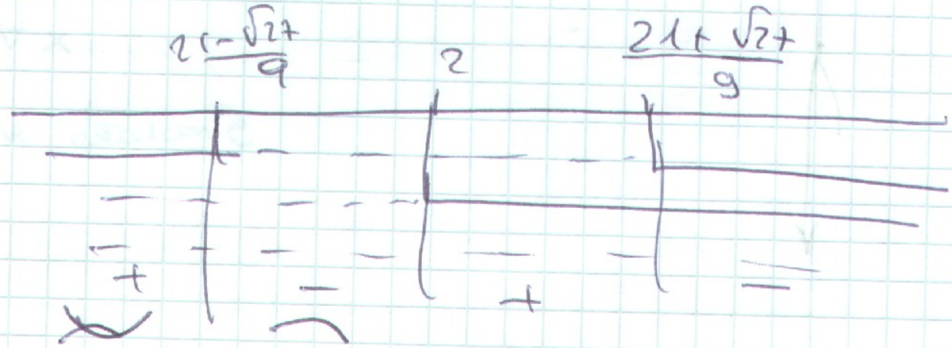
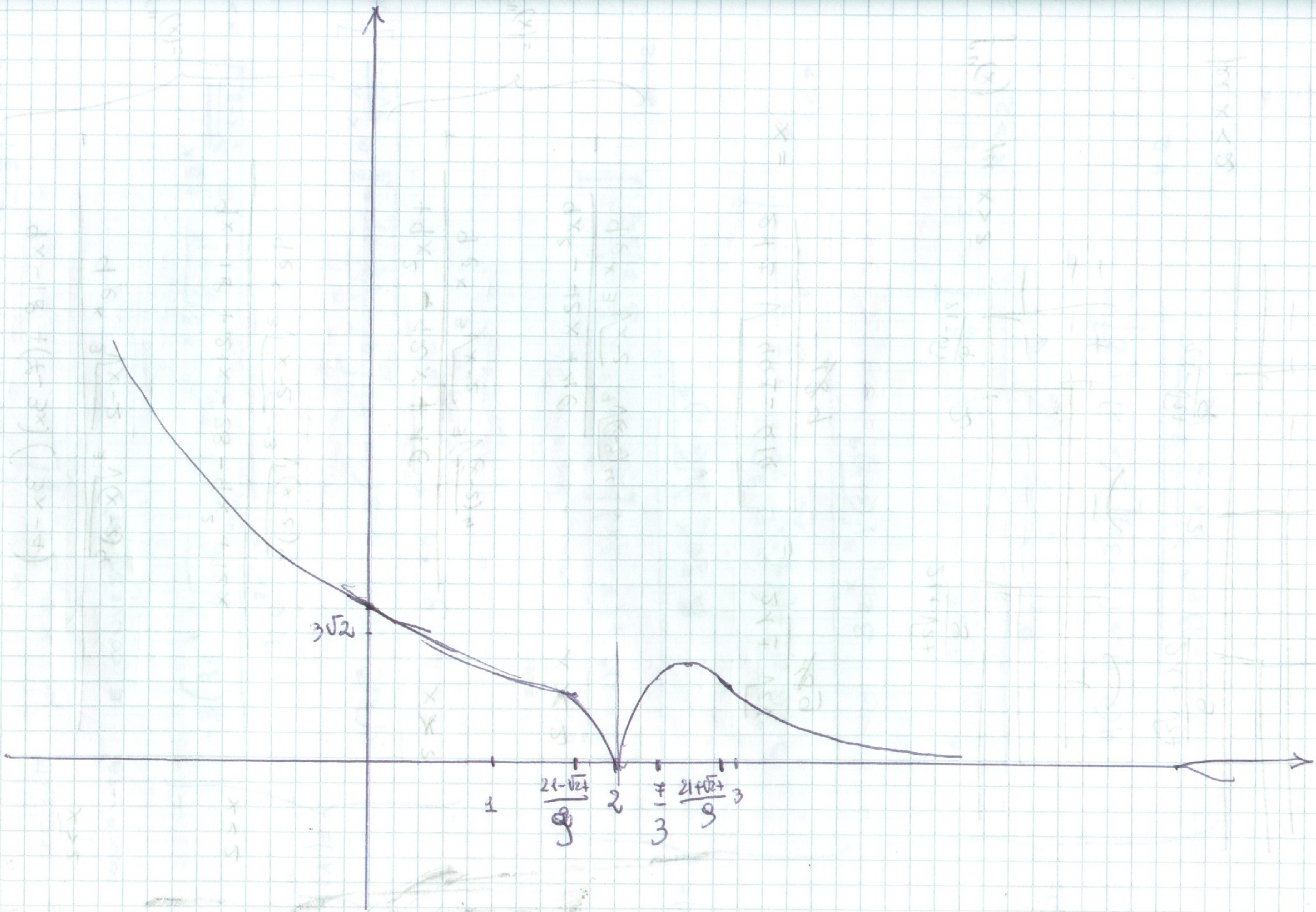
$$x=\frac{21\pm\sqrt{441-414}}{\cancel{18}\,9}=\frac{21\pm\sqrt{27}}{\cancel{18}\,9}$$

$f''(x)$ per $x>2$



per $x<2$





3√2
1
21−√27 / 9
2
7/3
21+√27 / 9
3