Del Prete - Febbraio 1994

$$f(x) = \log|1-e^{2x}|$$

CAMPO DI ESISTENZA

$|1-e^{2x}| \neq 0$; $1-e^{2x} \neq 0$; $e^{2x} \neq 1$; $2x \neq 0$; $x \neq 0$

CE: $\mathbb{R} - \{0\}$

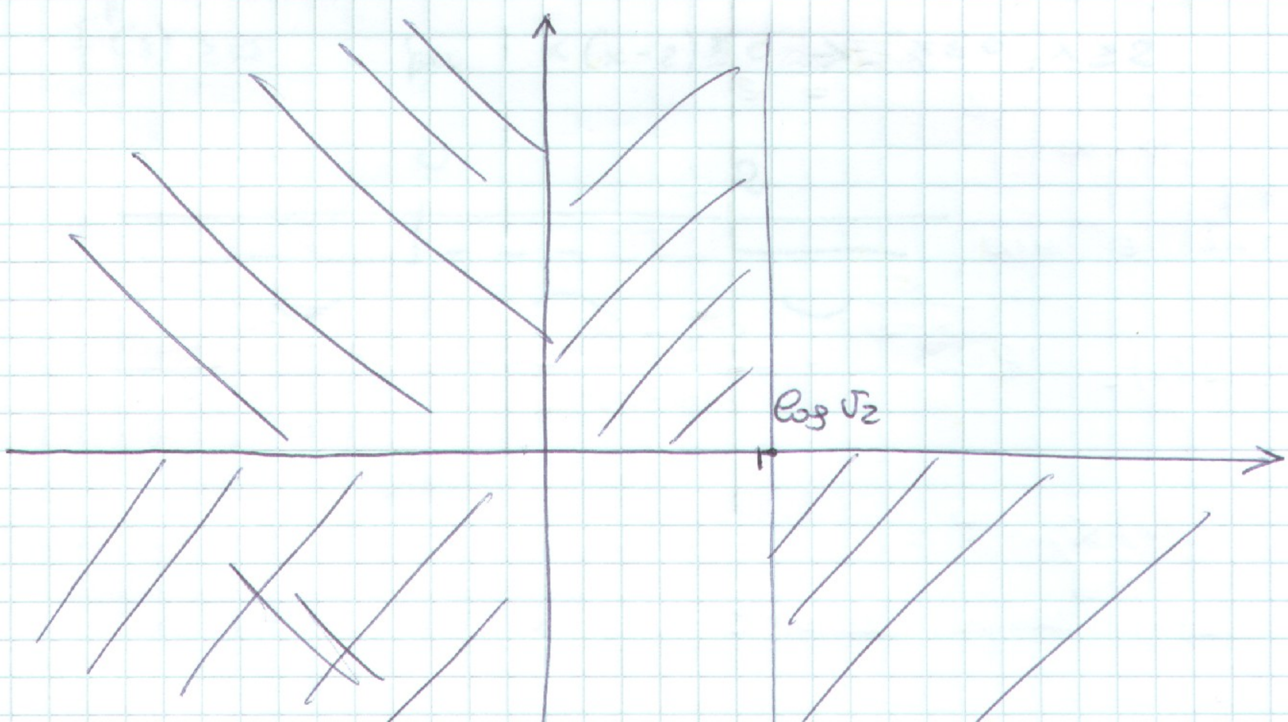
$$f(x) = \begin{cases} \log(1-e^{2x}) & \text{per } e^{2x} < 1 \quad ; \quad x<0 \\ \log(-1+e^{2x}) & \text{per } e^{2x} > 1 \quad ; \quad x>0 \end{cases}$$

POSITIVITÀ

$\log(1-e^{2x}) > 0$; $1-e^{2x} > 1$; $e^{2x} < 0$; mai

$\log(-1+e^{2x}) > 0$; $e^{2x} - 1 > 1$; $e^{2x} > 2$; $2x > \log 2$

$x > \log\sqrt{2}$



$$\lim_{x \to 0^-} \log(1-e^{2x}) = -\infty$$

$$\lim_{x \to 0^+} \log(-1+e^{2x}) = +\infty$$

$$\lim_{x \to -\infty} \log(1-e^{2x}) = \log 1 = 0$$

$$\lim_{x \to +\infty} \log(e^{2x}-1) = +\infty$$

INTERSEZIONI

$\cancel{y=0 \quad \log|1-e^{2x}|=0} \quad |1-e^{2x}|=1$ non $\in$ CE

$\cancel{x \neq 0}$

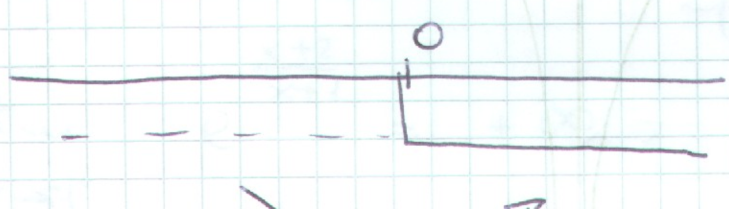
$y=0 \qquad 1-e^{2x}=1 \qquad e^{2x}=0 \qquad \cancel{x=0 \notin CE}$ mai

$\qquad\quad e^{2x}-1=1 \qquad e^{2x}=2 \qquad x=\log\sqrt{2}$

MAX E MIN

$$f'(x) = \begin{cases} \dfrac{-2e^{2x}}{1-e^{2x}} & x<0 \\[2ex] \dfrac{2e^{2x}}{e^{2x}-1} & x>0 \end{cases} \quad \rightarrow \quad \frac{2e^{2x}}{e^{2x}-1} \neq 0 \quad \forall x$$

$$f'(x)>0 \quad \begin{cases} e^{2x}>0 & \forall x \\[2ex] e^{2x}-1>0 & x>0 \end{cases}$$



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$$\lim_{x\to +\infty} \log(e^{2x}-1) = +\infty$$

INTERSEZIONI

$\cancel{y=0 \quad \log|1-e^{2x}|=0 \quad |1-e^{2x}|=1}$ non $\in$ CE

$x\neq 0$

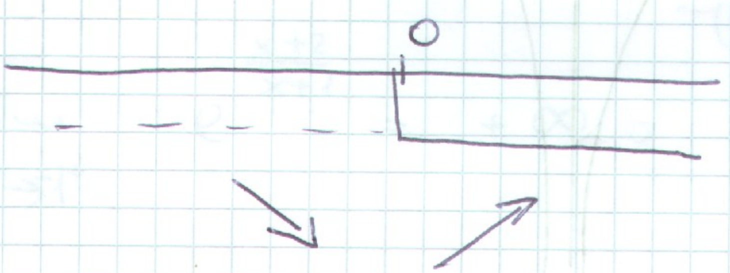
$y=0 \qquad 1-e^{2x}=1 \qquad e^{2x}=0 \qquad \cancel{x=0 \notin CE}$ mai

$\qquad\qquad e^{2x}-1=1 \qquad e^{2x}=2 \qquad x=\log\sqrt{2}$

MAX E MIN

$$f'(x)=\begin{cases} \dfrac{-2e^{2x}}{1-e^{2x}} & x<0 \\ \dfrac{2e^{2x}}{e^{2x}-1} & x>0 \end{cases} \rightarrow \frac{2e^{2x}}{e^{2x}-1} \neq 0 \quad \forall x$$

$$f'(x)>0 \quad \begin{cases} e^{2x}>0 & \forall x \\ e^{2x}-1>0 & x>0 \end{cases}$$



$$f''(x) = \frac{4e^{2x}(e^{2x}-1) - 2e^{2x} \cdot 2e^{2x}}{(e^{2x}-1)^2} = \frac{\cancel{4e^{4x}} - 4e^{2x} - \cancel{4e^{4x}}}{(e^{2x}-1)^2}$$

$$= -\frac{4e^{2x}}{(e^{2x}-1)^2} < 0 \quad \forall x$$

$f(x)$ sempre concava