Del Prete luglio '90

$$f(x) = |x|\, e^{\frac{1}{1-x^2}}$$

CAMPO DI ESISTENZA

$1-x^2 \neq 0$; $x \neq \pm 1$ CE : $\mathbb{R} - \{-1, 1\}$

POSITIVITA'

$f(x) > 0 \quad \forall x \in CE$

$$f(x) = \begin{cases} -x\, e^{\frac{1}{1-x^2}} & x < 0 \\ x\, e^{\frac{1}{1-x^2}} & x > 0 \end{cases}$$

$$\lim_{x \to -\infty} -x\, e^{\frac{1}{1-x^2}} = +\infty\, e^{0} = +\infty$$

$1-x^2 > 0$ per $-1 < x < 1$

$$\lim_{x \to -1^-} -x\, e^{\frac{1}{1-x^2}} = 1 \cdot e^{-\infty} = 0$$

$$\lim_{x \to -1^+} -x\, e^{\frac{1}{1-x^2}} = 1\, e^{+\infty} = +\infty$$

$$\lim_{x \to 1^-} x\, e^{\frac{1}{1-x^2}} = 1 \cdot e^{+\infty} = +\infty$$

$$\lim_{x \to 1^+} x\, e^{\frac{1}{1-x^2}} = 1\, e^{-\infty} = 0$$

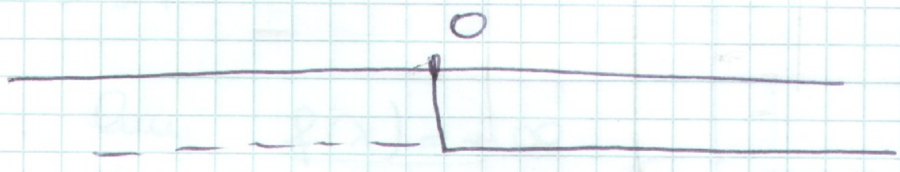
MAX E MIN.

$$f'(x)=\begin{cases} -e^{\frac{1}{1-x^2}} - x\, e^{\frac{1}{1-x^2}} \cdot \dfrac{2x}{(1-x^2)^2} & x<0 \\[2ex] e^{\frac{1}{1-x^2}} + x\, e^{\frac{1}{1-x^2}} \dfrac{2x}{(1-x^2)^2} & x>0 \end{cases}$$

$$f'(x)=\begin{cases} -e^{\frac{1}{1-x^2}}\left(1+\dfrac{2x^2}{(1-x^2)^2}\right) & x<0 \\[2ex] e^{\frac{1}{1-x^2}}\left(1+\dfrac{2x^2}{(1-x^2)^2}\right) & x>0 \end{cases}$$

$$f'(x)=\begin{cases} -e^{\frac{1}{1-x^2}}\,\dfrac{1+x^4}{(1-x^2)^2} & x<0 \\[2ex] e^{\frac{1}{1-x^2}}\,\dfrac{1+x^4}{(1-x^2)^2} & x>0 \end{cases}$$

$f'(x)>0$ per $x>0$



$x=0$ punto di min

$f(0)=0$

$$f'(x)=\begin{cases} -e^{\frac{1}{1-x^2}} - x\,e^{\frac{1}{1-x^2}} \cdot \dfrac{2x}{(1-x^2)^2} & x<0 \\[2ex] e^{\frac{1}{1-x^2}} + x\,e^{\frac{1}{1-x^2}} \dfrac{2x}{(1-x^2)^2} & x>0 \end{cases}$$

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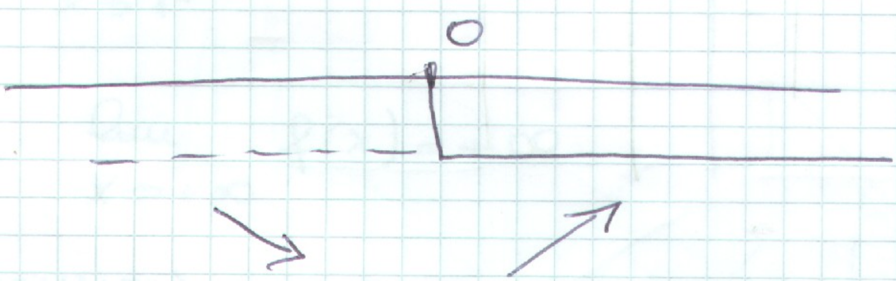
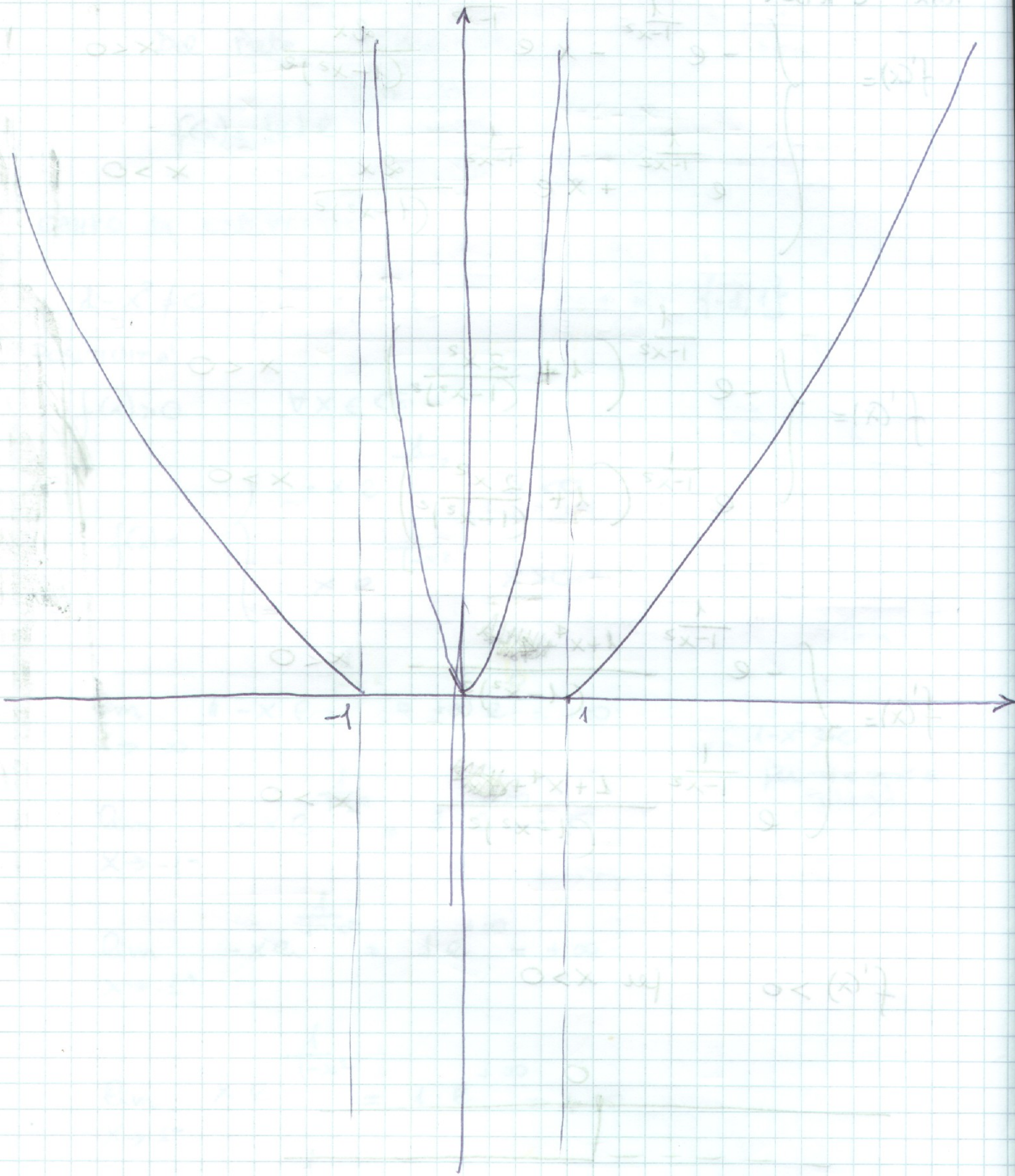
$$f'(x)=\begin{cases} -e^{\frac{1}{1-x^2}}\,\dfrac{1+x^4+}{(1-x^2)^2} & x<0 \\[2ex] e^{\frac{1}{1-x^2}}\,\dfrac{1+x^4+}{(1-x^2)^2} & x>0 \end{cases}$$

$f'(x)>0$ per $x>0$



$x=0$ punto di min

$f(0)=0$

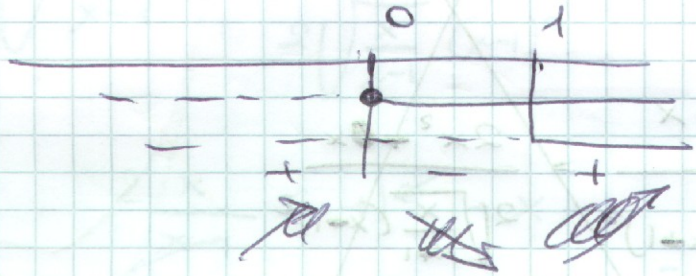

-1
1

Del Prete

$$f(x) = x\sqrt{\frac{x}{x-1}}$$

CAMPO DI ESISTENZA

$$\frac{x}{x-1} \geq 0 \qquad \begin{cases} x \geq 0 \\ x-1>0 \end{cases} \qquad \begin{cases} x \geq 0 \\ x>1 \end{cases}$$



C.E.: $\left]-\infty, 0\right] \cup \left]1, +\infty\right[$

$$\lim_{x \to -\infty} f(x) = -\infty \cdot 1 = -\infty$$

$$\lim_{x \to 0^-} f(x) = f(0) = 0$$

$$\lim_{x \to 1^+} f(x) = +\infty$$

$$\lim_{x \to +\infty} f(x) = +\infty$$

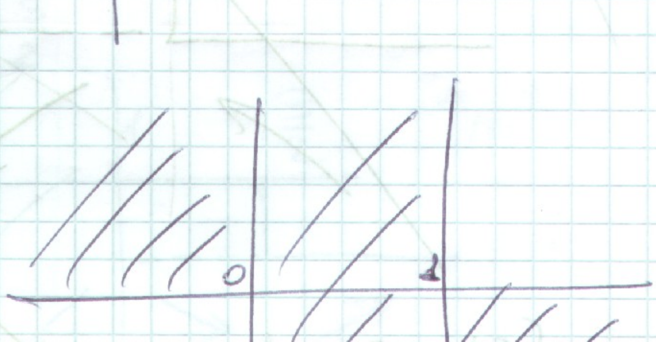
$$m = \lim_{x \to +\infty} \sqrt{\frac{x}{x-1}} = 1$$

$$n = \lim_{x \to +\infty} x\sqrt{\frac{x}{x-1}} - x =$$

$$= \lim_{x \to +\infty} x$$

POSITIVITA'

$f(x) > 0$ per $x > 0$



MAX E MIN

$$f(x) = x\sqrt{\frac{x}{x-1}}$$

$$f'(x) = \sqrt{\frac{x}{x-1}} + x\,\frac{1}{2\sqrt{\frac{x}{x-1}}} \cdot \frac{\cancel{x}-1-\cancel{x}}{(x-1)^2}$$

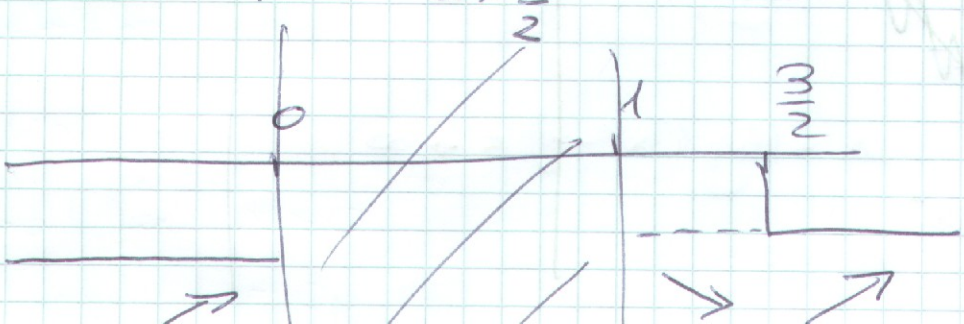
$$= \sqrt{\frac{x}{x-1}} - \frac{x}{2\sqrt{\frac{x}{x-1}}\,(x-1)^2} =$$

$$= \frac{2\frac{x}{\cancel{x-1}}(x-1)^{\cancel{2}} - x}{2\sqrt{\frac{x}{x-1}}\,(x-1)^2} =$$

$$= \frac{2x^2 - 3x}{2\sqrt{\frac{x}{x-1}}\,(x-1)^2}$$

$f'(x) = 0 \qquad x(2x-3) = 0$

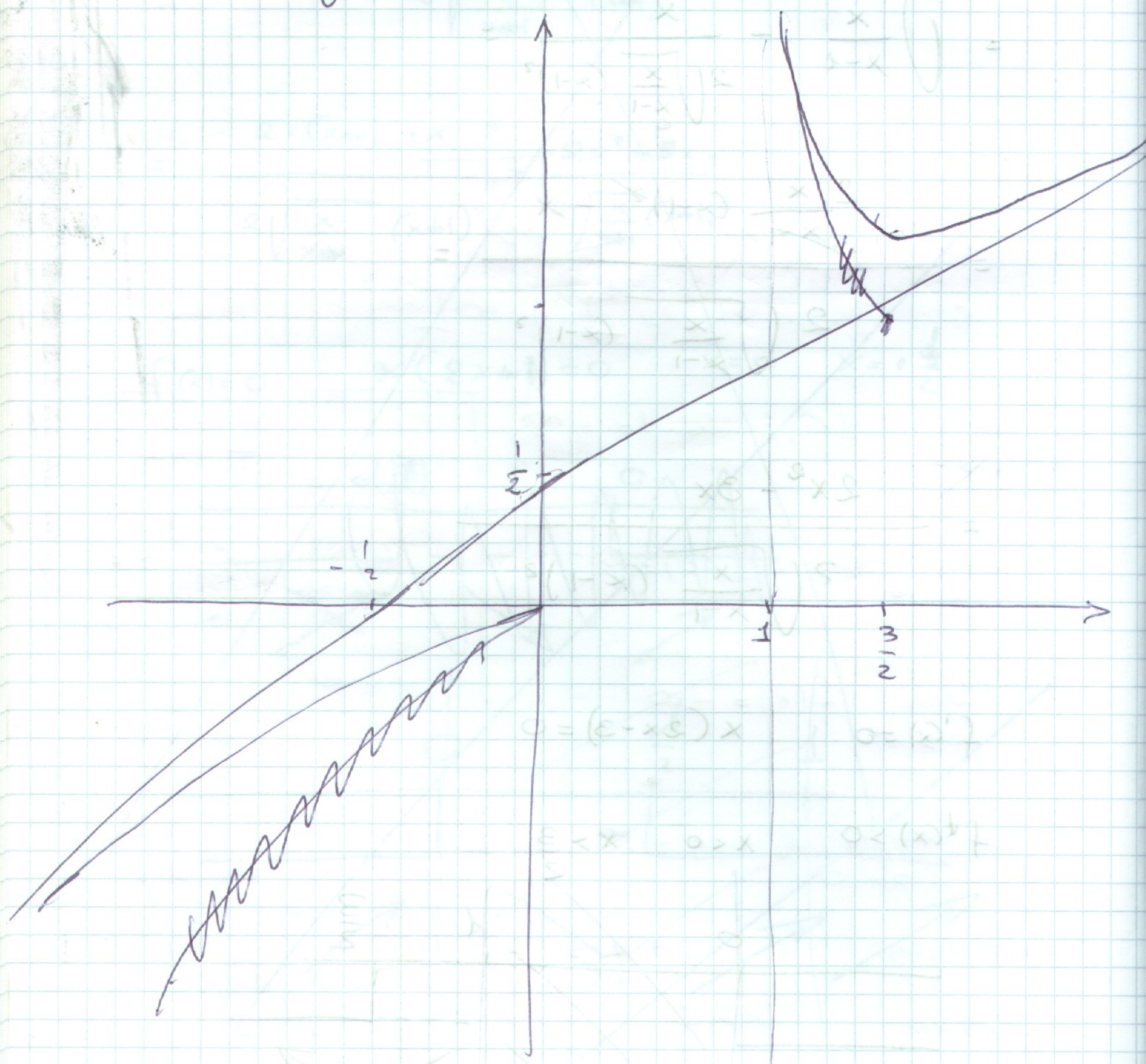
$f'(x) > 0 \qquad x < 0 \qquad x > \frac{3}{2}$



$x = \frac{3}{2}$ punto di minimo

$$f\left(\frac{3}{2}\right) = \frac{3}{2}\sqrt{\frac{\frac{3}{2}}{\frac{3}{2}-1}} = \frac{3}{2}\sqrt{\frac{\frac{3}{2}}{\frac{1}{2}}} =$$

$$= \frac{3}{2}\sqrt{3}$$



Asintoti obliqui

$$m = \lim_{x \to +\infty} \sqrt{\frac{x}{x-1}} = 1$$

$$n = \lim_{x \to +\infty} x\sqrt{\frac{x}{x-1}} - x =$$

$$= \lim_{x \to +\infty} x\left(\sqrt{\frac{x}{x-1}} - 1\right) = \lim_{x \to +\infty} x \frac{\frac{x}{x-1} - 1}{\sqrt{\frac{x}{x-1}} + 1} =$$

$$= \lim_{x \to +\infty} x \frac{\frac{\cancel{x} - \cancel{x} + 1}{x-1}}{\sqrt{\frac{x}{x-1}} + 1} = \lim_{x \to +\infty} \frac{\frac{x}{x-1}}{\sqrt{\frac{x}{x-1}} + 1} = \frac{1}{2}$$

$$y = x + \frac{1}{2}$$

$$m = \lim_{x \to -\infty} \cancel{x}\sqrt{\frac{x}{x-1}} = 1$$

$$n = \lim_{x \to -\infty} x\sqrt{\frac{x}{x-1}} - x = \lim_{x \to -\infty} x\left(\sqrt{\frac{x}{x-1}} - 1\right)$$

$$= \frac{1}{2}$$