$$f(x) = x + \log(x^2-1)$$

CAMPO DI ESISTENZA

$x^2-1>0$; $x<-1$, $x>1$

~~POSTI~~ $y \gtrless 0$ Non possiamo calcolare la positività

$$\lim_{x\to-\infty} x+\log(x^2-1) = -\infty+\infty$$

$$\lim_{x\to-\infty} x\left(1+\frac{\log(x^2-1)}{x}\right) =$$

$$= \lim_{x\to-\infty} x\left(1+\frac{\log(x-1)}{x}+\frac{\log(x+1)}{x}\right] = -\infty(1) = -\infty$$

$$\left[\begin{array}{l} \lim_{x\to-\infty} \frac{\log(x-1)}{x} = \lim_{x\to-\infty} \frac{1}{x-1} = 0 \\ \lim_{x\to-\infty} \frac{\log(x+1)}{x} = 0 \end{array}\right]$$

$$\lim_{x\to+\infty} x+\log(x^2-1) = +\infty$$

$$\lim_{x\to-1^-} x+\log(x^2-1) = -\infty$$

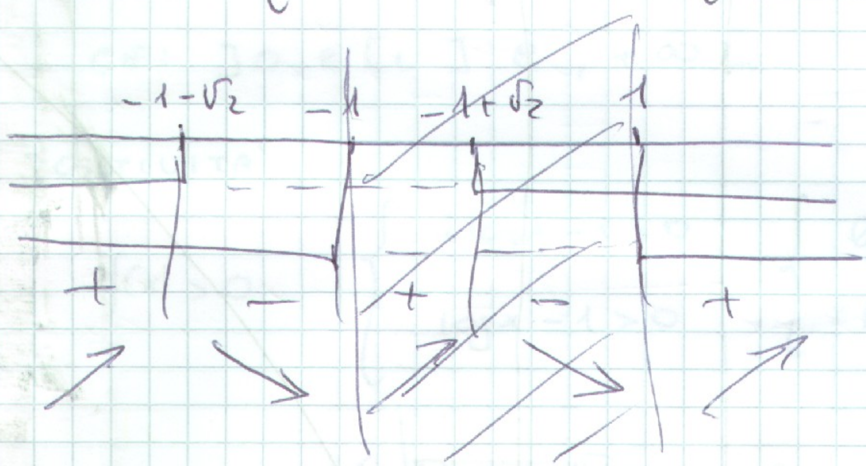
$$\lim_{x\to 1^+} x+\log(x^2-1) = -\infty$$

MAX E MIN

$$f'(x) = 1 + \frac{2x}{x^2-1} = \frac{x^2+2x-1}{x^2-1}$$

$$x = -1 \pm \sqrt{1+1} = -1 \pm \sqrt{2}$$

$$f'(x) > 0 \quad \begin{cases} x^2+2x-1 > 0 \\ x^2-1 > 0 \end{cases} \quad \begin{cases} x < -1-\sqrt{2}, \; x > -1+\sqrt{2} \\ x < -1, \; x > 1 \end{cases}$$



$x = -1-\sqrt{2}$ punto di max

$x = -1$ ~~punto di minimo~~ $\notin CE$

$x = -1+\sqrt{2}$ ~~punto di max~~ $\notin CE$

$x = 1$ ~~punto di minimo~~ $\notin CE$

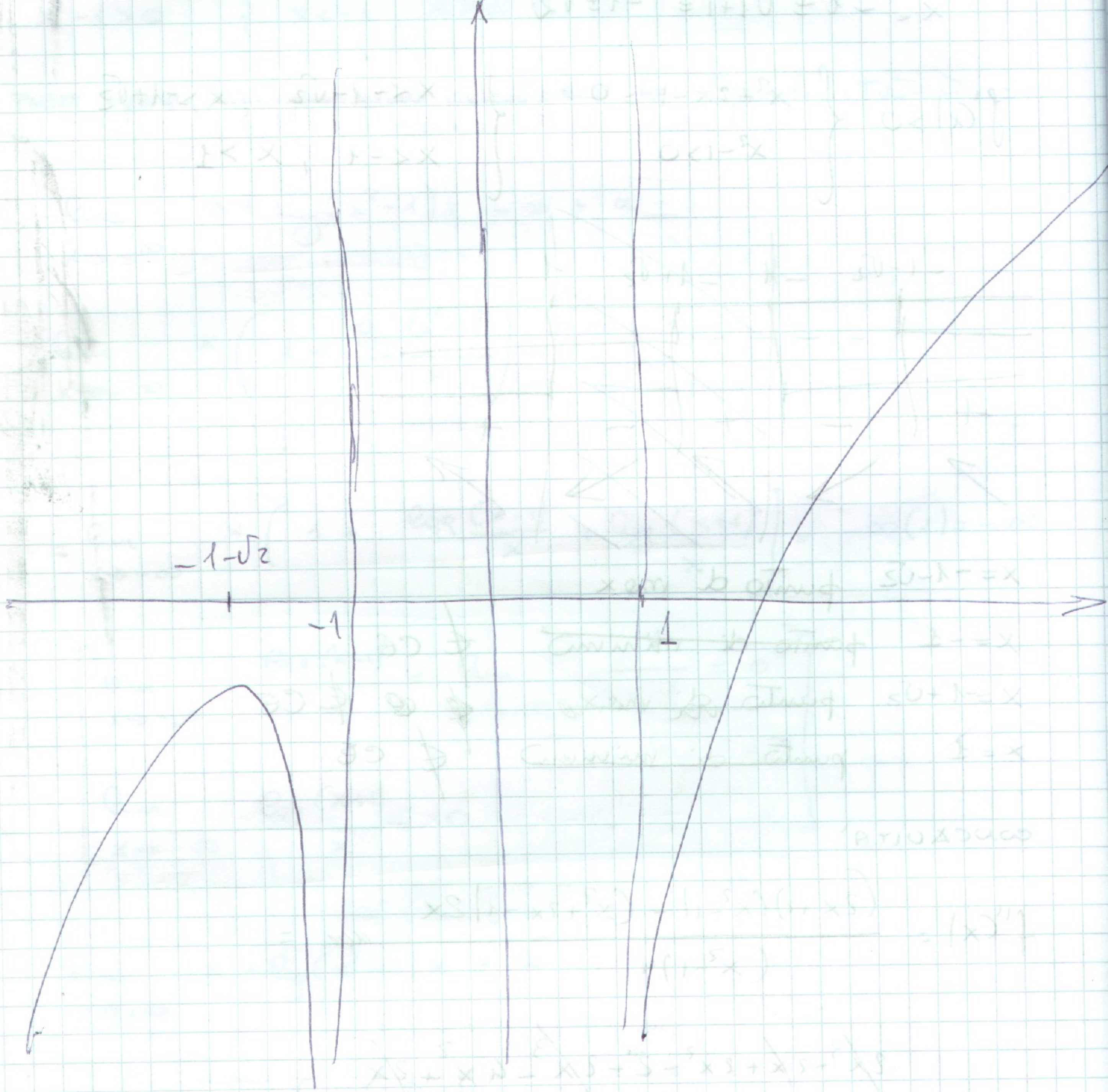
CONCAVITA'

$$f''(x) = \frac{(2x+2)(x^2-1) - (x^2+2x-1)2x}{(x^2-1)^4} =$$

$$= \frac{2x^3 - 2x + 2x^2 - 2 - 2x^3 - 4x^2 + 2x}{(x^2-1)^4}$$

$$= \frac{-2x^2-2}{(x^2-1)^4} = -2\frac{x^2-1}{(x^2-1)^4} = -\frac{2}{(x^2-1)^3}$$

$f''(x) < 0 \quad \forall x \in CE \quad f(x)$ sempre concava



-1-√2
-1
1