$$f(x)=\frac{xe^x}{\sqrt{2x-1}}$$

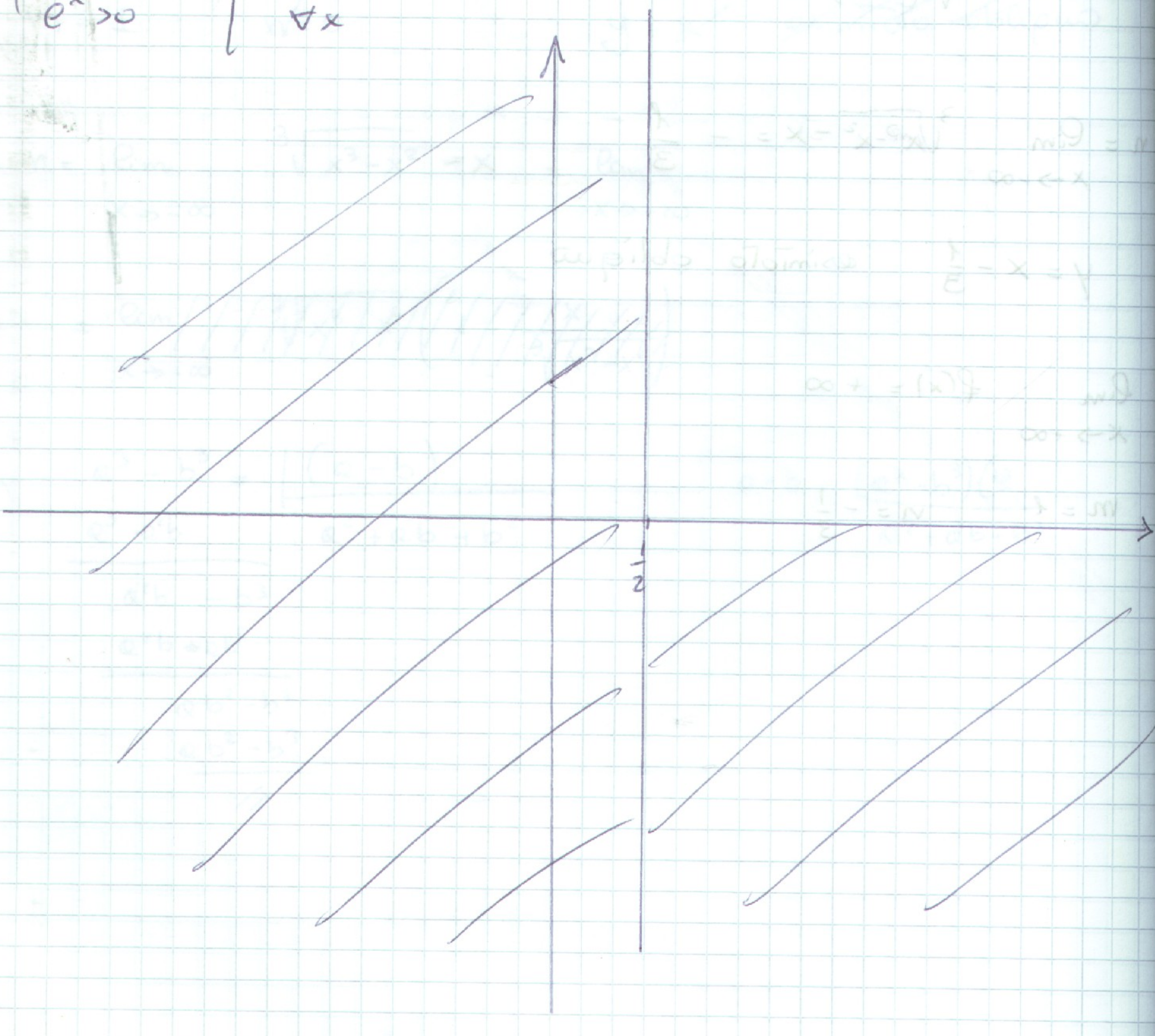
CAMPO DI ESISTENZA

$2x-1>0 \qquad x>\frac{1}{2}$

CE: $\left]\frac{1}{2};+\infty\right[$

POSITIVITÀ

$$\begin{cases} x>0 \\ e^x>0 \end{cases} \Rightarrow \begin{cases} x>0 \\ \forall x \end{cases} \Rightarrow f(x)>0 \quad \forall x \in CE$$



$$\lim_{x \to \frac{1}{2}^+} \frac{x e^x}{\sqrt{2x-1}} = +\infty$$

$$\lim_{x \to +\infty} \frac{x e^x}{\sqrt{2x-1}} = +\infty$$

$$m = \lim_{x \to +\infty} \frac{e^x}{\sqrt{2x-1}} = +\infty$$

MAX E MIN

$$f'(x) = \frac{(e^x + x e^x)\sqrt{2x-1} - \frac{x e^x}{2\sqrt{2x-1}} \cdot 2}{2x-1} =$$

$$= e^x \frac{(1+x)\sqrt{2x-1} - \frac{x}{\sqrt{2x-1}}}{2x-1}$$

$$= e^x \frac{(1+x)(2x-1) - x}{\sqrt{2x-1}\,(2x-1)} =$$

$$= e^x \frac{2x - 1 + 2x^2 - x - x}{\sqrt{2x-1}\,(2x-1)} = e^x \frac{2x^2-1}{\sqrt{2x-1}\,(2x-1)}$$

$$f'(x) > 0 \quad \begin{cases} 2x^2 - 1 > 0 \\ 2x - 1 > 0 \end{cases} \qquad \begin{cases} x < -\frac{\sqrt{2}}{2} \;;\; x > \frac{\sqrt{2}}{2} \\ x > \frac{1}{2} \end{cases}$$

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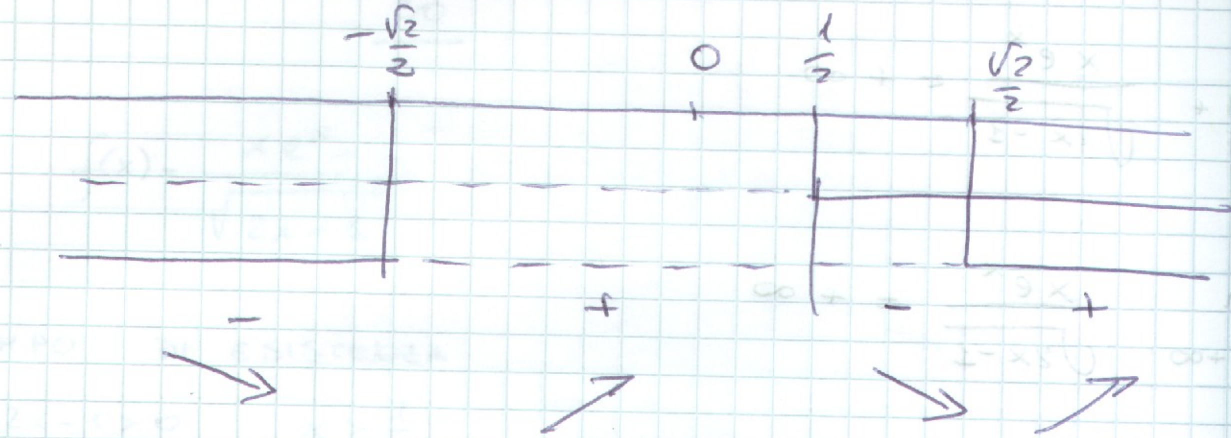


$x = -\frac{\sqrt{2}}{2}$ min

$x = \frac{1}{2}$ max

$x = \frac{\sqrt{2}}{2}$ min

non fanno parte del CE

CONCAVITÀ E CONVESSITÀ

$$f'(x) = \frac{\left[e^x(2x^2-1)+e^x 4x\right](2x-1)^{\frac{3}{2}} - e^x(2x^2-1)\frac{3}{2}(2x-1)^{\frac{3}{2}-1}}{(2x-1)^3} =$$

$$= e^x \frac{(2x^2+4x-1)(2x-1)^{\frac{3}{2}} - \frac{3}{2}(2x^2-1)\sqrt{2x-1}}{(2x-1)^3} =$$

$$= e^x\sqrt{2x-1}\ \frac{2(2x^2+4x-1)(2x-1)-3(2x^2-1)}{2\,\cancel{\sqrt{2x-1}}\,(2x-1)^3} =$$

$$= e^x\sqrt{2x-1}\ \frac{8x^3-4x^2+16x^2-8x-4x+2-6x^2+6}{(2x-1)^3} =$$

$$= e^x\sqrt{2x-1}\ \frac{4x^3+3x^2-6x+4}{(2x-1)^3}$$

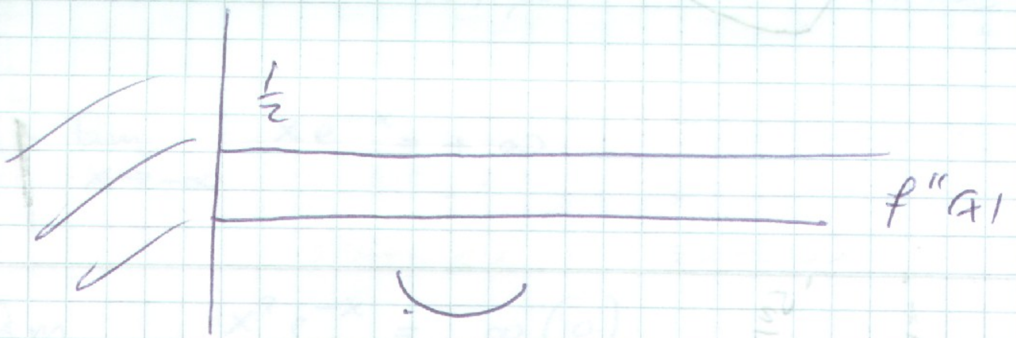
$$g\left(\frac{1}{2}\right) = 4\cdot\frac{1}{8} + 3\frac{1}{4} - \frac{6}{2} + 4 = \frac{5}{4} + 1 > 0$$

$$g'(x) = 12x^2 + 6x - 6 = 6(2x^2 + x - 1)$$

$$x = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4} \begin{cases} -1 \\ \frac{1}{2} \end{cases}$$

$g'(x) > 0$ per $x < -1$, $x > \frac{1}{2}$

allora $g(x)$ è crescente per $x > \frac{1}{2}$ e poiché $g(\frac{1}{2}) > 0$, allora $g(x)$ è crescente $\forall x > \frac{1}{2}$ per cui $f''(x) > 0$ se $2x - 1 > 0 \Rightarrow x > \frac{1}{2}$



e la f è sempre convessa

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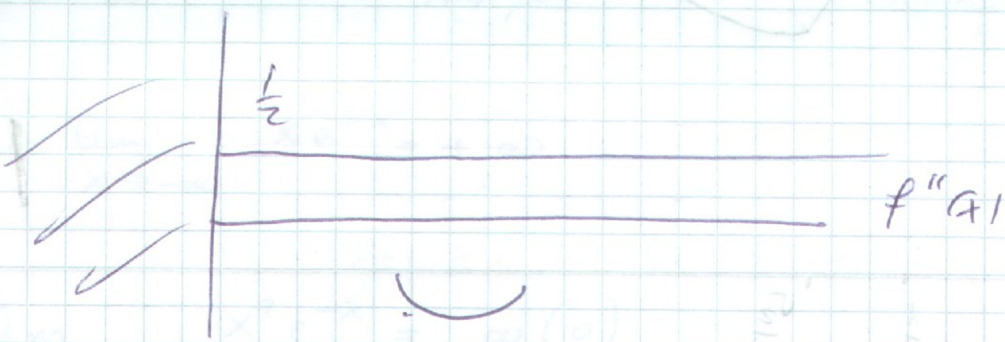

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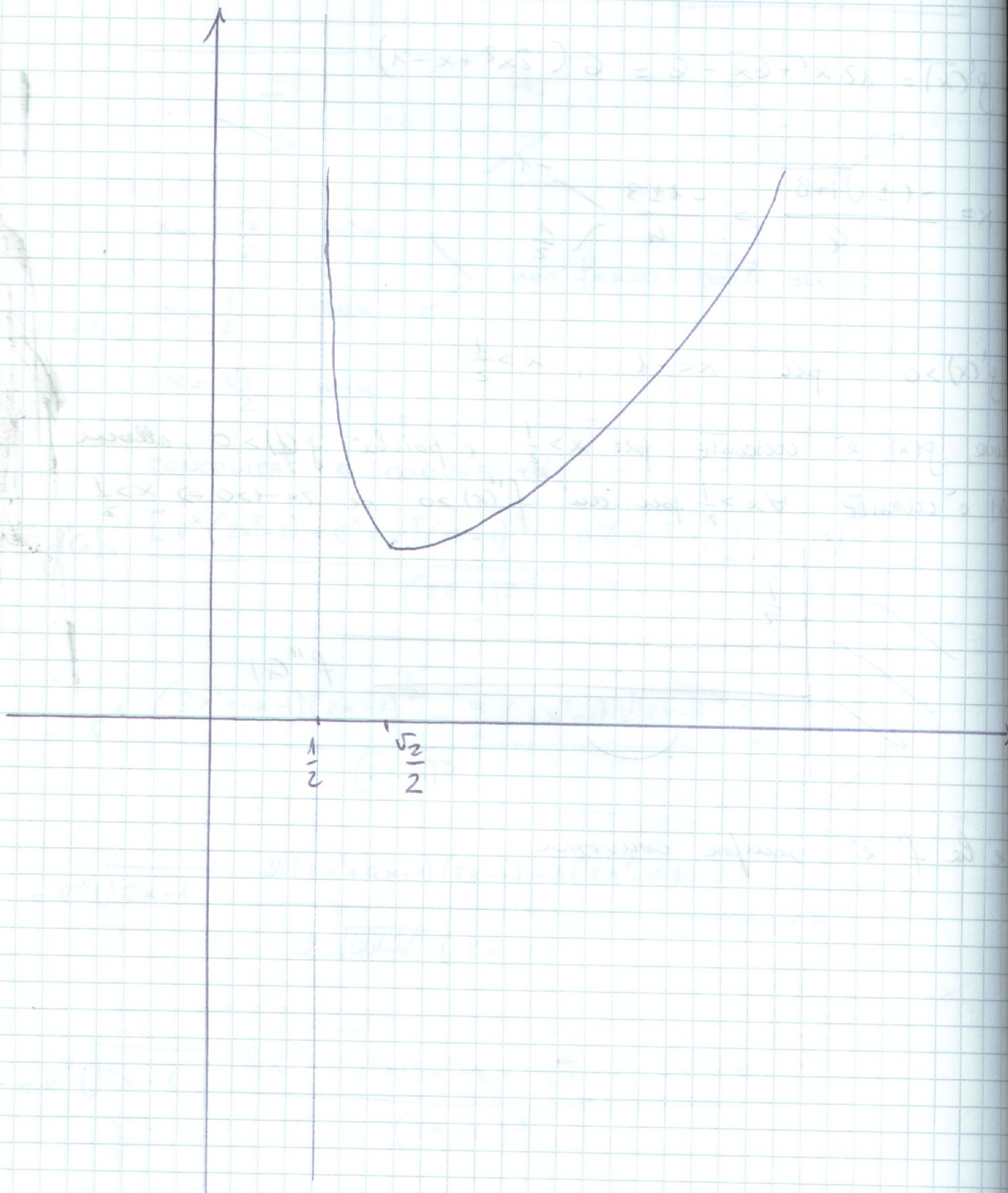

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