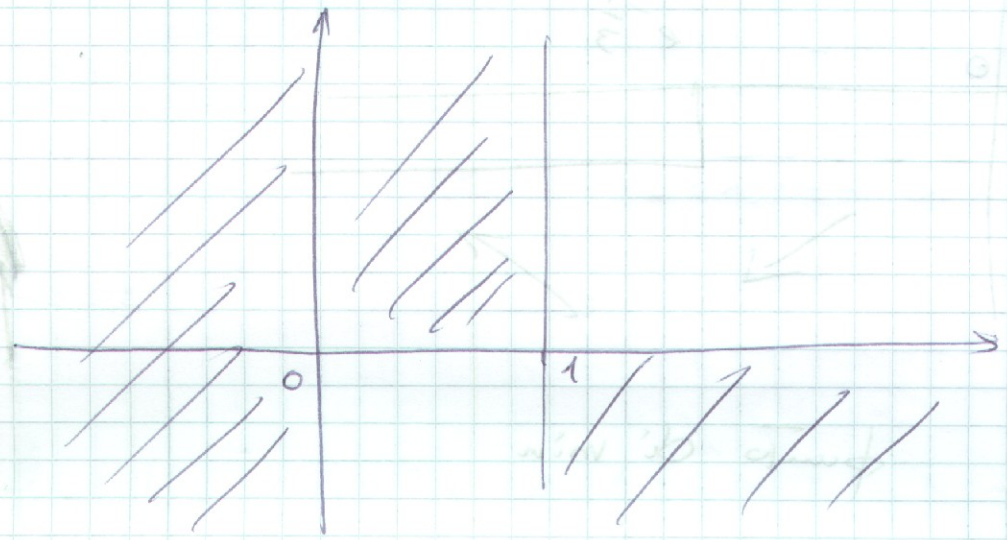
$$f(x) = x^3 \ln x$$

CAMPO DI ESISTENZA $x > 0$ $\quad CE:]0, +\infty[$

POSITIVITÀ $\begin{cases} x > 0 \\ \ln x > 0 \end{cases} \quad \begin{cases} x > 0 \\ x > 1 \end{cases} \quad x > 1$



Intersezione con l'asse x $\Rightarrow$ ~~$x = 0$~~ $\quad x = 1$

↑ non $\in CE$

$$\lim_{x \to 0} x^3 \ln x = 0$$

$$\lim_{x \to +\infty} x^3 \ln x = +\infty$$

$$m = \lim_{x \to +\infty} x^2 \ln x = +\infty$$ non c'è asintoto obliquo

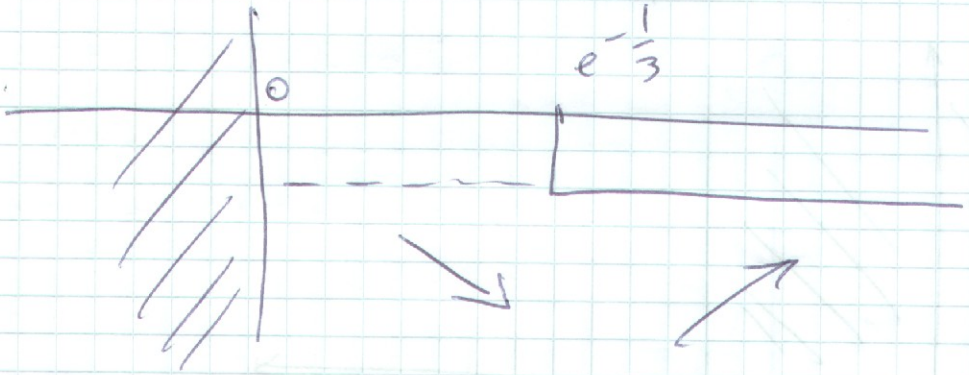
MAX E MIN

$$f'(x) = 3x^2 \ln x + \frac{x^3}{x} = 3x^2 \ln x + x^2 =$$

$$= x^2 (3 \ln x + 1)$$

$f'(x)=0 \quad \begin{cases} x=0 \\ \ln x = -\frac{1}{3} \end{cases} \qquad \begin{cases} x=0 \\ x = e^{-\frac{1}{3}} \end{cases}$

$f'(x)>0 \quad \begin{cases} x>0 \\ x > e^{-\frac{1}{3}} \end{cases}$



$x = e^{-\frac{1}{3}}$ punto di min

$$f\left(e^{-\frac{1}{3}}\right) = e^{-1} \ln e^{-\frac{1}{3}} = -\frac{1}{3e}$$

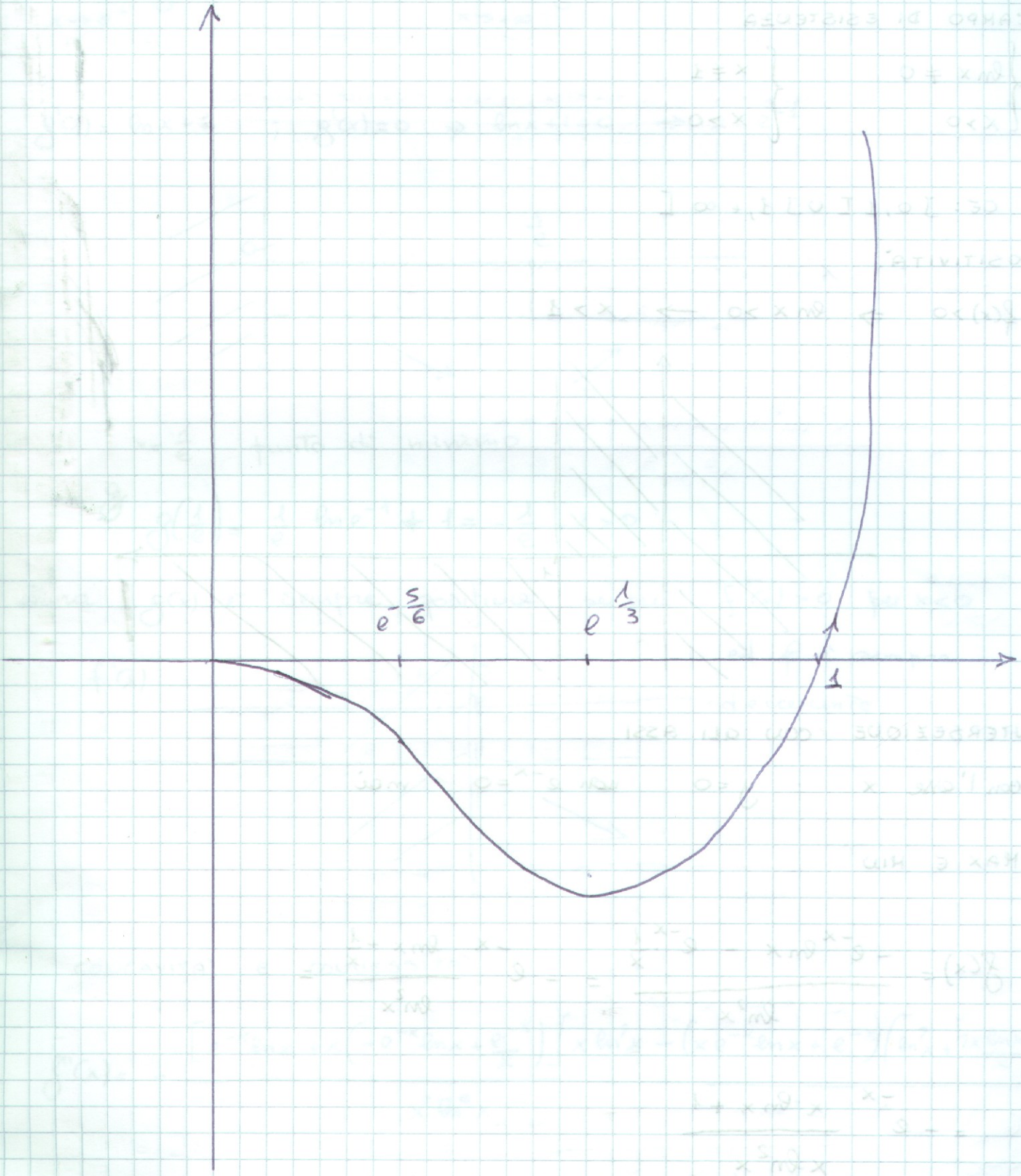
CONCAVITÀ E CONVESSITÀ

$f''(x) = 2x(3\ln x + 1) + x^2\left(\frac{3}{x}\right) =$

$= 6x\ln x + 2x + 3x =$

$= 6x\ln x + 5x = x(6\ln x + 5)$

$f''(x)=0 \quad \begin{cases} x=0 \\ x = e^{-\frac{5}{6}} \end{cases}$


$e^{-\frac{5}{6}}$
$e^{\frac{1}{3}}$
1