$$f(x) = \frac{e^{-x}}{\ln x}$$

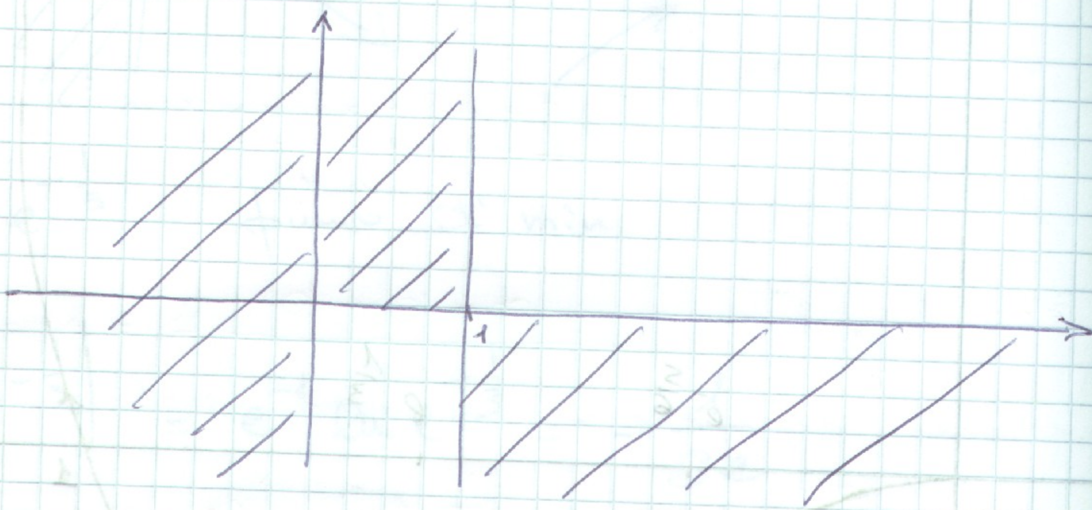
CAMPO DI ESISTENZA

$$\begin{cases} \ln x \neq 0 \\ x > 0 \end{cases} \qquad \begin{cases} x \neq 1 \\ x > 0 \end{cases}$$

CE: $]0,1[\cup]1,+\infty[$

POSITIVITÀ

$f(x) > 0 \Rightarrow \ln x > 0 \longrightarrow x > 1$



INTERSEZIONE CON GLI ASSI

con l'asse x : $y = 0$ non $e^{-x} = 0$ mai

MAX E MIN

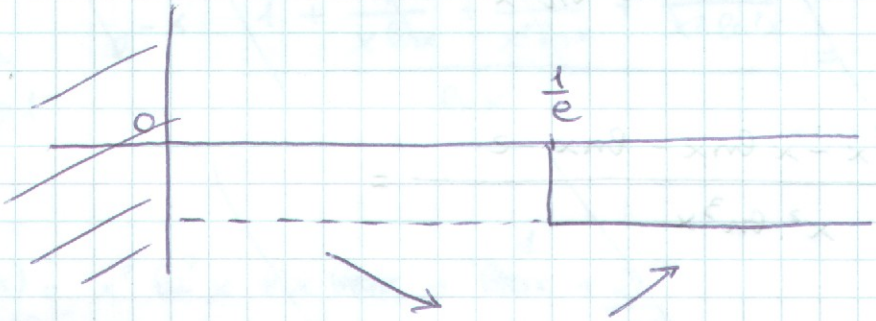
$$f'(x) = \frac{-e^{-x}\ln x - e^{-x}\cdot\frac{1}{x}}{\ln^2 x} = -e^{-x}\,\frac{\ln x + \frac{1}{x}}{\ln^2 x} =$$

$$= -e^{-x}\,\frac{x\ln x + 1}{x\ln^2 x}$$

Poniamo $g(x) = x \ln x + 1$

$\lim_{x \to 0^+} g(x) = 0 + 1 = 1$ $\qquad$ $\lim_{x \to +\infty} g(x) = +\infty$

$g'(x) = \ln x + 1$; $g'(x) = 0 \Rightarrow \ln x + 1 = 0 \rightarrow x = e^{-1}$

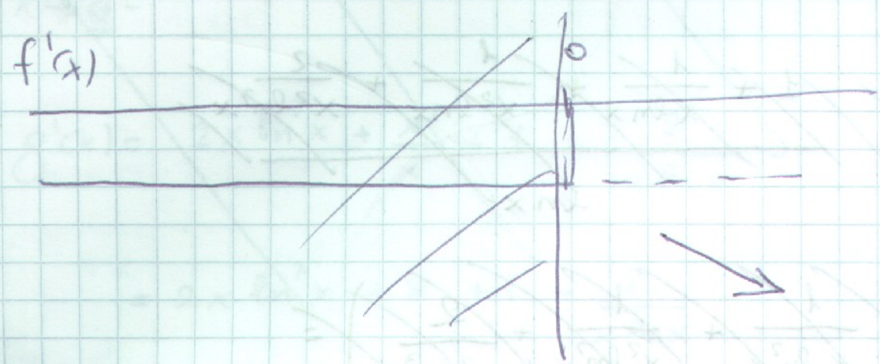


$x = \frac{1}{e}$ punto di minimo

$g\left(\frac{1}{e}\right) = \frac{1}{e} \ln e^{-1} + 1 = -\frac{1}{e} + 1 > 0$

allora $g(x)$ è sempre positiva per cui $f'(x) > 0$ per $x<0$

ed f è sempre decrescente



CONCAVITÀ E CONVESSITÀ

$$f''(x) = -\frac{\left[e^{-x}\ln x + x\left(-e^{-x}\ln x + \frac{e^{-x}}{x}\right)\right] x \ln^2 x - \left(x e^{-x}\ln x + e^{-x}\right)\left(\ln^2 x + 2x\frac{\ln x}{x}\right)}{x^2 \ln^4 x} =$$

$$= -\frac{\left(e^{-x}\ln x - x e^{-x}\ln x + e^{-x}\right) x \ln^2 x - \left(x e^{-x}\ln x + e^{-x}\right)\left(\ln x + 2\right)\ln x}{x^2 \ln^4 x}$$

$$f''(x) = -e^{-x}\,\frac{(\ln x - x\ln x + 1)x\ln x - (x\ln x + 1)(\ln x + 2)}{x^2\ln^3 x}$$

$$f''(x) = -e^{-x}\,\frac{x\ln^2 x - x^2\ln^2 x + x\ln x - x\ln^2 x - 2x\ln x - \ln x - 2}{x^2\ln^3 x} =$$

$$= -e^{-x}\,\frac{-x^2\ln^2 x - x\ln x - \ln x - 2}{x^2\ln^3 x} =$$

$$= e^{-x}\,\frac{x^2\ln^2 x + x\ln x + \ln x + 2}{x^2\ln^3 x}$$

$$\lim_{x\to 0^+} f''(x) = \cancel{1\,[0 + 0 + \infty]}$$

$$= \lim_{x\to 0^+} \cancel{f''(x) = e^{-x}\,\frac{1 + \frac{1}{x\ln x} + \frac{1}{x^2\ln x} + \frac{2}{x^2\ln^2 x}}{\ln x}}$$

$$= \lim_{x\to 0^+} \cancel{e^{-x}\left(\frac{1}{\ln x} + \frac{1}{x\ln^2 x} + \frac{1}{x^2\ln^2 x} + \frac{2}{x^2\ln^3 x}\right)} =$$

$$= \cancel{1\,(0 + \infty + \infty)}$$

$$= \lim_{x\to 0^+} \cancel{e^{-x}\,\frac{x^2\ln x + x + 1 + \frac{2}{\ln x}}{x^2\ln^2 x}} = \cancel{+\infty}$$

~~Su~~

$$\lim_{x\to 1^-} f''(x) = 1/\ \cancel{0/+\mathcal{F}-2}$$

$$= \lim_{x\to 1^-} e^{-x}\,\frac{\ln x + \frac{1}{x} + \frac{1}{x^2} + \frac{2}{x^2\ln x}}{\ln^2 x} =$$

$$= \lim_{x\to 1^-} e^{-x}\,\frac{1 + \frac{1}{x\ln x} + \frac{1}{x^2\ln x} + \frac{2}{x^2\ln^2 x}}{\ln x} = -\infty$$

$$g(x) = x^2\ln^2 x + x\,\cancel{\ln x} + \overset{1}{\cancel{\ln}}\,x + \frac{2}{\ln x}$$

$$\lim_{x\to 0^+} g(x) = 0 + 0 + \frac{1}{\infty} + 0 = \cancel{\infty}\ 1$$

$$\lim_{x\to 1^-} g(x) = 0 + 1 + 1 + \infty = -\infty$$

$$g'(x) = 2x\ln^2 x + \frac{x^2}{x} + 1 - \frac{2}{x\ln x} =$$

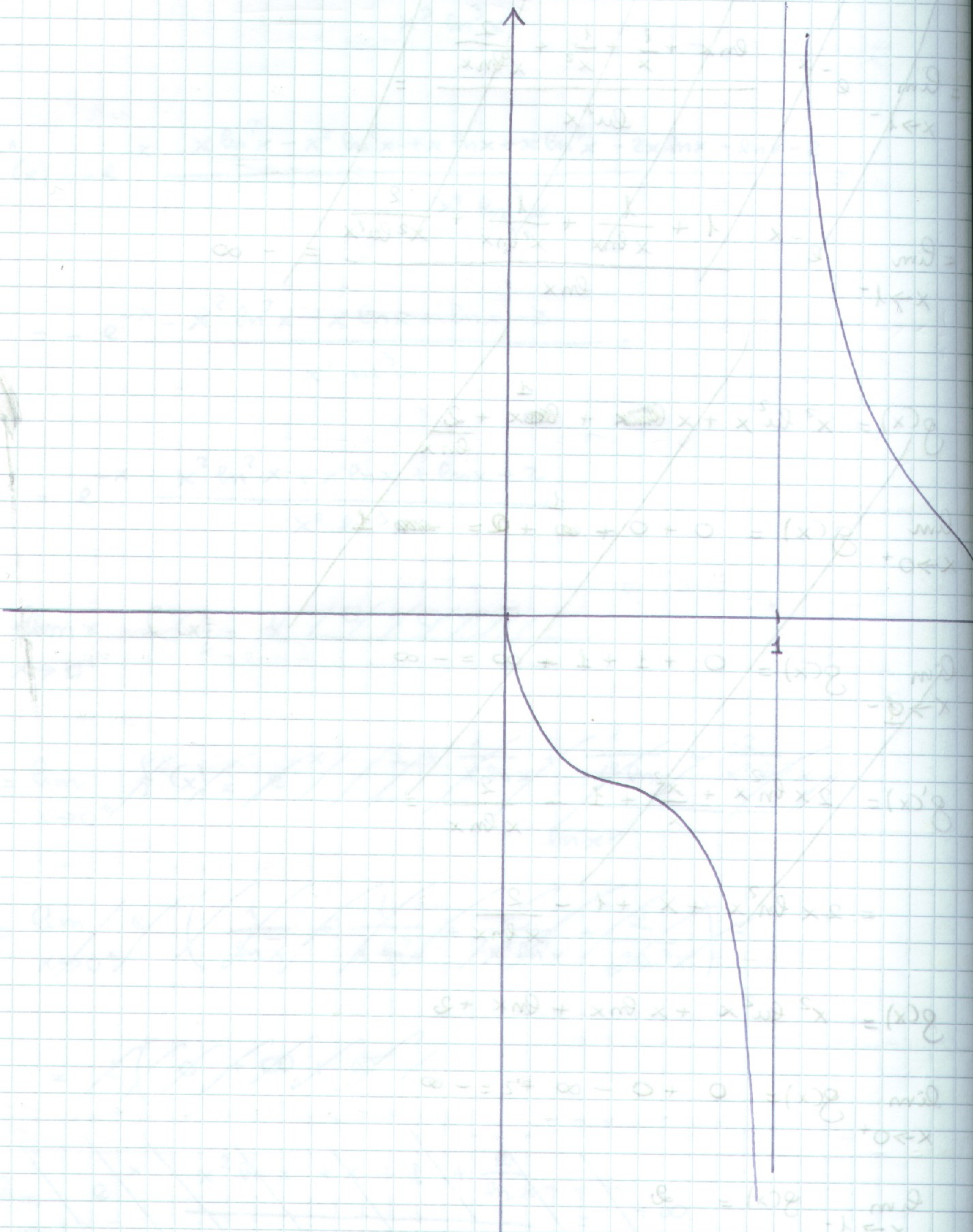
$$= 2x\ln^2 x + x + 1 - \frac{2}{x\ln x}$$

$$g(x) = x^2\ln^2 x + x\ln x + \ln x + 2$$

$$\lim_{x\to 0^+} g(x) = 0 + 0 - \infty + 2 = -\infty$$

$$\lim_{x\to 1^-} g(x) = 2$$

$$g'(x) = 2x\ln^2 x + \frac{2x^2\ln x}{x} + \ln x + \frac{x}{x} + \frac{1}{x} =$$


1