$$f(x) = \sqrt{1+x} - |x|$$

CAMPO DI ESISTENZA

$1+x > 0 \qquad x > -1$

C.E.: $]-1, +\infty[$

$$f(x) = \begin{cases} \sqrt{1+x} - x & x > 0 \\ \sqrt{1+x} + x & x < 0 \end{cases}$$

POSITIVITÀ

per $x > 0$

$\sqrt{1+x} - x > 0 \quad ; \quad \sqrt{1+x} > x$

essendo $x > 0$ allora è soddisfatta per $1+x > x^2$

$x^2 - x - 1 < 0 \qquad x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2} \rightarrow x < \frac{1+\sqrt{5}}{2}$

per $x < 0$

$\sqrt{1+x} + x > 0 \qquad \sqrt{1+x} > -x \qquad$ essendo $-x > 0$

allora deve essere

$1+x > x^2 \qquad x^2 + x - 1 < 0 \ ; \ -1 < \frac{1-\sqrt{5}}{2} < x$

Interseca l'asse x in $x = \frac{1 \pm \sqrt{5}}{2}$

interseca l'asse y in $y = 1$

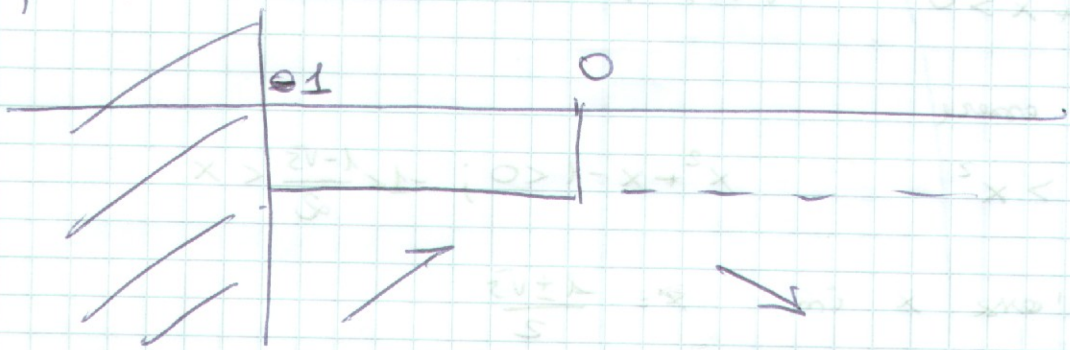
MAX E MIN

$$f'(x) = \begin{cases} \dfrac{1}{2\sqrt{1+x}} - 1 & x > 0 \\ \\ \dfrac{1}{2\sqrt{1+x}} + 1 & x < 0 \end{cases}$$

$$f'(x) > 0 \Rightarrow \begin{cases} \dfrac{1}{2\sqrt{1+x}} > 1 \\ \\ \dfrac{1}{2\sqrt{1+x}} > -1 \end{cases} \quad \begin{cases} \sqrt{1+x} < \dfrac{1}{2} \\ \\ \forall x \end{cases} \quad \begin{cases} 1+x < \dfrac{1}{4} \\ \\ \forall x \end{cases}$$

$$\begin{cases} x < -\dfrac{3}{4} \\ \\ \forall x \end{cases}$$

complessivamente



$x = 0$ punto di max

$$\lim_{x \to 0^+} f'(x) = \frac{1}{2} - 1 = -\frac{1}{2}$$

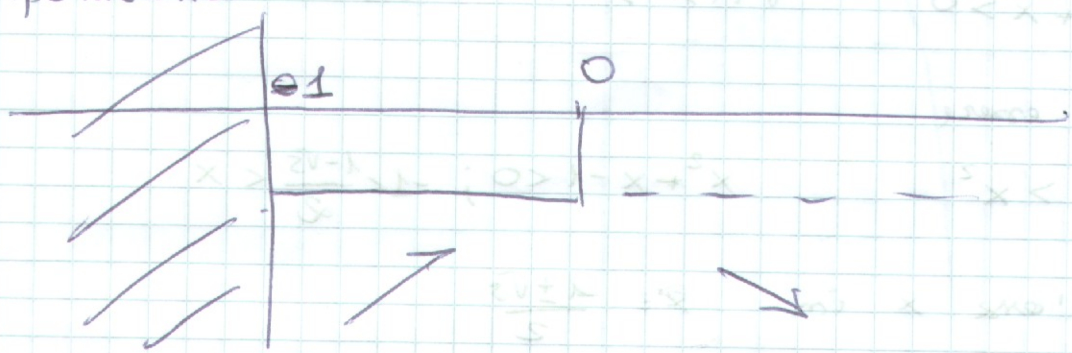
MAX E MIN

$$f'(x) = \begin{cases} \dfrac{1}{2\sqrt{1+x}} - 1 & x > 0 \\[2ex] \dfrac{1}{2\sqrt{1+x}} + 1 & x < 0 \end{cases}$$

$$f'(x) > 0 \Rightarrow \begin{cases} \dfrac{1}{2\sqrt{1+x}} > 1 \\[2ex] \dfrac{1}{2\sqrt{1+x}} > -1 \end{cases} \qquad \begin{cases} \sqrt{1+x} < \dfrac{1}{2} \\[2ex] \forall x \end{cases} \qquad \begin{cases} 1+x < \dfrac{1}{4} \\[2ex] \forall x \end{cases}$$

$$\begin{cases} x < -\dfrac{3}{4} \\[2ex] \forall x \end{cases}$$

complessivamente



$x=0$ punto di max

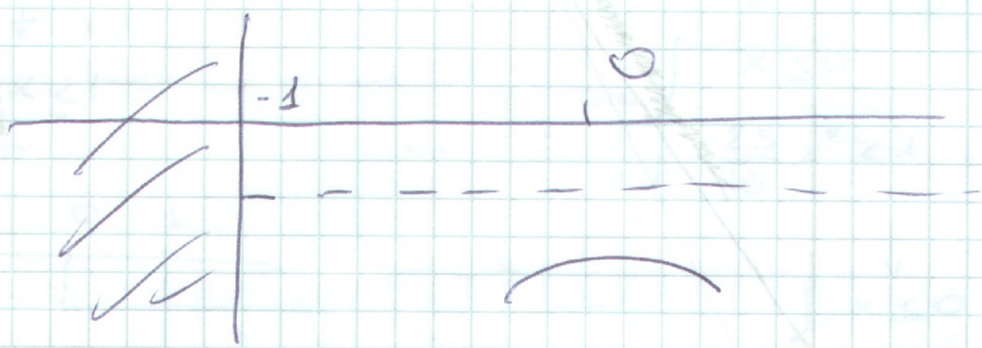
$$\lim_{x \to 0^+} f'(x) = \frac{1}{2} - 1 = -\frac{1}{2}$$

CONCAVITÀ E CONVESSITÀ

$$f''(x) = \begin{cases} \frac{1}{2} D(1+x)^{-\frac{1}{2}} \cancel{+} 0 & x > 0 \\ \frac{1}{2} D(1+x)^{-\frac{1}{2}} + 0 & x < 0 \end{cases}$$

$$f''(x) = -\frac{1}{4}(1+x)^{-\frac{3}{2}} = -\frac{1}{4\sqrt{(1+x)^3}} = -\frac{1}{4(1+x)\sqrt{1+x}}$$

$f''(x) > 0 \quad 1+x < 0 \quad \Rightarrow x < -1$

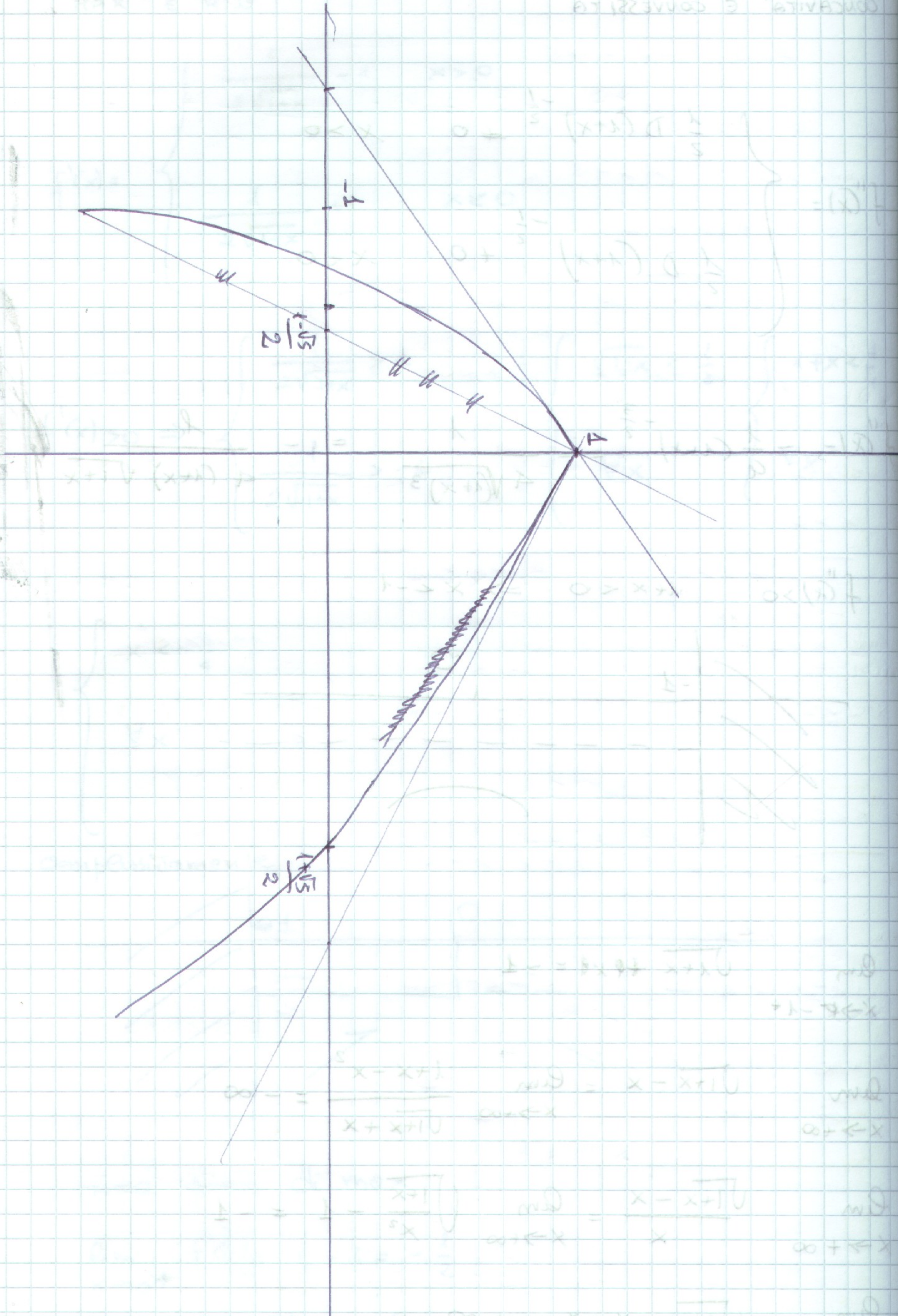


$$\lim_{x \to -1^+} \sqrt{1+x} \cancel{+} x = -1$$

$$\lim_{x \to +\infty} \sqrt{1+x} - x = \lim_{x \to +\infty} \frac{1+x-x^2}{\sqrt{1+x}+x} = -\infty$$

$$\lim_{x \to +\infty} \frac{\sqrt{1+x} - x}{x} = \lim_{x \to +\infty} \sqrt{\frac{1+x}{x^2}} - 1 = -1$$

$q = \lim \sqrt{1+x} - x + x = +\infty$



-1
1-√5 / 2
1+√5 / 2
1