$$f(x) = 3x + 4\sqrt{1-x^2}$$

CAMPO DI ESISTENZA

$1-x^2 \geq 0 \qquad -1 \leq x \leq 1 \qquad$ CE: $[-1, 1]$

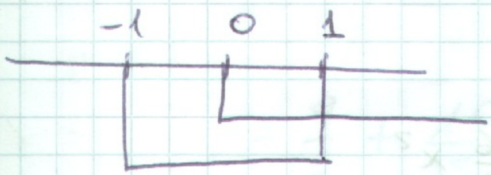
POSITIVITÀ

$3x + 4\sqrt{1-x^2} > 0 \quad ; \quad \sqrt{1-x^2} > -\frac{3}{4}x$

$$\begin{cases} x > 0 \\ 1-x^2 \geq 0 \end{cases} \qquad \begin{cases} x \leq 0 \\ 1-x^2 > \frac{9}{16}x^2 \end{cases} \; ; \; \begin{cases} x \leq 0 \\ \left(\frac{9}{16}+1\right)x^2 < 1 \end{cases}$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\begin{cases} -1 \leq x \leq 1 \\ x > 0 \end{cases} \qquad \begin{cases} x \leq 0 \\ \frac{25}{16}x^2 < 1 \end{cases} \qquad \begin{cases} x \leq 0 \\ x^2 < \frac{16}{25} \end{cases}$$



$$\begin{cases} x \leq 0 \\ -\frac{4}{5} < x < \frac{4}{5} \end{cases}$$

$$\downarrow \qquad\qquad \downarrow$$

$$0 < x \leq 1 \qquad\qquad -\frac{4}{5} < x \leq 0$$

Allora

$f(x) > 0$ per $-\frac{4}{5} < x \leq 1$

Intersezione con l'asse y ; $x = 0 \to y = 4$

Intersezione con l'asse x ; $y = 0$

$3x + 4\sqrt{1-x^2} = 0 \qquad \sqrt{1-x^2} = -\frac{3}{4}x \quad ; \quad 1-x^2 = \frac{9}{16}x^2$

$$\begin{cases} \frac{25}{16}x^2 = 1 \end{cases} \qquad \begin{cases} x^2 = \frac{16}{25} \end{cases} \qquad \begin{cases} x = \pm\frac{4}{5} \end{cases} \qquad x = -\frac{4}{5}$$

$f(x) = 3x + 4\sqrt{1-x^2}$

CAMPO DI ESISTENZA

$1 - x^2 \geq 0 \qquad -1 \leq x \leq 1 \qquad ce: [-1, 1]$

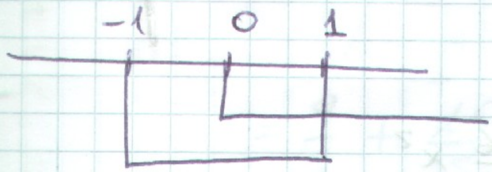
POSITIVITÀ

$3x + 4\sqrt{1-x^2} > 0 \quad ; \quad \sqrt{1-x^2} > -\frac{3}{4}x$

$\begin{cases} x > 0 \\ 1 - x^2 \geq 0 \end{cases} \qquad \begin{cases} x \leq 0 \\ 1 - x^2 > \frac{9}{16}x^2 \end{cases} \; ; \; \begin{cases} x \leq 0 \\ \left(\frac{9}{16}+1\right)x^2 < 1 \end{cases}$

$\downarrow \qquad\qquad \downarrow$

$\begin{cases} -1 \leq x \leq 1 \\ x > 0 \end{cases} \qquad \begin{cases} x \leq 0 \\ \frac{25}{16}x^2 < 1 \end{cases} \qquad \begin{cases} x \leq 0 \\ x^2 < \frac{16}{25} \end{cases}$



$\begin{cases} x \leq 0 \\ -\frac{4}{5} < x < \frac{4}{5} \end{cases}$

$\downarrow$

$0 < x \leq 1 \qquad\qquad -\frac{4}{5} < x \leq 0$

Allora

$f(x) > 0$ per $-\frac{4}{5} < x \leq 1$

Intersezione con l'asse y ; $x = 0 \rightarrow y = 4$

Intersezione con l'asse x ; $y = 0$

$3x + 4\sqrt{1-x^2} = 0 \qquad \sqrt{1-x^2} = -\frac{3}{4}x \quad ; \quad 1 - x^2 = \frac{9}{16}x^2$

$\begin{cases} \frac{25}{16}x^2 = 1 \\ x \leq 0 \end{cases} \qquad \begin{cases} x^2 = \frac{16}{25} \\ x \leq 0 \end{cases} \qquad \begin{cases} x = \pm\frac{4}{5} \\ x \leq 0 \end{cases} \qquad x = -\frac{4}{5}$

$f(-1) = \dots \qquad f(1) = 3$

CAMPO DI ESISTENZA

$1-x^2 \geq 0 \qquad -1 \leq x \leq 1 \qquad \text{ce}: [-1, 1]$

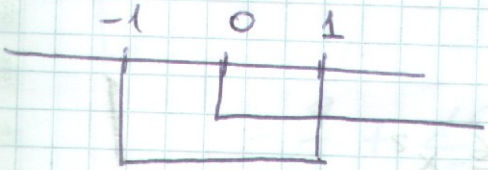
POSITIVITÀ

$3x + 4\sqrt{1-x^2} > 0 \quad ; \quad \sqrt{1-x^2} > -\frac{3}{4}x$

$\begin{cases} x > 0 \\ 1-x^2 \geq 0 \end{cases} \qquad \begin{cases} x \leq 0 \\ 1-x^2 > \frac{9}{16}x^2 \end{cases} \qquad \begin{matrix} x \leq 0 \\ ; \left(\frac{9}{16}+1\right)x^2 < 1 \end{matrix}$

$\downarrow \qquad\qquad \downarrow$

$\begin{cases} -1 \leq x \leq 1 \\ x > 0 \end{cases} \qquad \begin{cases} x \leq 0 \\ \frac{25}{16}x^2 < 1 \end{cases} \qquad \begin{cases} x \leq 0 \\ x^2 < \frac{16}{25} \end{cases}$



$\downarrow$

$0 < x \leq 1$

$\downarrow$

$\begin{cases} x \leq 0 \\ -\frac{4}{5} < x < \frac{4}{5} \end{cases}$

$\downarrow$

$-\frac{4}{5} < x \leq 0$

Allora

$f(x) > 0$ per $-\frac{4}{5} < x \leq 1$

Intersezione con l'asse y ; $x = 0 \rightarrow y = 4$

Intersezione con l'asse x ; $y = 0$

$3x + 4\sqrt{1-x^2} = 0 \qquad \sqrt{1-x^2} = -\frac{3}{4}x \quad ; \quad 1-x^2 = \frac{9}{16}x^2$

$\begin{cases} \frac{25}{16}x^2 = 1 \\ x \leq 0 \end{cases} \qquad \begin{cases} x^2 = \frac{16}{25} \\ x \leq 0 \end{cases} \qquad \begin{cases} x = \pm\frac{4}{5} \\ x \leq 0 \end{cases} \qquad x = -\frac{4}{5}$

$f(-1) = -3 \qquad f(1) = 3$

MAX E MIN.

$$f'(x) = 3 + \frac{4}{2\sqrt{1-x^2}}(-2x) = 3 - \frac{4x}{\sqrt{1-x^2}}$$

$$f'(x)=0 \rightarrow \frac{3\sqrt{1-x^2} - 4x}{\sqrt{1-x^2}} = 0$$

$3\sqrt{1-x^2} = 4x$; $9(1-x^2) = 16x^2$; $9 - 9x^2 = 16x^2$

$25x^2 = 9$; $x^2 = \frac{9}{25}$; $x = \pm\frac{3}{5}$

$f'(x) > 0$ $\quad 3\sqrt{1-x^2} - 4x > 0$

$\sqrt{1-x^2} > \frac{4}{3}x$ che dà luogo ai due sistemi

$$\begin{cases} x < 0 \\ 1-x^2 > 0 \end{cases} \qquad \begin{cases} x \geq 0 \\ 1-x^2 > \frac{16}{9}x^2 \end{cases}$$

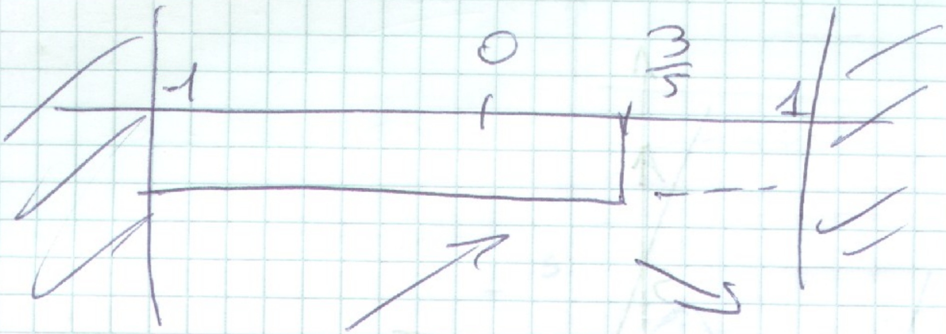
$$\downarrow \qquad \downarrow$$

$$\begin{cases} x < 0 \\ -1 < x < 1 \end{cases} \qquad \begin{cases} x \geq 0 \\ x^2 < \frac{9}{25} \end{cases} \qquad \begin{cases} x \geq 0 \\ -\frac{3}{5} < x < \frac{3}{5} \end{cases}$$

$$\downarrow \qquad \downarrow$$

$-1 < x < 0$ $\qquad 0 \leq x < \frac{3}{5}$

$f'(x) > 0$ per $-1 < x < \frac{3}{5}$



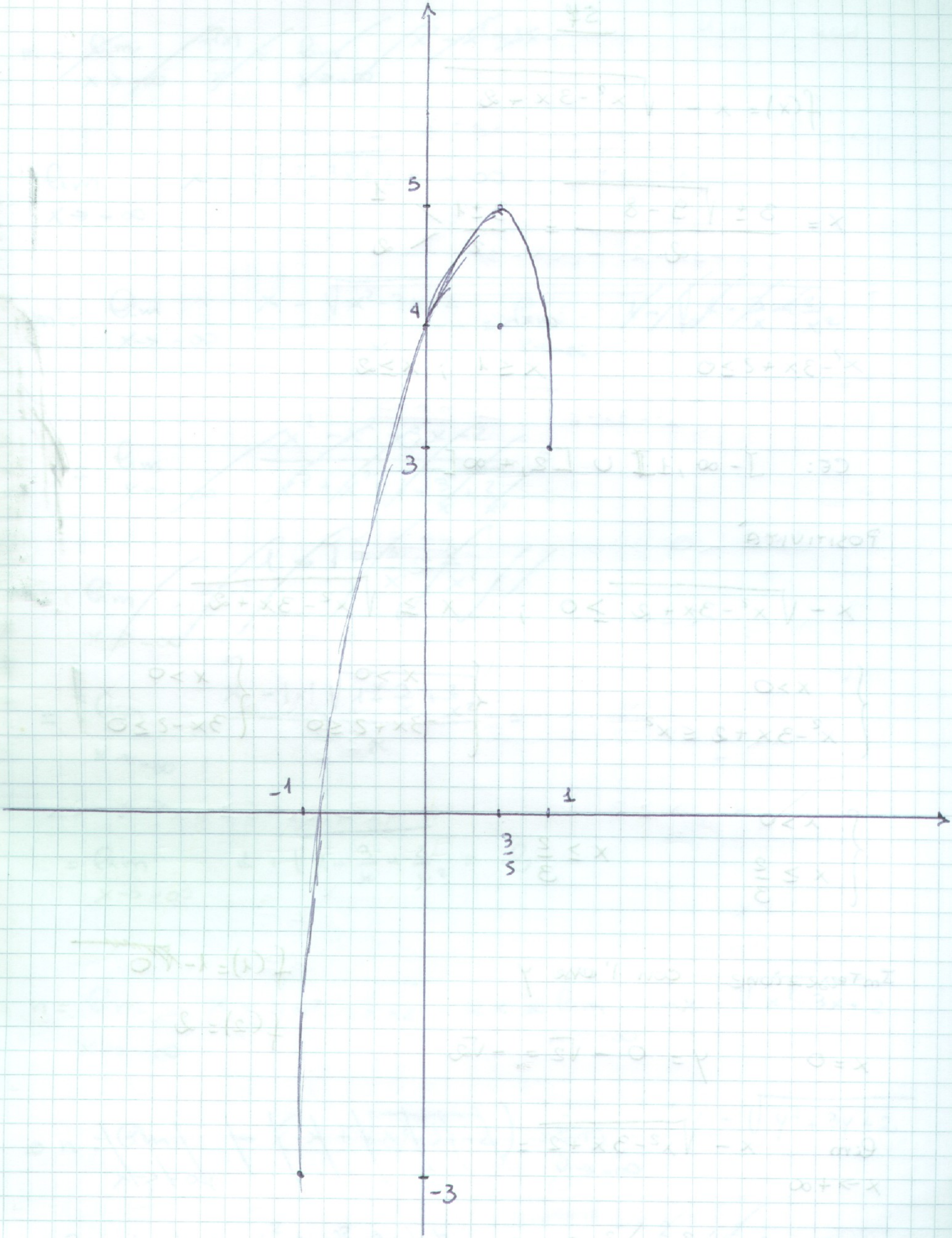
$x = \frac{3}{5}$ punto di max

$$f\left(\frac{3}{5}\right) = 3 \cdot \frac{3}{5} + 4\sqrt{1 - \frac{9}{25}} =$$

$$= \frac{9}{5} + 4\sqrt{\frac{25-9}{25}} =$$

$$= \frac{9}{5} + 4\sqrt{\frac{16}{25}} = \frac{9}{5} + 4 \cdot \frac{4}{5} =$$

$$= \frac{9}{5} + \frac{16}{5} = \frac{25}{5} = 5$$


5
4
3
-3
-1
1
$\frac{3}{5}$