$$f(x) = x + \log(x^2 - 1)$$

CAMPO DI ESISTENZA

$x^2 - 1 > 0$ ; $x < -1$ , $x > 1$

~~POSTI~~ ~~y≠0~~ Non possiamo calcolare la positività

$$\lim_{x \to -\infty} x + \log(x^2-1) = -\infty + \infty$$

$$\lim_{x \to -\infty} x\left(1 + \frac{\log(x^2-1)}{x}\right) =$$

$$= \lim_{x \to -\infty} x\left(1 + \frac{\log(x-1)}{x} + \frac{\log(x+1)}{x}\right] = -\infty(1) = -\infty$$

$$\left[\begin{array}{l} \lim_{x \to -\infty} \frac{\log(x-1)}{x} = \lim_{x \to -\infty} \frac{1}{x-1} = 0 \\ \lim_{x \to -\infty} \frac{\log(x+1)}{x} = 0 \end{array}\right]$$

$$\lim_{x \to +\infty} x + \log(x^2-1| = +\infty$$

$$\lim_{x \to -1^-} x + \log(x^2-1) = -\infty$$

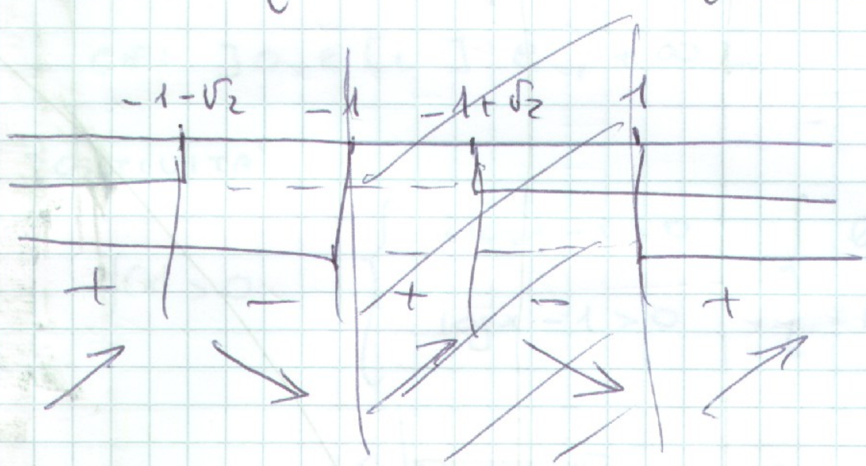
$$\lim_{x \to 1^+} x + \log(x^2-1) = \cancel{+}\infty$$

MAX E MIN

$$f'(x) = 1 + \frac{2x}{x^2-1} = \frac{x^2+2x-1}{x^2-1}$$

$$x = -1 \pm \sqrt{1+1} = -1 \pm \sqrt{2}$$

$$f'(x) > 0 \quad \begin{cases} x^2+2x-1 > 0 \\ x^2-1 > 0 \end{cases} \quad \begin{cases} x < -1-\sqrt{2} \ , \ x > -1+\sqrt{2} \\ x < -1 \ , \ x > 1 \end{cases}$$




$x = -1-\sqrt{2}$ punto di max

$x = -1$ ~~punto di minimo~~ $\notin CE$

$x = -1+\sqrt{2}$ ~~punto di max~~ $\notin CE$

$x = 1$ ~~punto di minimo~~ $\notin CE$

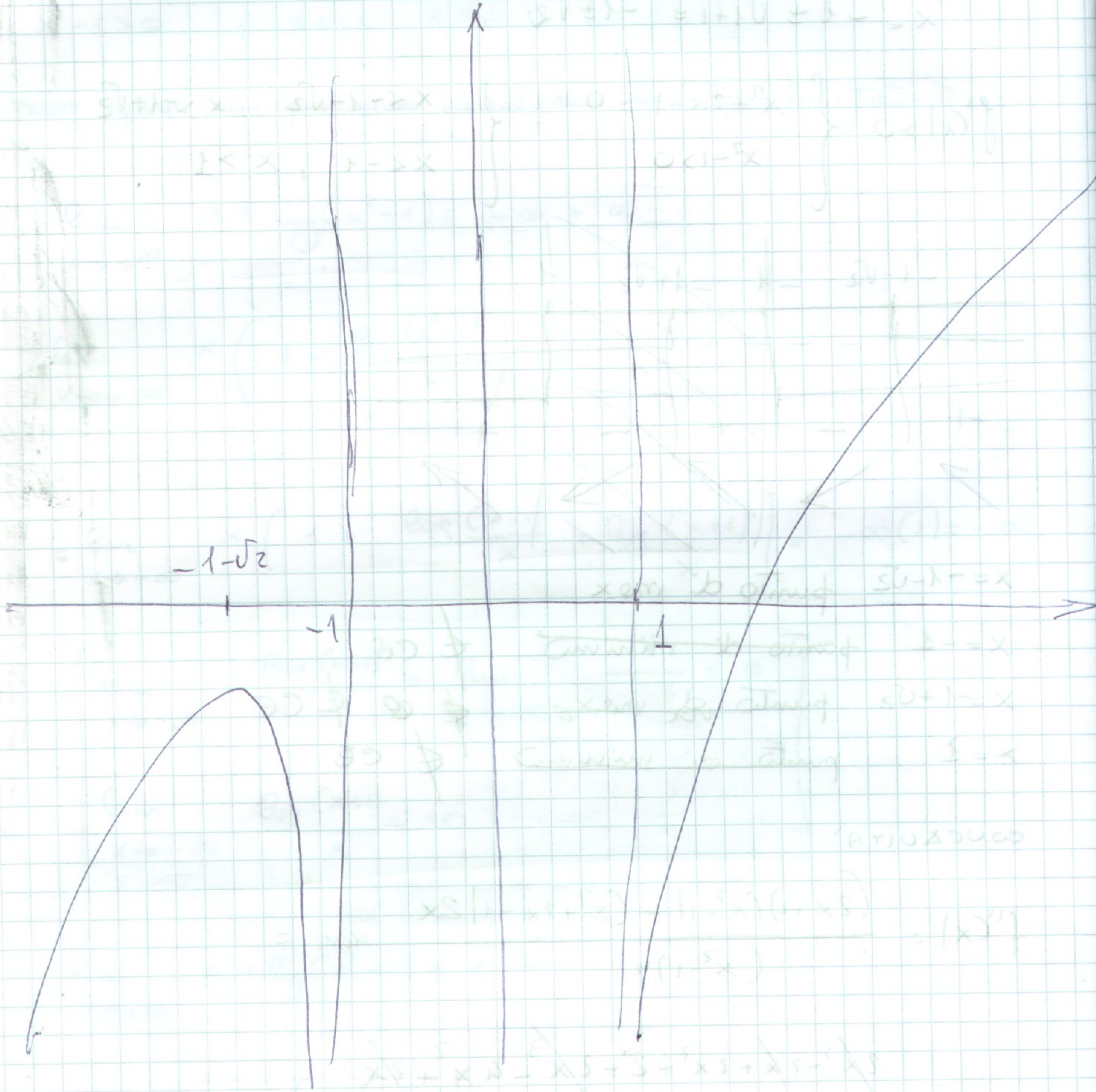
CONCAVITA'

$$f''(x) = \frac{(2x+2)(x^2-1) - (x^2+2x-1)2x}{(x^2-1)^4} =$$

$$= \frac{\cancel{2x^3} - \cancel{2x} + 2x^2 - 2 - \cancel{2x^3} - 4x^2 + \cancel{2x}}{(x^2-1)^4}$$

$$= \frac{-2x^2-2}{(x^2-1)^4} = -2\frac{x^2-1}{(x^2-1)^4} = -\frac{2}{(x^2-1)^3}$$

$f''(x) < 0 \quad \forall x \in CE \quad f(x)$ sempre concava



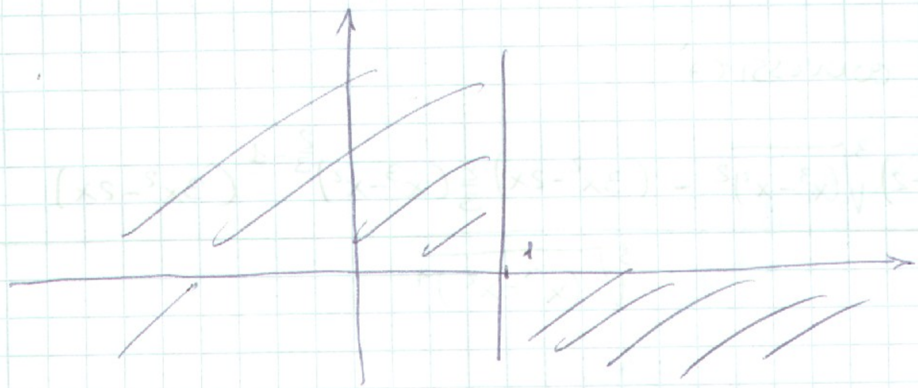
y
-1-√2
-1
1

$f(x) = \sqrt[3]{x^3 - x^2}$

CE: $\mathbb{R}$

POSITIVITÀ

$f(x) > 0 \qquad x^3 - x^2 > 0 \; ; \; x^2(x-1) > 0 \; ; \; x > 1$



INTERSEZIONI CON GLI ASSI

asse x $\Rightarrow$ $y = 0 \qquad x = 0 \; , \; x = 1$

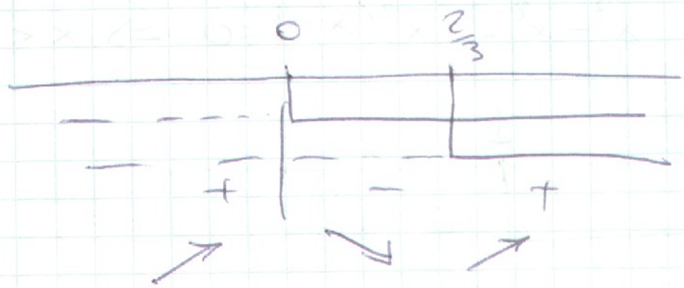
asse y - $x = 0 \qquad y = 0$

MAX E MIN

$$f'(x) = \frac{1}{3}(x^3 - x^2)^{\frac{1}{3}-1}(3x^2 - 2x) =$$

$$= \frac{1}{3} \frac{3x^2 - 2x}{\sqrt[3]{(x^3 - x^2)^2}}$$

$f'(x) = 0 \qquad x(3x - 2) = 0 \qquad x = 0 \; ; \; x = \frac{2}{3}$



$x = \frac{2}{3}$ min $\qquad f\left(\frac{2}{3}\right) = \sqrt[3]{\left(\frac{2}{3}\right)^3 - \left(\frac{2}{3}\right)^2} = \sqrt[3]{\frac{8}{27} - \frac{4}{9}} =$

$= \sqrt[3]{\frac{8-12}{27}} = \sqrt[3]{-\frac{4}{27}} = -\frac{\sqrt[3]{4}}{3}$

in $x=0$ e $x=1$

la derivata è infinita

$\lim_{x\to 0^-} f'(x) = +\infty \qquad \lim_{x\to 0^+} f'(x) = -\infty$

CONCAVITÀ E CONVESSITÀ

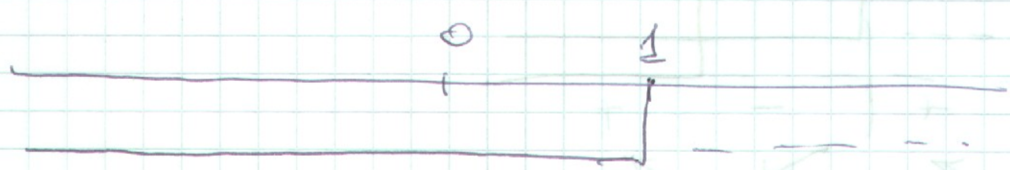
$$f''(x) = \frac{1}{3}\,\frac{(6x-2)\sqrt[3]{(x^3-x^2)^2} - (3x^2-2x)\frac{2}{3}(x^3-x^2)^{\frac{2}{3}-1}(3x^2-2x)}{\sqrt[3]{(x^3-x^2)^4}} =$$

$$= \frac{1}{3}\,\frac{(6x-2)\sqrt[3]{(x^3-x^2)^2} - x^2(9x^2+4-12x)\frac{2}{3}\frac{1}{\sqrt{x^3-x^2}}}{\sqrt[3]{(x^3-x^2)^4}} =$$

$$= \frac{1}{3}\,\frac{(6x-2)(x^3-x^2) - 6x^4 - \frac{8}{3}x^2 + 8x^3}{\sqrt[3]{x^3-x^2}\,\sqrt[3]{(x^3-x^2)^4}}$$

$$= \frac{1}{3}\,\frac{6x^4-6x^3-2x^3+2x^2-6x^4-\frac{8}{3}x^2+8x^3}{\sqrt[3]{x^3-x^2}\,\sqrt[3]{(x^3-x^2)^4}} = \frac{1}{3}\,\frac{-\frac{8}{3}x^2}{\sqrt[3]{x^3-x^2}\,\sqrt[3]{(x^3-x^2)^4}}$$

$f''(x) > 0 \Rightarrow x^3 - x^2 = x^2(x-1) < 0 \Rightarrow x < 1$



$x=\frac{2}{3}$ min $\qquad f\left(\frac{2}{3}\right)=\sqrt[3]{\left(\frac{2}{3}\right)^3-\left(\frac{2}{3}\right)^2}=\sqrt[3]{\frac{8}{27}-\frac{4}{9}}=$

$$=\sqrt[3]{\frac{8-12}{27}}=\sqrt[3]{-\frac{4}{27}}=-\frac{\sqrt[3]{4}}{3}$$

in $x=0$ e $x=1$

la derivata è infinita

$\lim_{x\to 0^-} f'(x)=+\infty \qquad \lim_{x\to 0^+} f'(x)=-\infty$

CONCAVITÀ E CONVESSITÀ

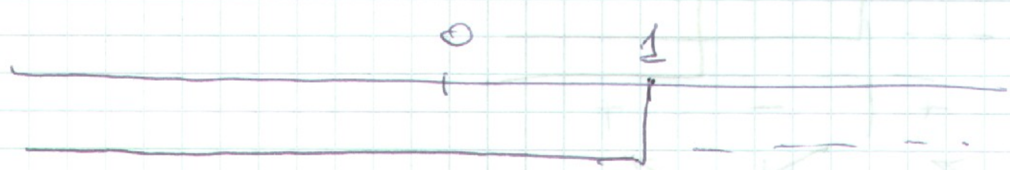
$$f''(x)=\frac{1}{3}\,\frac{(6x-2)\sqrt[3]{(x^3-x^2)^2}-(3x^2-2x)\frac{2}{3}(x^3-x^2)^{\frac{2}{3}-1}(3x^2-2x)}{\sqrt[3]{(x^3-x^2)^4}}=$$

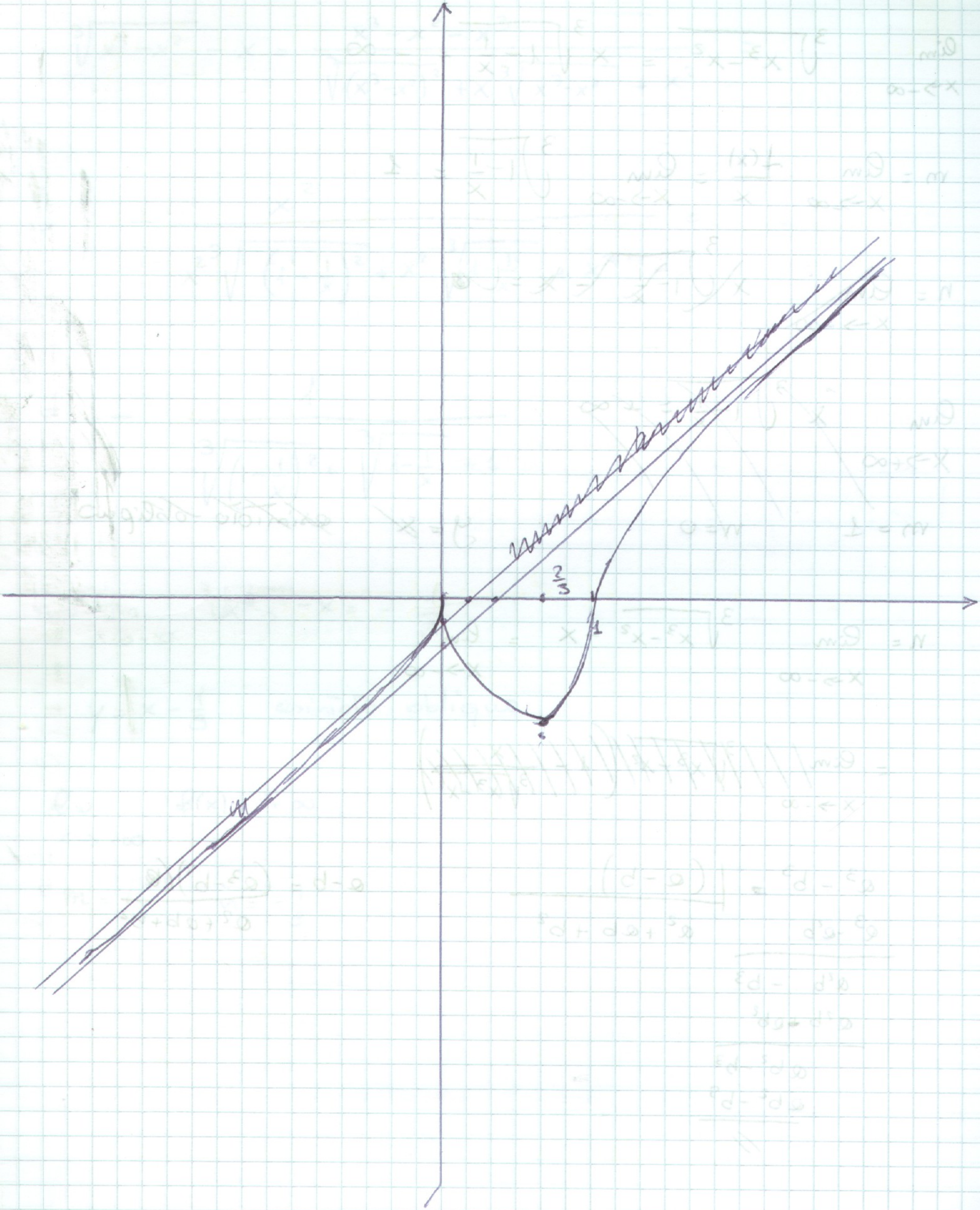
$$=\frac{1}{3}\,\frac{(6x-2)\sqrt[3]{(x^3-x^2)^2}-x^2(9x^2+4-12x)\frac{2}{3}\frac{1}{\sqrt{x^3-x^2}}}{\sqrt[3]{(x^3-x^2)^4}}=$$

$$=\frac{1}{3}\,\frac{(6x-2)(x^3-x^2)-6x^4-\frac{8}{3}x^2+8x^3}{\sqrt[3]{x^3-x^2}\,\sqrt[3]{(x^3-x^2)^4}}$$

$$=\frac{1}{3}\,\frac{6x^4-6x^3-2x^3+2x^2-6x^4-\frac{8}{3}x^2+8x^3}{\sqrt[3]{x^3-x^2}\,\sqrt[3]{(x^3-x^2)^4}}=\frac{1}{3}\,\frac{-\frac{8}{3}x^2}{\sqrt[3]{x^3-x^2}\,\sqrt[3]{(x^3-x^2)^4}}$$

$f'(x)>0 \Rightarrow \cancel{6x}\quad x^3-x^2=x^2(x-1)<0 \Rightarrow x<1$




1

$$\lim_{x\to-\infty} \sqrt[3]{x^3-x^2} = x\sqrt[3]{1-\frac{1}{x}} = -\infty$$

$$m = \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \sqrt[3]{1-\frac{1}{x}} = 1$$

$$\cancel{n = \lim_{x\to-\infty} x\sqrt[3]{1-\frac{1}{x}} - x = \infty}$$

$$\cancel{\lim_{x\to+\infty} x\sqrt[3]{1-\frac{1}{x}} = +\infty}$$

$m = 1$ $\qquad$ $n = 0$ $\qquad\qquad$ $y = x$ ~~asíntota oblicua~~

$$n = \lim_{x\to-\infty} \sqrt[3]{x^3-x^2} - x = \lim_{x\to-\infty}$$

$$= \lim_{x\to-\infty} \cancel{\sqrt[3]{x^3-x^2}\left(x - \frac{x}{\sqrt[3]{x^3-x^2}}\right)}$$

$$\begin{array}{l} a^3 - b^3 \\ a^3 - a^2b \\ \hline \quad a^2b - b^3 \\ \quad a^2b + ab^2 \\ \hline \qquad ab^2 - b^3 \\ \qquad ab^2 - b^3 \\ \hline \qquad // \end{array} = \frac{(a-b)}{a^2+ab+b^2}$$

$$a - b = \frac{(a^3-b^3)(}{a^2+ab+b^2}$$

$$\sqrt[3]{x^3-x^2}-x=\frac{x^3-x^2-x^3}{\sqrt[3]{(x^3-x^2)^2}+x\sqrt[3]{x^3-x^2}+x^2}=$$

$$=-\frac{x^2}{x^2\sqrt[3]{\left(1-\frac{1}{x}\right)^2}+x^2\sqrt[3]{1-\frac{1}{x}}+x^2}=$$

$$=-\frac{1}{\sqrt[3]{\left(1-\frac{1}{x}\right)^2}+\sqrt[3]{1-\frac{1}{x}}+1}$$

$$n=\lim_{x\to-\infty}\sqrt[3]{x^3-x^2}-x=-\frac{1}{3}$$

$y=x-\frac{1}{3}$ asintoto obliquo

$$\lim_{x\to+\infty}f(x)=+\infty$$

$m=1$ $\quad n=-\frac{1}{3}$

$$f(x) = \frac{x e^x}{\sqrt{2x-1}}$$

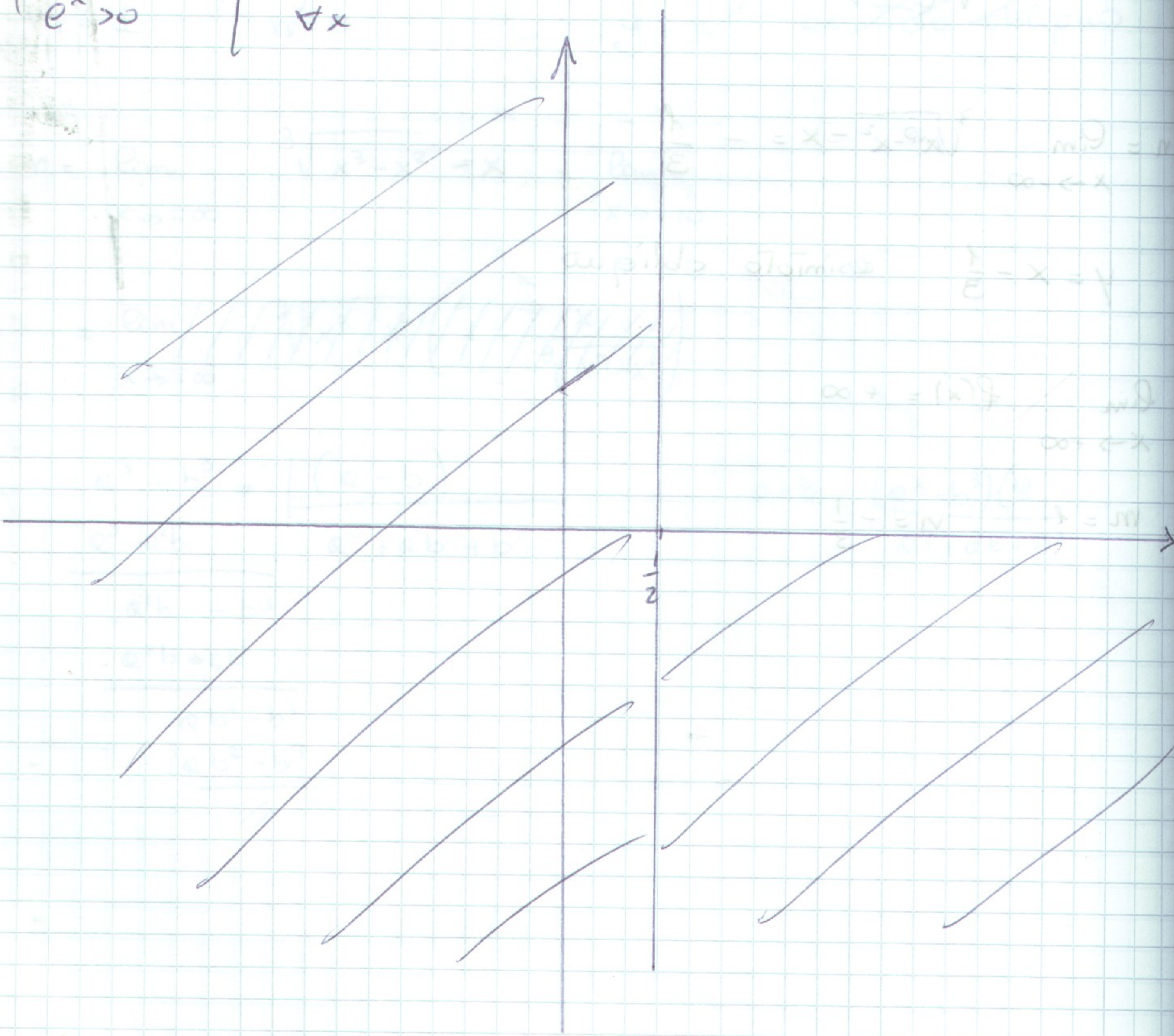
CAMPO DI ESISTENZA

$2x-1>0 \qquad x>\frac{1}{2}$

$CE: \ ]\frac{1}{2}; +\infty[$

POSITIVITÀ

$$\begin{cases} x>0 \\ e^x>0 \end{cases} \Rightarrow \begin{cases} x>0 \\ \forall x \end{cases} \Rightarrow f(x)>0 \quad \forall x \in CE$$



$$\lim_{x \to \frac{1}{2}^+} \frac{x e^x}{\sqrt{2x-1}} = +\infty$$

$$\lim_{x \to +\infty} \frac{x e^x}{\sqrt{2x-1}} = +\infty$$

$$m = \lim_{x \to +\infty} \frac{e^x}{\sqrt{2x-1}} = +\infty$$

MAX E MIN

$$f'(x) = \frac{(e^x + xe^x)\sqrt{2x-1} - \frac{xe^x}{2\sqrt{2x-1}} \cdot 2}{2x-1} =$$

$$= e^x \frac{(1+x)\sqrt{2x-1} - \frac{x}{\sqrt{2x-1}}}{2x-1}$$

$$= e^x \frac{(1+x)(2x-1) - x}{\sqrt{2x-1}\,(2x-1)} =$$

$$= e^x \frac{2x-1+2x^2-x-x}{\sqrt{2x-1}\,(2x-1)} = e^x \frac{2x^2-1}{\sqrt{2x-1}\,(2x-1)}$$

$$f'(x) > 0 \quad \begin{cases} 2x^2-1 > 0 \\ 2x-1 > 0 \end{cases} \quad \begin{cases} x < -\frac{\sqrt{2}}{2} \; ; \; x > \frac{\sqrt{2}}{2} \\ x > \frac{1}{2} \end{cases}$$

$$\lim_{x \to \frac{1}{2}^+} \frac{x e^x}{\sqrt{2x-1}} = +\infty$$

$$\lim_{x \to +\infty} \frac{x e^x}{\sqrt{2x-1}} = +\infty$$

$$m = \lim_{x \to +\infty} \frac{e^x}{\sqrt{2x-1}} = +\infty$$

MAX E MIN

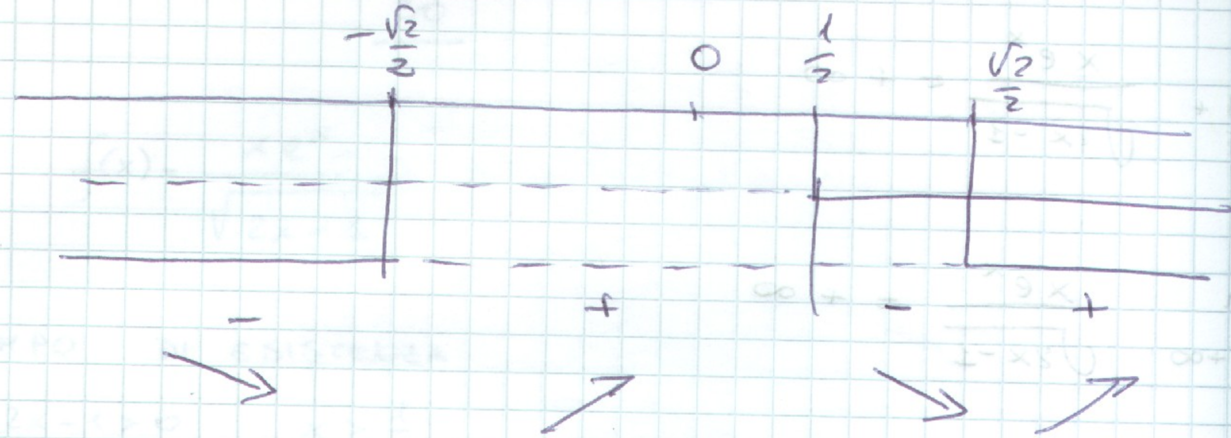
$$f'(x) = \frac{(e^x + xe^x)\sqrt{2x-1} - \frac{xe^x}{2\sqrt{2x-1}} \cdot 2}{2x-1} =$$

$$= e^x \frac{(1+x)\sqrt{2x-1} - \frac{x}{\sqrt{2x-1}}}{2x-1}$$

$$= e^x \frac{(1+x)(2x-1) - x}{\sqrt{2x-1}\,(2x-1)} =$$

$$= e^x \frac{2x-1+2x^2-x-x}{\sqrt{2x-1}\,(2x-1)} = e^x \frac{2x^2-1}{\sqrt{2x-1}\,(2x-1)}$$

$$f'(x) > 0 \quad \begin{cases} 2x^2-1>0 \\ 2x-1>0 \end{cases} \quad \begin{cases} x < -\frac{\sqrt{2}}{2} \ ; \ x > \frac{\sqrt{2}}{2} \\ x > \frac{1}{2} \end{cases}$$




$x = -\frac{\sqrt{2}}{2}$ min

$x = \frac{1}{2}$ max

non fanno parte del CE

$x = \frac{\sqrt{2}}{2}$ min

CONCAVITÀ E CONVESSITÀ

$$f'(x) = \frac{\left[e^x(2x^2-1) + e^x 4x\right](2x-1)^{\frac{3}{2}} - e^x(2x^2-1)\frac{3}{2}(2x-1)^{\frac{3}{2}-1}}{(2x-1)^3} =$$

$$= e^x \frac{(2x^2+4x-1)(2x-1)^{\frac{3}{2}} - \frac{3}{2}(2x^2-1)\sqrt{2x-1}}{(2x-1)^3} =$$

$$= e^x\sqrt{2x-1}\ \frac{2(2x^2+4x-1)(2x-1) - 3(2x^2-1)}{2\cancel{\sqrt{\ }}\,(2x-1)^3} =$$

$$= e^x\sqrt{2x-1}\ \frac{8x^3-4x^2+16x^2-8x-4x+2-6x^2+6}{(2x-1)^3} =$$

$$= e^x\sqrt{2x-1}\ \frac{4x^3+3x^2-6x+4}{(2x-1)^3}$$

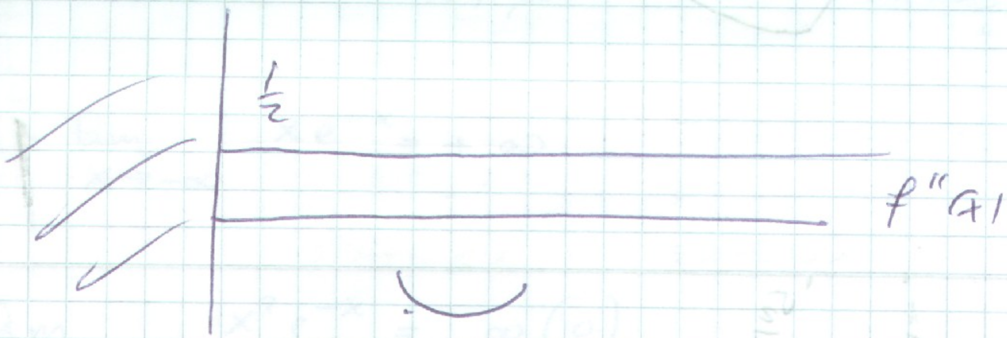
$$g\left(\frac{1}{2}\right) = 4\cdot\frac{1}{8} + 3\frac{1}{4} - \frac{6}{2} + 4 = \frac{5}{4} + 1 > 0$$

$$g'(x) = 12x^2 + 6x - 6 = 6(2x^2 + x - 1)$$

$$x = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1\pm 3}{4} \begin{cases} -1 \\ \frac{1}{2} \end{cases}$$

$g'(x) > 0$ per $x < -1$, $x > \frac{1}{2}$

allora $g(x)$ è crescente per $x > \frac{1}{2}$ e poiché $g\left(\frac{1}{2}\right) > 0$, allora $g(x)$ è crescente $\forall x > \frac{1}{2}$ per cui $f''(x) > 0$ se $2x - 1 > 0 \Rightarrow x > \frac{1}{2}$




e la $f$ è sempre convessa

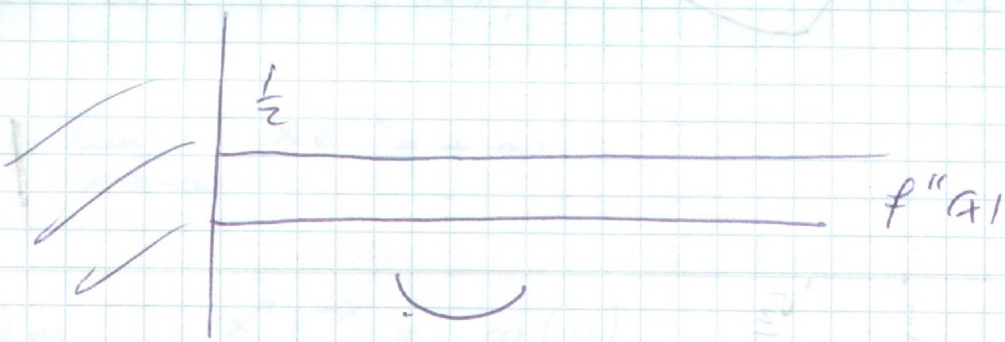
$$g\left(\frac{1}{2}\right) = 4\cdot\frac{1}{8} + 3\frac{1}{4} - \frac{6}{2} + 4 = \frac{5}{4} + 1 > 0$$

$$g'(x) = 12x^2 + 6x - 6 = 6(2x^2 + x - 1)$$

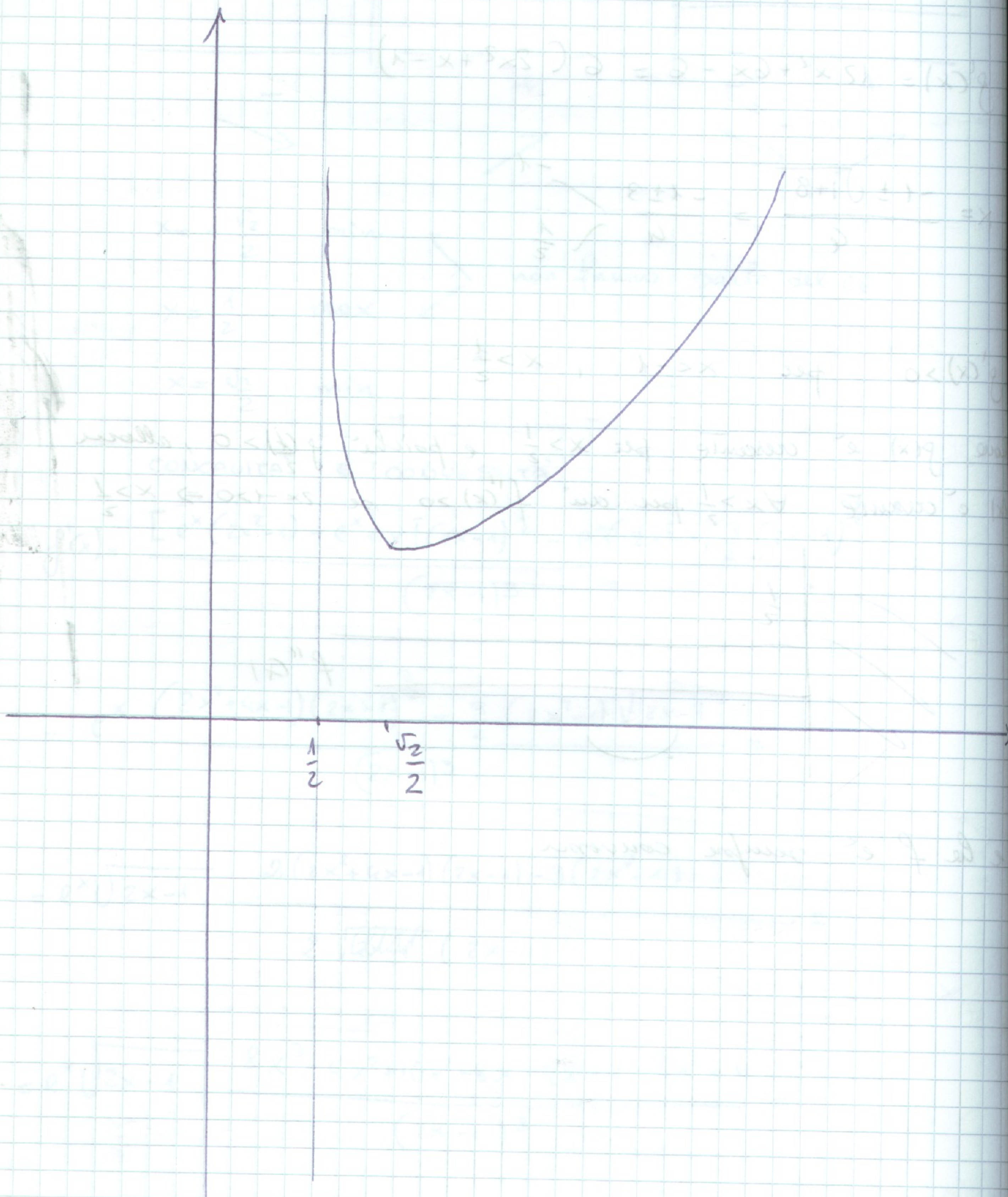
$$x = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4} \begin{cases} -1 \\ \frac{1}{2} \end{cases}$$

$g'(x) > 0$ per $x < -1$, $x > \frac{1}{2}$

allora $g(x)$ è crescente per $x > \frac{1}{2}$ e poichè $g(\frac{1}{2}) > 0$, allora $g(x)$ è crescente $\forall x > \frac{1}{2}$ per cui $f''(x) > 0$ se $2x - 1 > 0 \Rightarrow x > \frac{1}{2}$




e la $f$ è sempre convessa



$\frac{1}{2}$
$\frac{\sqrt{2}}{2}$

$f(x) = x^2 e^{-x}$

CE : $\mathbb{R}$

Positività : $f(x) > 0 \quad \forall x \neq 0$

Intersezione con gli assi

asse y $\quad x = 0 \Rightarrow y = 0 \cdot e^0 = 0$

asse x $\quad y = 0 \Rightarrow x = 0$

~~MAX E MIN~~

$\lim_{x \to -\infty} x^2 e^{-x} = \cancel{\lim_{x \to -\infty}} \quad \cancel{x^2} + \infty$

$m = \lim_{x \to -\infty} x e^{-x} = +\infty$

$\lim_{x \to +\infty} x^2 e^{-x} = \infty (0)$

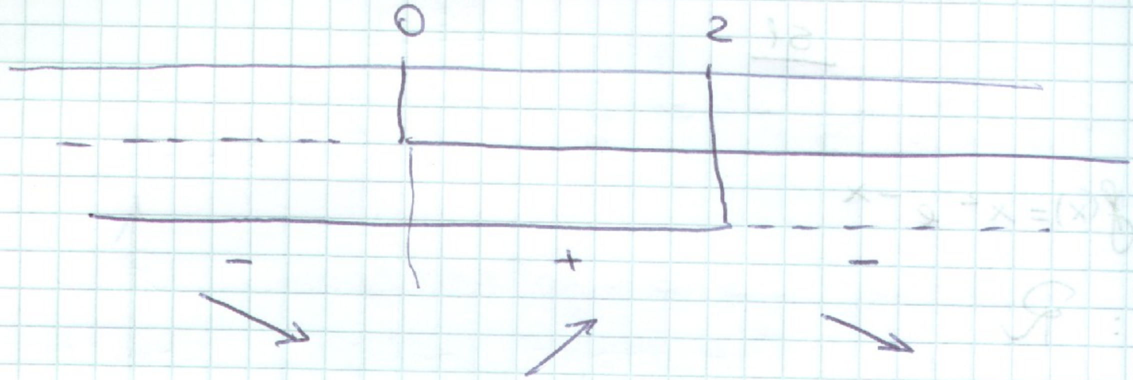
$\lim_{x \to +\infty} \frac{x^2}{e^x} = 0$

MAX E MIN

$f'(x) = 2x e^{-x} - x^2 e^{-x} = e^{-x} x (2 - x)$

$f'(x) = 0 \quad x = 0, \ x = 2$

$f'(x) > 0 \quad \begin{cases} x > 0 \\ 2 - x > 0 \end{cases} \quad \begin{cases} x > 0 \\ x < 2 \end{cases}$




$x=0$ punto di min $f(0) = 0$

$x=2$ punto di max $f(2) = 4e^{-2}$

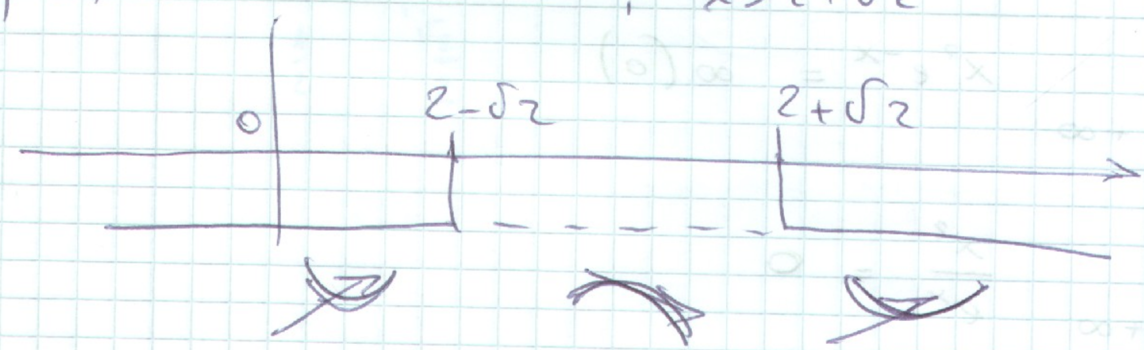
CONCAVITÀ E CONVESSITÀ

$f''(x) = -e^{-x}(2x - x^2) + e^{-x}(2-2x) =$

$= e^{-x}(-2x + x^2 + 2 - 2x) = e^{-x}(x^2 - 4x + 2)$

$x = 2 \pm \sqrt{2}$

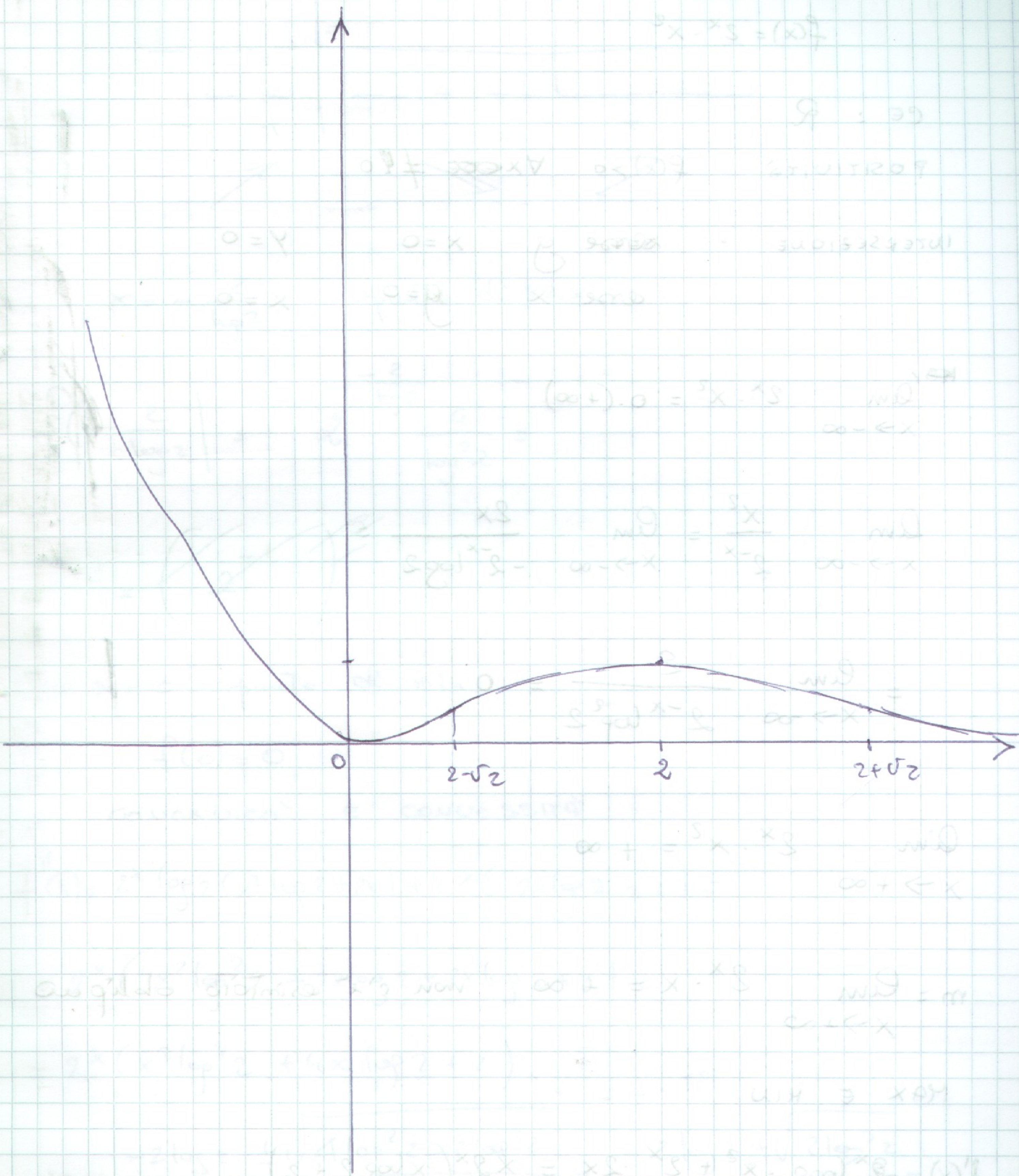
$f''(x) > 0 \qquad x < 2-\sqrt{2} \quad ; \quad x > 2+\sqrt{2}$




$x = 2-\sqrt{2}$ punto di max

$f(2-\sqrt{2}) = (2-\sqrt{2})\, e^{-2+\sqrt{2}}$

$x = 2+\sqrt{2}$ punto di min



0
2-√2
2
2+√2

$f(x) = 2^x \cdot x^2$

CE : $\mathbb{R}$

POSITIVITÀ $f(x) > 0 \quad \forall x \neq 0$

INTERSEZIONE - ~~asse~~ y $\quad x = 0 \quad y = 0$

asse x $\quad y = 0 \quad x = 0$

$$\lim_{x \to -\infty} 2^x \cdot x^2 = 0 \cdot (+\infty)$$

$$\lim_{x \to -\infty} \frac{x^2}{2^{-x}} = \lim_{x \to -\infty} \frac{2x}{-2^{-x} \log 2} =$$

$$= \lim_{x \to -\infty} \frac{2}{2^{-x} \log^2 2} = 0$$

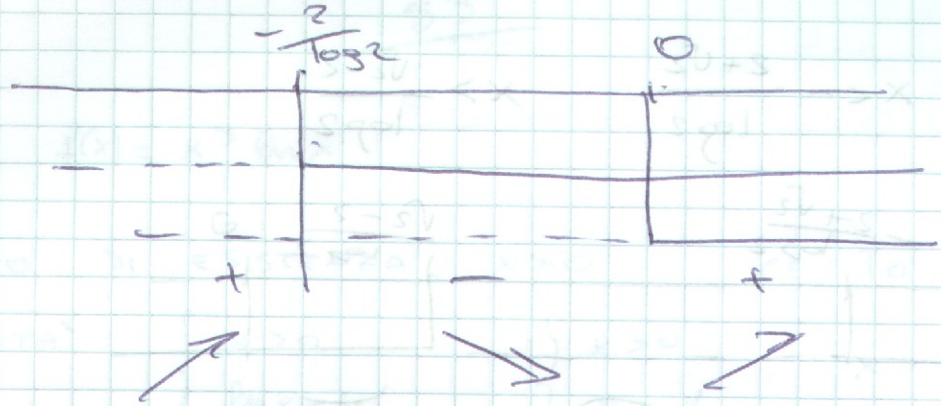
$$\lim_{x \to +\infty} 2^x \cdot x^2 = +\infty$$

$$m = \lim_{x \to +\infty} 2^x \cdot x = +\infty$$ non c'è asintoto obliquo

MAX E MIN

$$f'(x) = 2^x \log 2 \cdot x^2 + 2^x \cdot 2x = x 2^x (x \log 2 + 2)$$

$f'(x) = 0 \quad \begin{cases} x = 0 \\ x = -\dfrac{2}{\log 2} \end{cases} \qquad f'(x) > 0 \quad \begin{cases} x > 0 \\ x > -\dfrac{2}{\log 2} \end{cases}$



$x = -\frac{2}{\log 2}$ punto di MAX

$$f\left(-\frac{2}{\log 2}\right) = 2^{-\frac{2}{\log 2}} \frac{4}{\log^2 2} =$$

$$= \cancel{\left(2^{-1}\right)^2}$$

$x = 0$ punto di min

$$f(0) = 0$$

CONCAVITÀ E CONVESSITÀ

$$f''(x) = 2^x \log 2 (x^2 \log 2 + 2x) + 2^x (2x \log 2 + 2) =$$

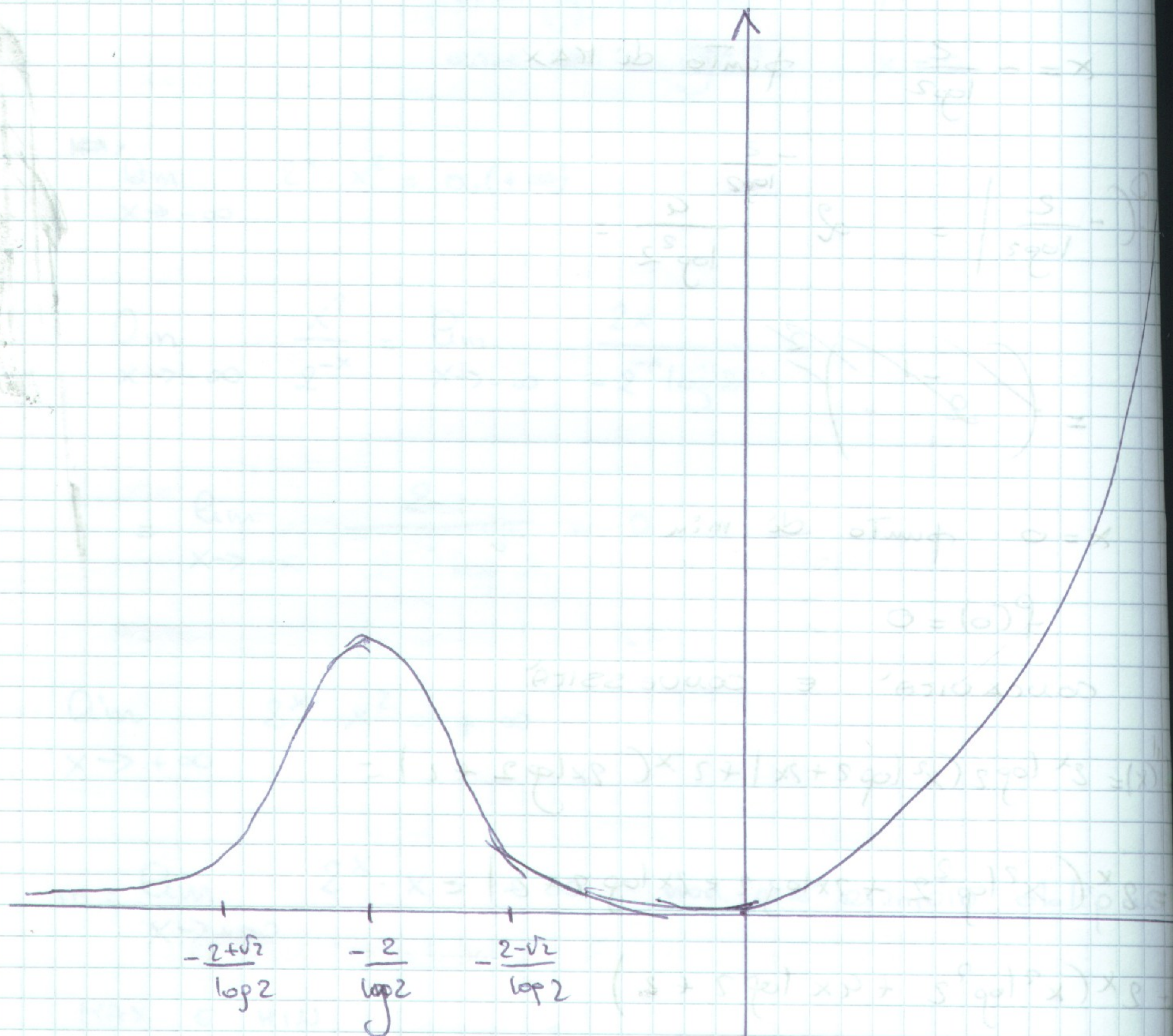
$$= 2^x (x^2 \log^2 2 + 2x \log 2 + 2x \log 2 + 2) =$$

$$= 2^x (x^2 \log^2 2 + 4x \log 2 + 2)$$

$$x = \frac{-2\log 2 \pm \sqrt{4\log^2 2 - 2\log^2 2}}{\log^2 2} = \frac{-2\log 2 \pm \sqrt{2\log^2 2}}{\log^2 2} =$$

$$= \frac{-2\log 2 \pm \log 2 \sqrt{2}}{\log^2 2} = \frac{-2 \pm \sqrt{2}}{\log 2}$$

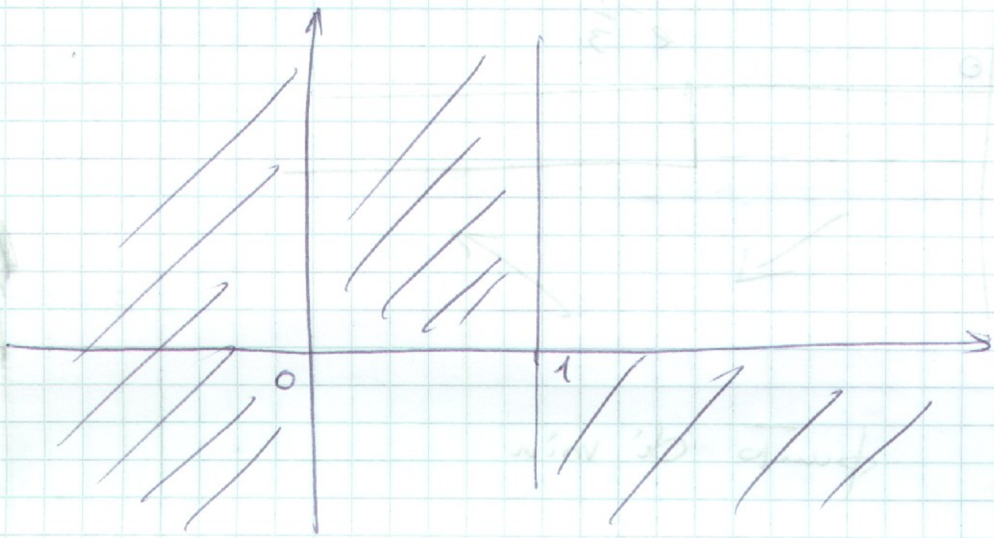
$f''(x)>0 \qquad x < -\frac{2+\sqrt{2}}{\log 2} \qquad x > \frac{\sqrt{2}-2}{\log 2}$

$f(x) = x^3 \ln x$

CAMPO DI ESISTENZA $x > 0$ CE : $]0, +\infty[$

POSITIVITA' $\begin{cases} x > 0 \\ \ln x > 0 \end{cases}$ $\begin{cases} x > 0 \\ x > 1 \end{cases}$ $x > 1$



Intersezione con l'asse x $\Rightarrow$ $\cancel{x = 0}$ $x = 1$

non $\in$ CE

$\lim\limits_{x \to 0} x^3 \ln x = 0$

$\lim\limits_{x \to +\infty} x^3 \ln x = +\infty$

$m = \lim\limits_{x \to +\infty} x^2 \ln x = +\infty$ non c'e' asintoto obliquo

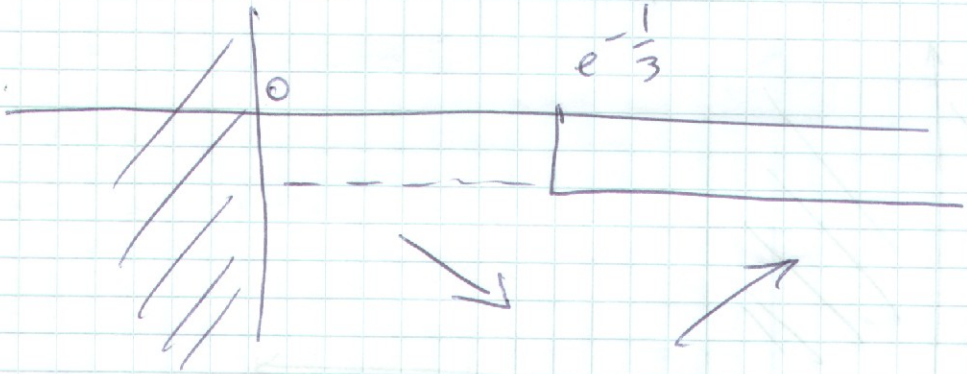
MAX E MIN

$f'(x) = 3x^2 \ln x + \dfrac{x^3}{x} = 3x^2 \ln x + x^2 =$

$= x^2 (3 \ln x + 1)$

$f'(x)=0 \quad \begin{cases} x=0 \\ \ln x = -\frac{1}{3} \end{cases} \quad \begin{cases} x=0 \\ x=e^{-\frac{1}{3}} \end{cases}$

$f'(x)>0 \quad \begin{cases} x>0 \\ x>e^{-\frac{1}{3}} \end{cases}$




$x=e^{-\frac{1}{3}}$ punto di min

$f\left(e^{-\frac{1}{3}}\right) = e^{-1} \ln e^{-\frac{1}{3}} = -\frac{1}{3e}$

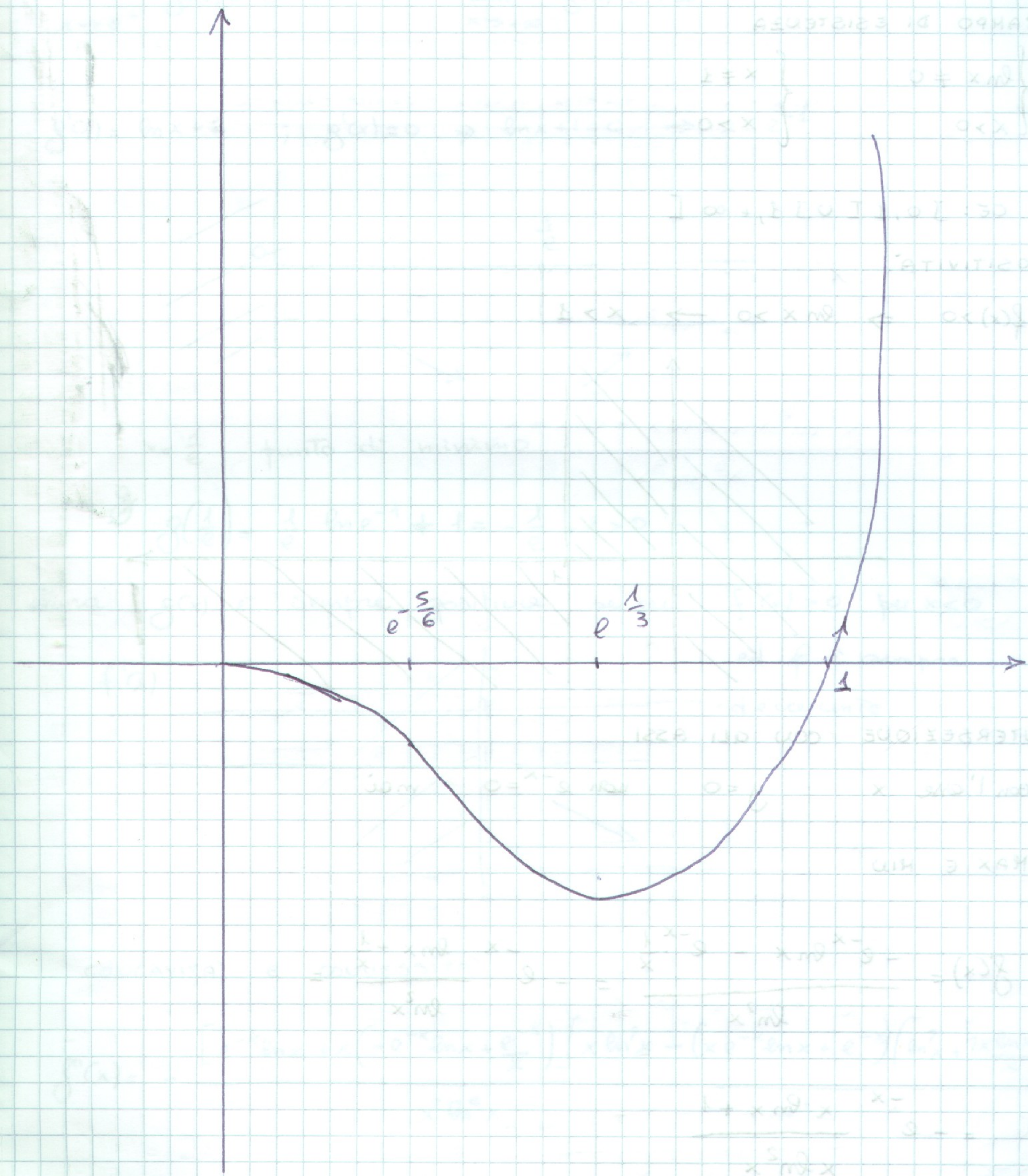
CONCAVITÀ E CONVESSITÀ

$f''(x) = 2x(3\ln x+1) + x^2\left(\frac{3}{x}\right) =$

$= 6x\ln x + 2x + 3x =$

$= 6x\ln x + 5x = x(6\ln x+5)$

$f''(x)=0 \quad \begin{cases} x=0 \\ x=e^{-\frac{5}{6}} \end{cases}$



$e^{-\frac{5}{6}}$
$e^{\frac{1}{3}}$
1

$$f(x) = \frac{e^{-x}}{\ln x}$$

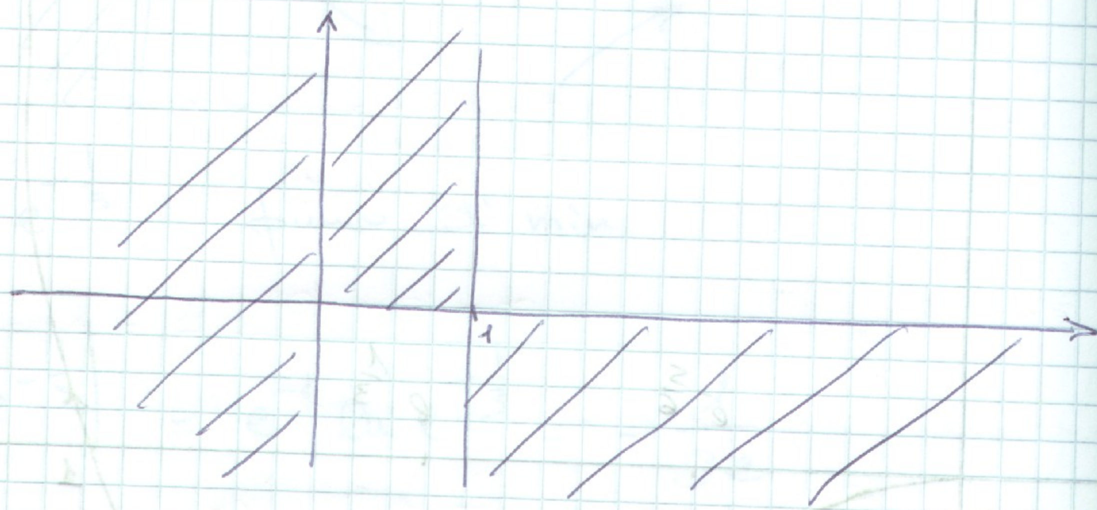
CAMPO DI ESISTENZA

$$\begin{cases} \ln x \neq 0 \\ x > 0 \end{cases} \qquad \begin{cases} x \neq 1 \\ x > 0 \end{cases}$$

CE: $]0, 1[ \cup ]1, +\infty[$

POSITIVITÀ

$f(x) > 0 \Rightarrow \ln x > 0 \longrightarrow x > 1$



INTERSEZIONE CON GLI ASSI

con l'asse x : $y = 0$ non $e^{-x} = 0$ mai

MAX E MIN

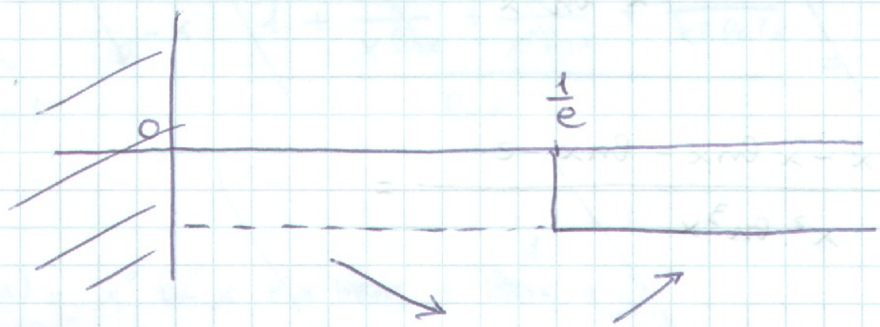
$$f'(x) = \frac{-e^{-x}\ln x - e^{-x}\cdot\frac{1}{x}}{\ln^2 x} = -e^{-x}\,\frac{\ln x + \frac{1}{x}}{\ln^2 x} =$$

$$= -e^{-x}\,\frac{x\ln x + 1}{x\ln^2 x}$$

Poniamo $g(x) = x\ln x + 1$

$\lim_{x \to 0^+} g(x) = 0 + 1 = 1$  $\lim_{x \to +\infty} g(x) = +\infty$

$g'(x) = \ln x + 1$ ; $g'(x) = 0 \Rightarrow \ln x + 1 = 0 \rightarrow x = e^{-1}$

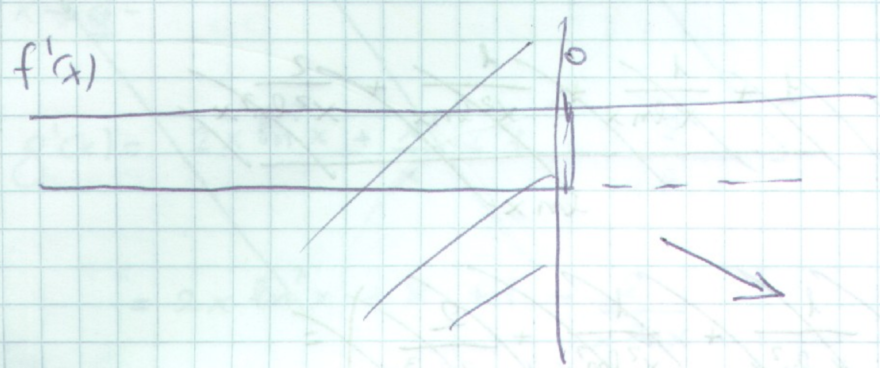



$x = \frac{1}{e}$ punto di minimo

~~g~~ $g\left(\frac{1}{e}\right) = \frac{1}{e}\ln e^{-1} + 1 = -\frac{1}{e} + 1 > 0$

allora $g(x)$ è sempre positiva per cui $f'(x) > 0$ per $x < 0$

ed $f$ è sempre decrescente




CONCAVITÀ E CONVESSITÀ

$$f''(x) = -\frac{\left[e^{-x}\ln x + x\left(-e^{-x}\ln x + \frac{e^{-x}}{x}\right)\right] x\ln^2 x - \left(xe^{-x}\ln x + e^{-x}\right)\left(\ln^2 x + 2x\frac{\ln x}{x}\right)}{x^2\ln^4 x} =$$

$$= -\frac{\left(e^{-x}\ln x - xe^{-x}\ln x + e^{-x}\right) x\ln^2 x - \left(xe^{-x}\ln x + e^{-x}\right)(\ln x + 2)\ln x}{x^2\ln^4 x}$$

$$f''(x) = -e^{-x}\,\frac{(\ln x - x\ln x + 1)x\ln x - (x\ln x + 1)(\ln x + 2)}{x^2\ln^3 x}$$

$$f''(x) = -e^{-x}\,\frac{x\ln^2 x - x^2\ln^2 x + x\ln x - x\ln^2 x - 2x\ln x - \ln x - 2}{x^2\ln^3 x} =$$

$$= -e^{-x}\,\frac{-x^2\ln^2 x - x\ln x - \ln x - 2}{x^2\ln^3 x} =$$

$$= e^{-x}\,\frac{x^2\ln^2 x + x\ln x + \ln x + 2}{x^2\ln^3 x}$$

$$\lim_{x\to 0^+} f''(x) = \cancel{1\left(0 + 0 + \infty\right)}$$

$$= \cancel{\lim_{x\to 0^+} f''(x) = e^{-x}\,\frac{1 + \frac{1}{x\ln x} + \frac{1}{x^2\ln x} + \frac{2}{x^2\ln^2 x}}{\ln x}}$$

$$= \cancel{\lim_{x\to 0^+} e^{-x}\left(\frac{1}{\ln x} + \frac{1}{x\ln^2 x} + \frac{1}{x^2\ln^2 x} + \frac{2}{x^2\ln^3 x}\right)} =$$

$$= \cancel{1\left(0 + \infty + \infty\right)}$$

$$\cancel{= \lim_{x\to 0^+} e^{-x}\,\frac{x^2\ln x + x + 1 + \frac{2}{\ln x}}{x^2\ln^2 x} = +\infty}$$

~~Se~~

$$\lim_{x\to 1^-} f''(x) = \cancel{1/ \,0/+/-/2}$$

$$= \lim_{x\to 1^-} e^{-x}\, \frac{\ln x + \frac{1}{x} + \frac{1}{x^2} + \frac{2}{x^2 \ln x}}{\ln^2 x} =$$

$$= \lim_{x\to 1^-} e^{-x}\, \frac{1 + \frac{1}{x\ln x} + \frac{1}{x^2\ln x} + \frac{2}{x^2 \ln^2 x}}{\ln x} = -\infty$$

$$g(x) = x^2 \ln^2 x + x \cancel{\ln x} + \frac{1}{\cancel{\ln} x} + \frac{2}{\ln x}$$

$$\lim_{x\to 0^+} g(x) = 0 + 0 + \frac{1}{\infty} + 0 = \cancel{\infty}\ 1$$

$$\lim_{x\to 1^-} g(x) = 0 + 1 + 1 + \infty = -\infty$$

$$g'(x) = 2x\ln^2 x + \frac{x^2}{x} + 1 - \frac{2}{x\ln x} =$$

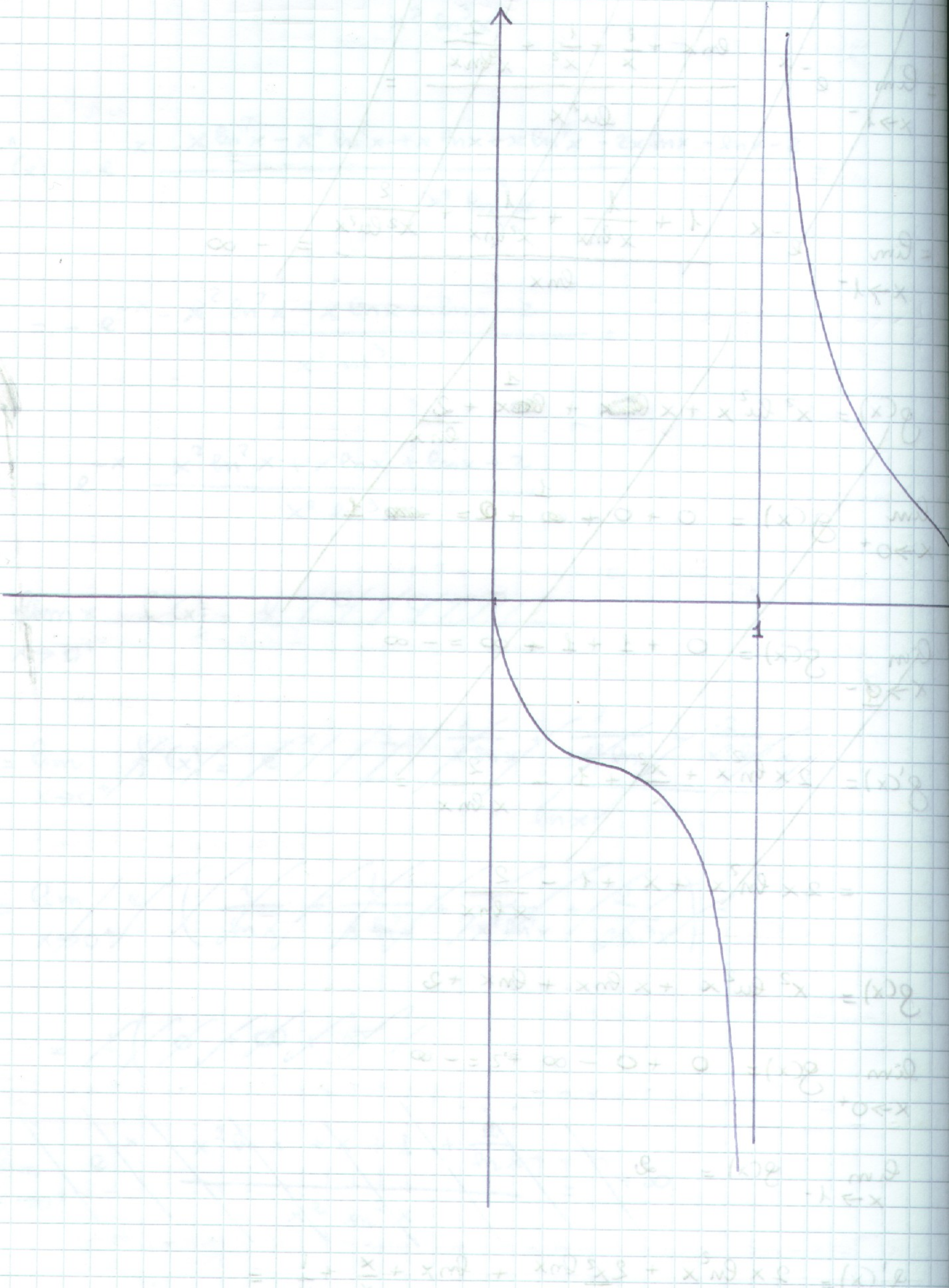
$$= 2x\ln^2 x + x + 1 - \frac{2}{x\ln x}$$

$$g(x) = x^2 \ln^2 x + x\ln x + \ln x + 2$$

$$\lim_{x\to 0^+} g(x) = 0 + 0 - \infty + 2 = -\infty$$

$$\lim_{x\to 1^-} g(x) = 2$$

$$g'(x) = 2x\ln^2 x + \frac{2x^2 \ln x}{x} + \ln x + \frac{x}{x} + \frac{1}{x} =$$


1

$$f(x) = \sqrt{1+x} - |x|$$

CAMPO DI ESISTENZA

$1+x > 0 \qquad x > -1$

C.E.: $]-1, +\infty[$

$$f(x) = \begin{cases} \sqrt{1+x} - x & x > 0 \\ \sqrt{1+x} + x & x < 0 \end{cases}$$

POSITIVITÀ

per $x > 0$

$\sqrt{1+x} - x > 0 \quad ; \quad \sqrt{1+x} > x$

essendo $x > 0$ allora è soddisfatta per $1+x > x^2$

$x^2 - x - 1 < 0 \qquad x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2} \rightarrow x < \frac{1+\sqrt{5}}{2}$

per $x < 0$

$\sqrt{1+x} + x > 0 \qquad \sqrt{1+x} > -x \quad$ essendo $-x > 0$

allora deve essere

$1+x > x^2 \qquad x^2 + x - 1 < 0 ; \; -1 < \frac{1-\sqrt{5}}{2} < x$

Interseca l'asse x in $x = \frac{1 \pm \sqrt{5}}{2}$

interseca l'asse y in $y = 1$

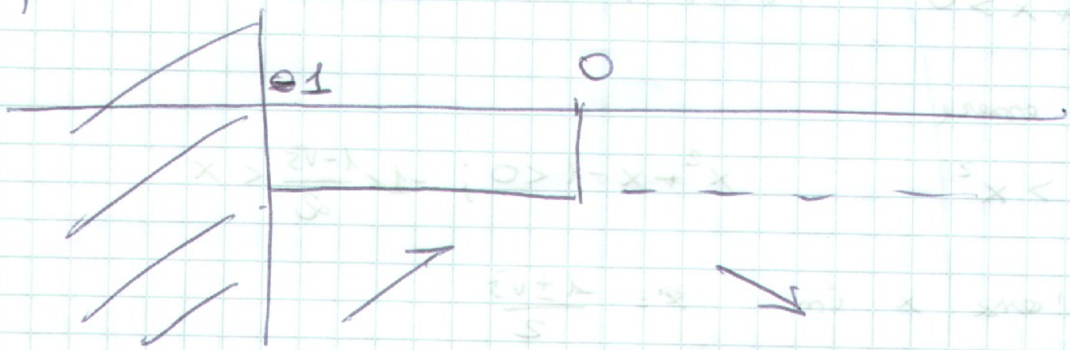
MAX E MIN

$$f'(x) = \begin{cases} \dfrac{1}{2\sqrt{1+x}} - 1 & x > 0 \\[2em] \dfrac{1}{2\sqrt{1+x}} + 1 & x < 0 \end{cases}$$

$$f'(x) > 0 \Rightarrow \begin{cases} \dfrac{1}{2\sqrt{1+x}} > 1 \\[2em] \dfrac{1}{2\sqrt{1+x}} > -1 \end{cases} \quad \begin{cases} \sqrt{1+x} < \dfrac{1}{2} \\[2em] \forall x \end{cases} \quad \begin{cases} 1+x < \dfrac{1}{4} \\[2em] \forall x \end{cases}$$

$$\begin{cases} x < -\dfrac{3}{4} \\[2em] \forall x \end{cases}$$

complessivamente




$x=0$ punto di max

$$\lim_{x \to 0^+} f'(x) = \frac{1}{2} - 1 = -\frac{1}{2}$$

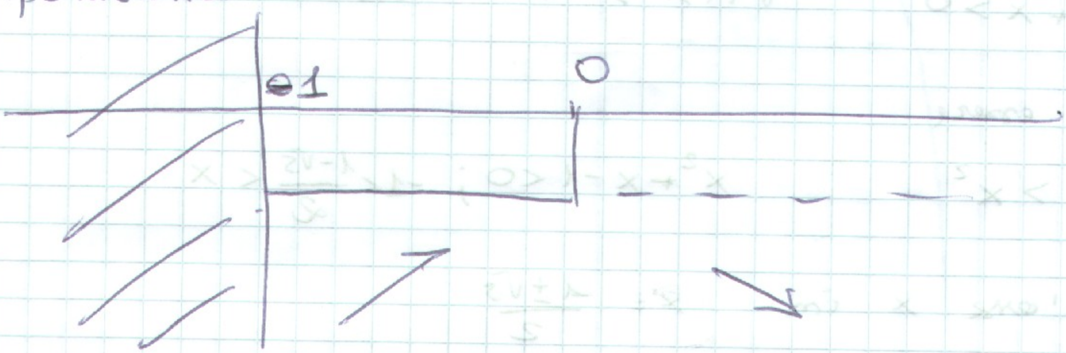
MAX E MIN

$$f'(x) = \begin{cases} \dfrac{1}{2\sqrt{1+x}} - 1 & x > 0 \\[2ex] \dfrac{1}{2\sqrt{1+x}} + 1 & x < 0 \end{cases}$$

$$f'(x) > 0 \Rightarrow \begin{cases} \dfrac{1}{2\sqrt{1+x}} > 1 \\[2ex] \dfrac{1}{2\sqrt{1+x}} > -1 \end{cases} \quad \begin{cases} \sqrt{1+x} < \dfrac{1}{2} \\[2ex] \forall x \end{cases} \quad \begin{cases} 1+x < \dfrac{1}{4} \\[2ex] \forall x \end{cases}$$

$$\begin{cases} x < -\dfrac{3}{4} \\[2ex] \forall x \end{cases}$$

complessivamente




$x=0$ punto di max

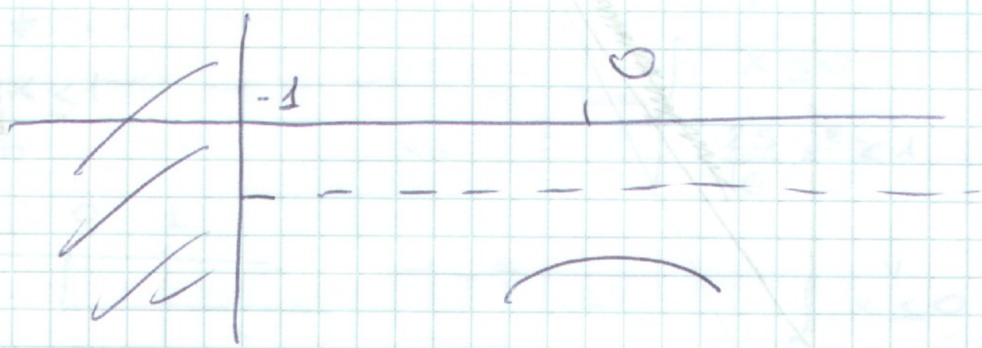
$$\lim_{x \to 0^+} f'(x) = \frac{1}{2} - 1 = -\frac{1}{2}$$

CONCAVITÀ E CONVESSITÀ

$$f''(x) = \begin{cases} \frac{1}{2} D(1+x)^{-\frac{1}{2}} + 0 & x > 0 \\ \frac{1}{2} D(1+x)^{-\frac{1}{2}} + 0 & x < 0 \end{cases}$$

$$f''(x) = -\frac{1}{4}(1+x)^{-\frac{3}{2}} = -\frac{1}{4\sqrt{(1+x)^3}} = -\frac{1}{4(1+x)\sqrt{1+x}}$$

$f''(x) > 0 \qquad 1+x < 0 \quad \Rightarrow \quad x < -1$

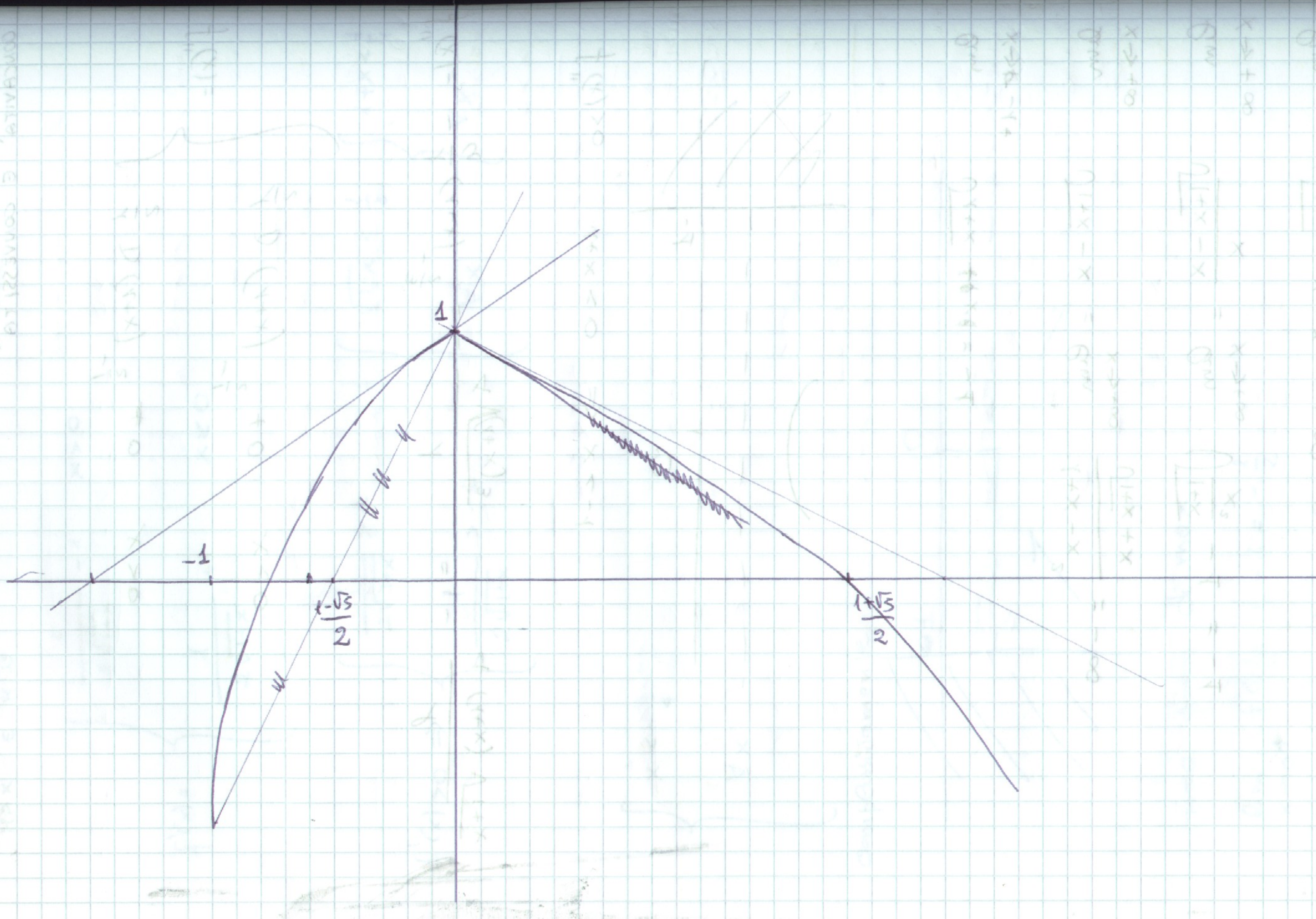



$$\lim_{x \to -1^+} \sqrt{1+x} + x = -1$$

$$\lim_{x \to +\infty} \sqrt{1+x} - x = \lim_{x \to +\infty} \frac{1+x-x^2}{\sqrt{1+x}+x} = -\infty$$

$$\lim_{x \to +\infty} \frac{\sqrt{1+x} - x}{x} = \lim_{x \to +\infty} \sqrt{\frac{1+x}{x^2}} - 1 = -1$$

$$q = \lim \sqrt{1+x} - x + x = +\infty$$



1
-1
1-√5 / 2
1+√5 / 2

$$f(x) = 3x + 4\sqrt{1-x^2}$$

CAMPO DI ESISTENZA

$1-x^2 \geq 0 \qquad -1 \leq x \leq 1 \qquad$ CE: $[-1, 1]$

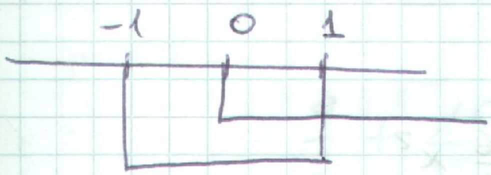
POSITIVITA'

$3x + 4\sqrt{1-x^2} > 0 \quad ; \quad \sqrt{1-x^2} > -\frac{3}{4}x$

$$\begin{cases} x > 0 \\ 1-x^2 \geq 0 \end{cases} \qquad \begin{cases} x \leq 0 \\ 1-x^2 > \frac{9}{16}x^2 \end{cases} ; \quad \begin{cases} x \leq 0 \\ \left(\frac{9}{16}+1\right)x^2 < 1 \end{cases}$$

$$\downarrow \qquad\qquad \downarrow$$

$$\begin{cases} -1 \leq x \leq 1 \\ x > 0 \end{cases} \qquad \begin{cases} x \leq 0 \\ \frac{25}{16}x^2 < 1 \end{cases} \qquad \begin{cases} x \leq 0 \\ x^2 < \frac{16}{25} \end{cases}$$




$$\downarrow$$

$$0 < x \leq 1$$

$$\begin{cases} x \leq 0 \\ -\frac{4}{5} < x < \frac{4}{5} \end{cases}$$

$$\downarrow$$

$$-\frac{4}{5} < x \leq 0$$

Allora

$f(x) > 0$ per $-\frac{4}{5} < x \leq 1$

Intersezione con l'asse $y$ ; $x = 0 \rightarrow y = 4$

Intersezione con l'asse $x$ ; $y = 0$

$3x + 4\sqrt{1-x^2} = 0 \qquad \sqrt{1-x^2} = -\frac{3}{4}x \quad ; \quad 1-x^2 = \frac{9}{16}x^2$

$$\begin{cases} \frac{25}{16}x^2 = 1 \end{cases} \qquad \begin{cases} x^2 = \frac{16}{25} \end{cases} \qquad \begin{cases} x = \pm \frac{4}{5} \end{cases} \qquad x = -\frac{4}{5}$$

$f(x) = 3x + 4\sqrt{1-x^2}$

CAMPO DI ESISTENZA

$1-x^2 \geq 0 \qquad -1 \leq x \leq 1 \qquad ce: [-1, 1]$

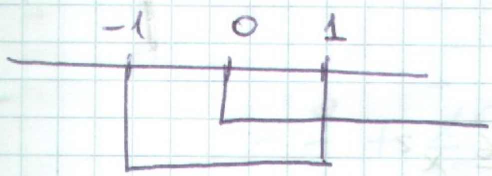
POSITIVITÀ

$3x + 4\sqrt{1-x^2} > 0 \quad ; \quad \sqrt{1-x^2} > -\frac{3}{4}x$

$\begin{cases} x > 0 \\ 1-x^2 \geq 0 \end{cases} \qquad \begin{cases} x \leq 0 \\ 1-x^2 > \frac{9}{16}x^2 \end{cases} \quad ; \quad \begin{cases} x \leq 0 \\ \left(\frac{9}{16}+1\right)x^2 < 1 \end{cases}$

$\downarrow \qquad\qquad \downarrow$

$\begin{cases} -1 \leq x \leq 1 \\ x > 0 \end{cases} \qquad \begin{cases} x \leq 0 \\ \frac{25}{16}x^2 < 1 \end{cases} \qquad \begin{cases} x \leq 0 \\ x^2 < \frac{16}{25} \end{cases}$




$\downarrow$

$0 < x \leq 1$

$\downarrow$

$\begin{cases} x \leq 0 \\ -\frac{4}{5} < x < \frac{4}{5} \end{cases}$

$\downarrow$

$-\frac{4}{5} < x \leq 0$

Allora

$f(x) > 0$ per $-\frac{4}{5} < x \leq 1$

Intersezione con l'asse y ; $x = 0 \rightarrow y = 4$

Intersezione con l'asse x ; $y = 0$

$3x + 4\sqrt{1-x^2} = 0 \qquad \sqrt{1-x^2} = -\frac{3}{4}x \quad ; \quad 1-x^2 = \frac{9}{16}x^2$

$\begin{cases} \frac{25}{16}x^2 = 1 \\ x \leq 0 \end{cases} \qquad \begin{cases} x^2 = \frac{16}{25} \\ x \leq 0 \end{cases} \qquad \begin{cases} x = \pm\frac{4}{5} \\ x \leq 0 \end{cases} \qquad x = -\frac{4}{5}$

$f(-1) = 3 \qquad f(1) = 3$

CAMPO DI ESISTENZA

$1-x^2 \ge 0 \qquad -1 \le x \le 1 \qquad CE: [-1, 1]$

POSITIVITÀ

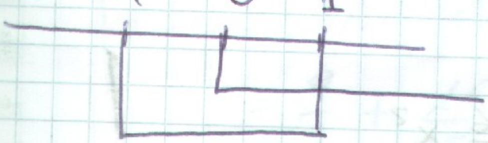
$3x + 4\sqrt{1-x^2} > 0 \quad ; \quad \sqrt{1-x^2} > -\frac{3}{4}x$

$$\begin{cases} x > 0 \\ 1-x^2 \ge 0 \end{cases} \qquad \begin{cases} x \le 0 \\ 1-x^2 > \frac{9}{16}x^2 \end{cases} ; \quad \begin{matrix} x \le 0 \\ \left(\frac{9}{16}+1\right)x^2 < 1 \end{matrix}$$

$\downarrow \qquad \downarrow$

$$\begin{cases} -1 \le x \le 1 \\ x > 0 \end{cases} \qquad \begin{cases} x \le 0 \\ \frac{25}{16}x^2 < 1 \end{cases} \qquad \begin{cases} x \le 0 \\ x^2 < \frac{16}{25} \end{cases}$$

-1 0 1



$\downarrow$

$0 < x \le 1$

$$\begin{cases} x \le 0 \\ -\frac{4}{5} < x < \frac{4}{5} \end{cases}$$

$\downarrow$

$-\frac{4}{5} < x \le 0$

Allora

$f(x) > 0$ per $-\frac{4}{5} < x \le 1$

Intersezione con l'asse $y$ ; $x=0 \rightarrow y = 4$

Intersezione con l'asse $x$ ; $y = 0$

$3x + 4\sqrt{1-x^2} = 0 \qquad \sqrt{1-x^2} = -\frac{3}{4}x \quad ; \quad 1-x^2 = \frac{9}{16}x^2$

$$\begin{cases} \frac{25}{16}x^2 = 1 \\ x \le 0 \end{cases} \qquad \begin{cases} x^2 = \frac{16}{25} \\ x \le 0 \end{cases} \qquad \begin{cases} x = \pm\frac{4}{5} \\ x \le 0 \end{cases} \qquad x = -\frac{4}{5}$$

$f(-1) = -3 \qquad f(1) = 3$

MAX E MIN.

$$f'(x) = 3 + \frac{4}{2\sqrt{1-x^2}}(-2x) = 3 - \frac{4x}{\sqrt{1-x^2}}$$

$$f'(x)=0 \rightarrow \frac{3\sqrt{1-x^2}-4x}{\sqrt{1-x^2}} = 0$$

$3\sqrt{1-x^2} = 4x$ ; $9(1-x^2) = 16x^2$ ; $9-9x^2 = 16x^2$

$25x^2 = 9$ ; $x^2 = \frac{9}{25}$ ; $x = \pm\frac{3}{5}$

$f'(x) > 0$ $\quad 3\sqrt{1-x^2} - 4x > 0$

$\sqrt{1-x^2} > \frac{4}{3}x$ che dà luogo ai due sistemi

$$\begin{cases} x < 0 \\ 1-x^2 > 0 \end{cases} \qquad \begin{cases} x \geq 0 \\ 1-x^2 > \frac{16}{9}x^2 \end{cases}$$

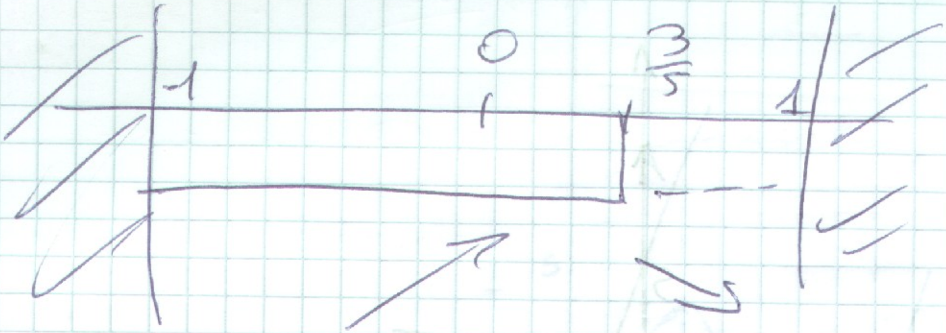
$$\downarrow \qquad\qquad \downarrow$$

$$\begin{cases} x < 0 \\ -1 < x < 1 \end{cases} \qquad \begin{cases} x \geq 0 \\ x^2 < \frac{9}{25} \end{cases} \qquad \begin{cases} x \geq 0 \\ -\frac{3}{5} < x < \frac{3}{5} \end{cases}$$

$$\downarrow \qquad\qquad \downarrow$$

$$-1 < x < 0 \qquad\qquad 0 \leq x < \frac{3}{5}$$

$f'(x) > 0$ per $-1 < x < \frac{3}{5}$



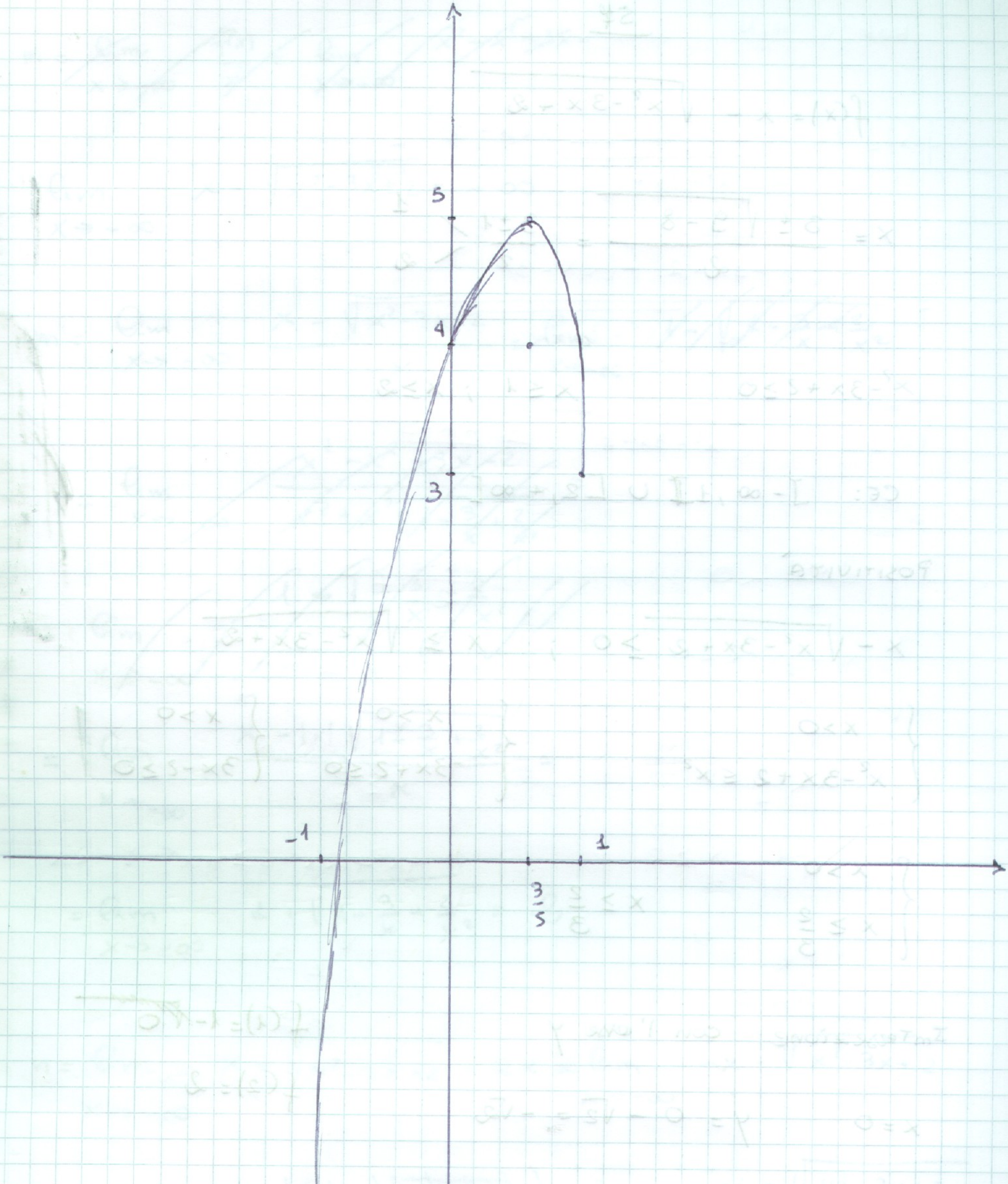

$x = \frac{3}{5}$ punto di max

$$f\left(\frac{3}{5}\right) = 3 \cdot \frac{3}{5} + 4\sqrt{1 - \frac{9}{25}} =$$

$$= \frac{9}{5} + 4\sqrt{\frac{25-9}{25}} =$$

$$= \frac{9}{5} + 4\sqrt{\frac{16}{25}} = \frac{9}{5} + 4 \cdot \frac{4}{5} =$$

$$= \frac{9}{5} + \frac{16}{5} = \frac{25}{5} = 5$$


5
4
3
-1
1
3/5
-3

57

$$f(x) = x - \sqrt{x^2 - 3x + 2}$$

$$x = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2} \begin{cases} 1 \\ 2 \end{cases}$$

$x^2 - 3x + 2 \geq 0$ $\qquad$ $x \leq 1 \;;\; x \geq 2$

CE: $]-\infty, 1] \cup [2, +\infty[$

POSITIVITÀ

$$x - \sqrt{x^2 - 3x + 2} \geq 0 \;; \quad x \geq \sqrt{x^2 - 3x + 2}$$

$$\begin{cases} x > 0 \\ x^2 - 3x + 2 \leq x^2 \end{cases} \qquad \begin{cases} x > 0 \\ -3x + 2 \leq 0 \end{cases} \qquad \begin{cases} x > 0 \\ 3x - 2 \geq 0 \end{cases}$$

$$\begin{cases} x > 0 \\ x \geq \frac{2}{3} \end{cases} \qquad x \geq \frac{2}{3}$$

Intersezione con l'asse y

$x = 0$ $\qquad y = 0 - \sqrt{2} = -\sqrt{2}$

$f(1) = 1 - \sqrt{0}$

$f(2) = 2$

$$\lim_{x \to +\infty} x - \sqrt{x^2 - 3x + 2} =$$

$$= \lim_{x \to +\infty} \frac{\cancel{x^2} - \cancel{x^2} + 3x - 2}{x + \sqrt{x^2 - 3x + 2}} = \frac{3}{2}$$

$$\cancel{m = \lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} x^2 - x^2 + 3x -}$$

$$\lim_{x \to -\infty} x - \sqrt{x^2 - 3x + 2} = -\infty$$

$$m = \lim_{x \to -\infty} \frac{x - \sqrt{x^2-3x+2}}{x} = \lim_{x \to -\infty} \cancel{1 - \sqrt{1 - \frac{3}{x} + \frac{2}{x^2}}}$$

$$\cancel{= \lim_{x \to -\infty} \frac{x^2 - x^2 + 3x - 2}{x^2 + x^2\sqrt{1 - \frac{3}{x} + \frac{2}{x^2}}} =}$$

$$\cancel{= \lim_{x \to -\infty} \frac{1 + 1 + \frac{3}{x} - \frac{2}{x^2}}{\quad}}$$

$$= \lim_{x \to -\infty} \frac{x - |x|\sqrt{1 - \frac{3}{x} + \frac{2}{x^2}}}{x} =$$

$$= \lim_{x \to -\infty} 1 + \sqrt{1 - \frac{3}{x} + \frac{2}{x^2}} = 2$$

$$n = \lim_{x \to -\infty} x - \sqrt{x^2 - 3x + 2} - 2x = \lim_{x \to -\infty} -x - \sqrt{x^2 - 3x + 2}$$

$$\cancel{n = \lim_{x \to +\infty} \left(-\left(x + \sqrt{x^2 - 3x + 2}\right)\right)} = \lim_{y \to +\infty} y - \sqrt{y^2 + 3y + 2}$$

$$\cancel{= \lim_{x \to -\infty} - \frac{x^2 + x^2 - 3x + 2}{x - \sqrt{\quad}}} = \lim_{y \to +\infty} \frac{\cancel{y^2} - \cancel{y^2} - 3y - 2}{y + \sqrt{y^2 + 3y + 2}} = -\frac{3}{2}$$

$$\cancel{m = \lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} x^2 - x^2 + 3x -}$$

$$\lim_{x \to -\infty} x - \sqrt{x^2 - 3x + 2} = -\infty$$

$$m = \lim_{x \to -\infty} \frac{x - \sqrt{x^2-3x+2}}{x} = \lim_{x \to -\infty} \cancel{1 - \sqrt{1 - \frac{3}{x} + \frac{2}{x^2}}}$$

$$= \cancel{\lim_{x \to -\infty} \frac{x^2 - x^2 + 3x - 2}{x^2 + x^2\sqrt{1 - \frac{3}{x} + \frac{2}{x^2}}}} =$$

$$= \cancel{\lim_{x \to -\infty} \frac{1 - 1 + \frac{3}{x} - \frac{2}{x^2}}{\phantom{x}}}$$

$$= \lim_{x \to -\infty} \frac{x - |x|\sqrt{1 - \frac{3}{x} + \frac{2}{x^2}}}{x} =$$

$$= \lim_{x \to -\infty} 1 + \sqrt{1 - \frac{3}{x} + \frac{2}{x^2}} = 2$$

$$n = \lim_{x \to -\infty} x - \sqrt{x^2 - 3x + 2} - 2x = \lim_{x \to -\infty} -x - \sqrt{x^2 - 3x + 2}$$

$$\cancel{n = \lim_{x \to +\infty} -\left(x + \sqrt{x^2 - 3x + 2}\right)} = \lim_{y \to +\infty} y - \sqrt{y^2 + 3y + 2}$$

$$= \cancel{\lim_{x \to -\infty} - \frac{x^2 + x^2 - 3x + 2}{x - \sqrt{\phantom{x}}}} = \lim_{y \to +\infty} \frac{y^2 - y^2 - 3y - 2}{y + \sqrt{y^2 + 3y + 2}} = -\frac{3}{2}$$

MAX E MIN

$$f'(x) = 1 - \frac{1}{2\sqrt{x^2-3x+2}}(2x-3) =$$

$$= \frac{2\sqrt{x^2-3x+2} - (2x-3)}{2\sqrt{x^2-3x+2}}$$

$$f'(x) \geq 0 \qquad \sqrt{x^2-3x+2} \geq \frac{2x-3}{2}$$

$$\begin{cases} \dfrac{2x-3}{2} < 0 \\ x^2-3x+2 \geq 0 \end{cases} \qquad \begin{cases} \dfrac{2x-3}{2} \geq 0 \\ x^2-3x+2 \geq \dfrac{(2x-3)^2}{4} \end{cases}$$

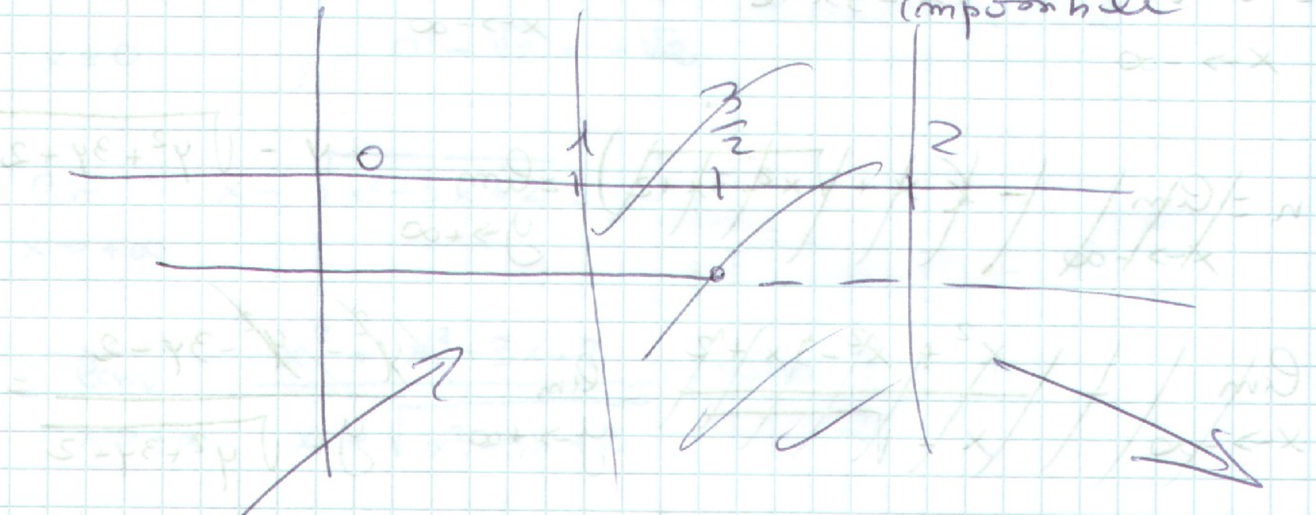
$$\downarrow \qquad\qquad \downarrow$$

$$\begin{cases} x < \frac{3}{2} \\ x \leq 1 \ , \ x \geq 2 \end{cases} \qquad \begin{cases} x \geq \frac{3}{2} \\ 4x^2-12x+8 \geq 4x^2+9-12x \end{cases}$$

$$\downarrow \qquad\qquad \downarrow$$

$$x < \frac{3}{2} \qquad\qquad \begin{cases} x \geq \frac{3}{2} \\ 8 \geq 9 \end{cases}$$

impossibile



MAX E MIN

$$f'(x) = 1 - \frac{1}{2\sqrt{x^2-3x+2}}(2x-3) =$$

$$= \frac{2\sqrt{x^2-3x+2} - (2x-3)}{2\sqrt{x^2-3x+2}}$$

$$f'(x) \geq 0 \qquad \sqrt{x^2-3x+2} \geq \frac{2x-3}{2}$$

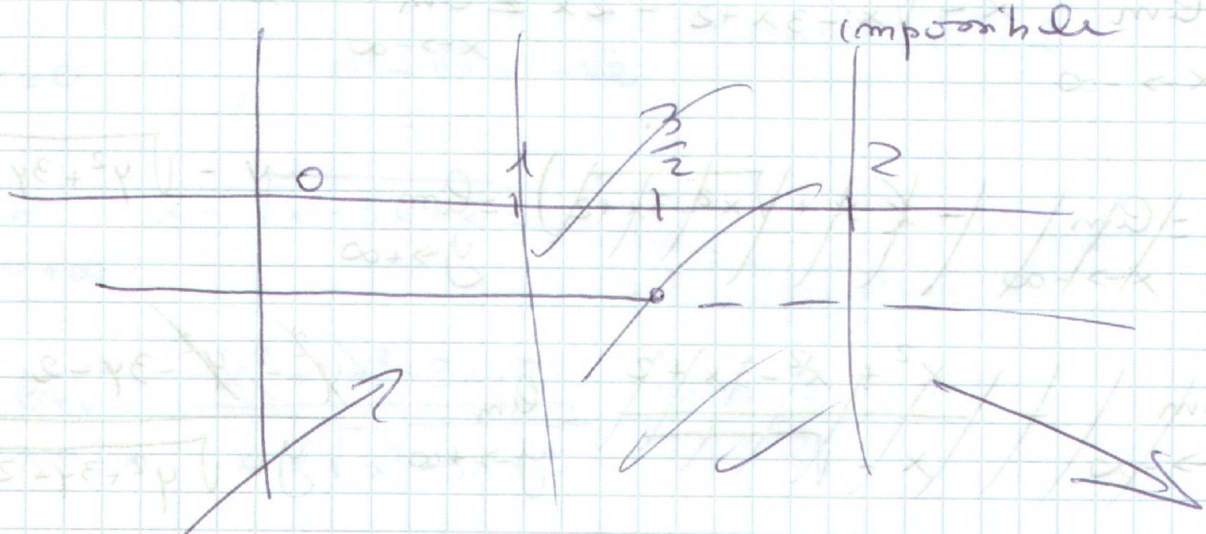
$$\begin{cases} \frac{2x-3}{2} < 0 \\ x^2-3x+2 \geq 0 \end{cases} \qquad \begin{cases} \frac{2x-3}{2} \geq 0 \\ x^2-3x+2 \geq \frac{(2x-3)^2}{4} \end{cases}$$

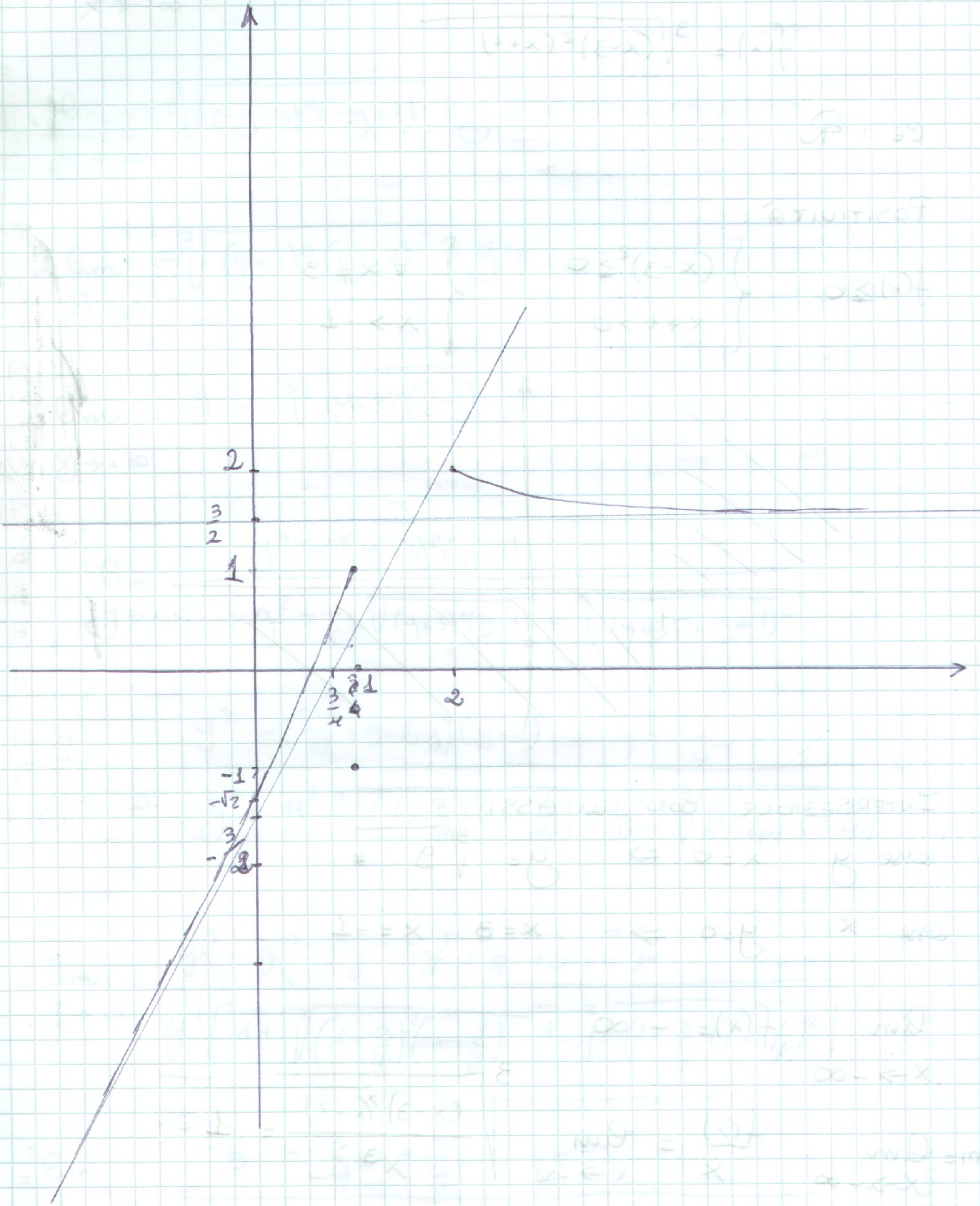
$$\begin{cases} x < \frac{3}{2} \\ x \leq 1 \;,\; x \geq 2 \end{cases} \qquad \begin{cases} x \geq \frac{3}{2} \\ 4x^2-12x+8 \geq 4x^2+9-12x \end{cases}$$

$$\downarrow \qquad\qquad \downarrow$$

$$x < \frac{3}{2} \qquad \begin{cases} x \geq \frac{3}{2} \\ 8 \geq 9 \end{cases}$$

impossibile



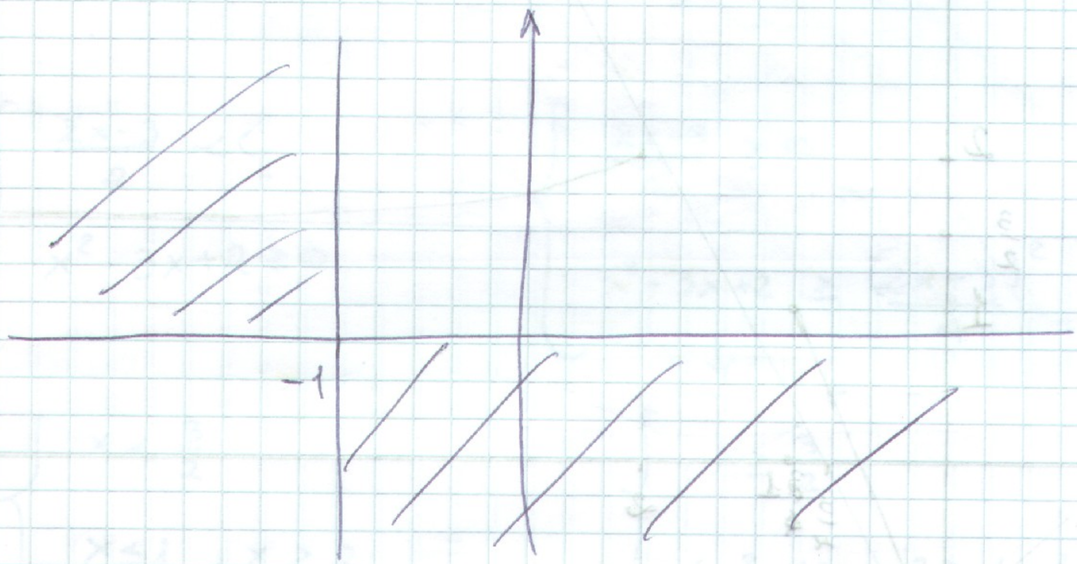

2
3/2
1
-1
-√2
-3/2
3/4
1
2

$$f(x) = \sqrt[3]{(x-3)^2(x+1)}$$

CE : $\mathbb{R}$

POSITIVITÀ:

$f(x) \geq 0 \quad \begin{cases} (x-3)^2 \geq 0 \\ x+1 > 0 \end{cases} \quad \begin{cases} \forall x \neq 3 \\ x > -1 \end{cases}$



INTERSEZIONE CON GLI ASSI

asse $y$ $\quad x=0 \Rightarrow \quad y = \sqrt[3]{9}$

asse $x$ $\quad y=0 \Rightarrow \quad x=3, \; x=-1$

$$\lim_{x \to -\infty} f(x) = -\infty$$

$$m = \lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \sqrt[3]{\frac{(x-3)^2(x+1)}{x^3}} = 1$$

$$n = \lim_{x \to -\infty} \sqrt[3]{(x-3)^2(x+1)} - x =$$

$$= \lim_{y \to \infty} \sqrt[3]{(-y-3)^2(-y+1)} + y =$$

$$= \lim_{y \to +\infty} -\sqrt[3]{(y+3)^2(y-1)} + y =$$

$$= \lim_{y \to +\infty} y - \sqrt[3]{(y+3)^2(y-1)} =$$

$$= \lim_{y \to +\infty} \frac{y^3 - (y+3)^2(y-1)}{y^2 + y\sqrt[3]{(y+3)^2(y-1)} + \sqrt[3]{(y+3)^4(y-1)^2}} =$$

$$= \lim_{y \to +\infty} \frac{y^3 - (y^2+9+6y)(y-1)}{y^2 + y^2\sqrt[3]{\left(1+\frac{3}{y}\right)^2\left(1-\frac{1}{y}\right)} + y^2\sqrt[3]{\left(1+\frac{3}{y}\right)^2\left(1-\frac{1}{y}\right)^2}} =$$

$$= \lim_{y \to +\infty} \frac{\cancel{y^3} - \cancel{y^3} - 9y - 6y^2 + y^2 + 9 + 6y}{y^2\left[1 + \sqrt[3]{\left(1+\frac{3}{y}\right)^2\left(1-\frac{1}{y}\right)} + \sqrt[3]{\left(1+\frac{3}{y}\right)^2\left(1-\frac{1}{y}\right)^2}\right]} =$$

$$= \lim_{y \to +\infty} \frac{-5 - \frac{3}{y} + \frac{9}{y^2}}{1 + \sqrt[3]{\left(1+\frac{3}{\ }\right)^2\left(1-\frac{1}{\ }\right)} + \sqrt[3]{\left(1+\frac{3}{\ }\right)^2\left(1-\frac{1}{\ }\right)^2}} = -\frac{5}{3}$$

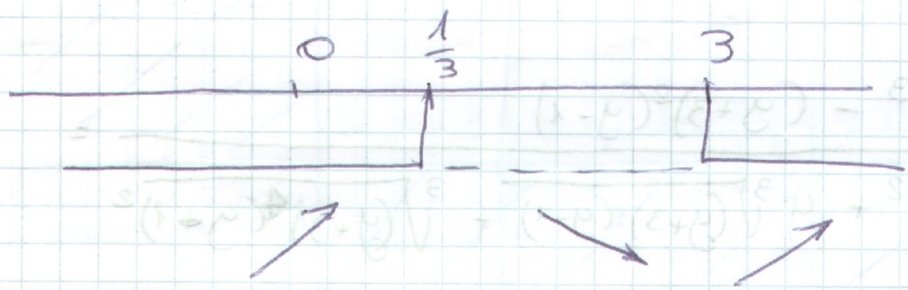
asintoto obliquo : $y = x - \frac{5}{3}$

MAX E MIN.

$$f'(x) = \frac{2(x-3)(x+1) + (x-3)^2}{3\sqrt[3]{(x-3)^4(x+1)^2}} = \frac{2x^2+2x-6x-6+x^2+9-6x}{3\sqrt[3]{(x-3)^4(x+1)^2}} =$$

$$= \frac{3x^2-10x+3}{3\sqrt[3]{(x-3)^4(x+1)^2}} \longrightarrow x = \frac{5 \pm \sqrt{25-9}}{3} = \frac{5 \pm 4}{3} \begin{cases} \frac{1}{3} \\ \frac{9}{3} = 3 \end{cases}$$

$f'(x) > 0 \quad x < \frac{1}{3}, \ x > 3$



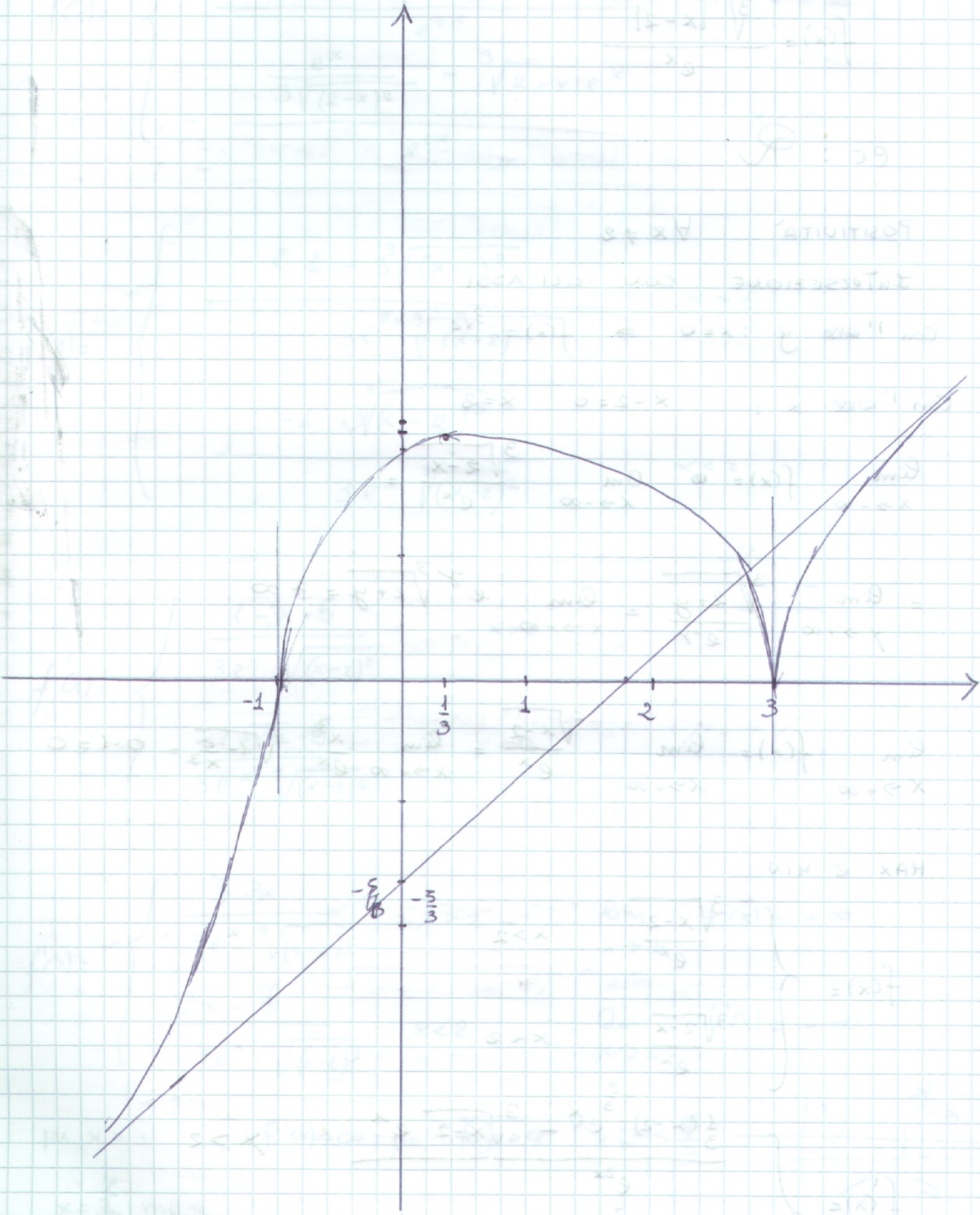

$\cancel{f(\frac{1}{3})}$ $x = \frac{1}{3}$ punto di MAX

in $x = 1$ e $x = 3$ la $f'(x) \to$ non è definita ~~e -∞ / e +∞~~

$$\lim_{x \to 1} f'(x) = -\infty \qquad \lim_{x \to 3^-} f'(x) = -\infty$$

$$\lim_{x \to 3^+} f'(x) = +\infty$$

$$f\left(\frac{1}{3}\right) = \sqrt[3]{\left(\frac{1}{3}-3\right)^2\left(\frac{1}{3}+1\right)} = \sqrt[3]{\frac{64}{9} \cdot \frac{4}{3}} = \frac{\sqrt[3]{256}}{3}$$



-1
1/3
1
2
3
-5/3

$$f(x) = \frac{\sqrt[3]{|x-2|}}{e^x}$$

C.E. : $\mathbb{R}$

POSITIVITÀ $\forall x \neq 2$

INTERSEZIONE CON GLI ASSI

Con l'asse y : $x = 0 \Rightarrow f(x) = \sqrt[3]{2}$

Con l'asse x : $x - 2 = 0 \quad x = 2$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{\sqrt[3]{2-x}}{e^x} =$$

$$= \lim_{y \to +\infty} \frac{\sqrt[3]{2+y}}{e^{-y}} = \lim_{x \to +\infty} e^y \sqrt[3]{2+y} = +\infty$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{\sqrt[3]{x-2}}{e^x} = \lim_{x \to +\infty} \frac{x^3}{e^x} \sqrt[3]{1 - \frac{2}{x^3}} = 0 \cdot 1 = 0$$

MAX E MIN

$$f(x) = \begin{cases} \dfrac{\sqrt[3]{x-2}}{e^x} & x > 2 \\[2ex] \dfrac{\sqrt[3]{2-x}}{e^x} & x < 2 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{\frac{1}{3}(x-2)^{-\frac{2}{3}} e^x - \sqrt[3]{x-2}\, e^x}{e^{2x}} & x > 2 \\[2ex] -\frac{1}{3}(2-x)^{-\frac{2}{3}} e^{x} - \sqrt[3]{2-x}\, e^x & x < 2 \end{cases}$$

$$f(x) = \frac{\quad}{e^x}$$

$C.E.: \mathbb{R}$

POSITIVITÀ $\forall x \neq 2$

INTERSEZIONE CON GLI ASSI

Con l'asse $y$ : $x = 0 \Rightarrow f(x) = \sqrt[3]{2}$

Con l'asse $x$ : $x - 2 = 0 \quad x = 2$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{\sqrt[3]{2-x}}{e^x} =$$

$$= \lim_{y \to +\infty} \frac{\sqrt[3]{2+y}}{e^{-y}} = \lim_{x \to +\infty} e^y \sqrt[3]{2+y} = +\infty$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{\sqrt[3]{x-2}}{e^x} = \lim_{x \to +\infty} \frac{x^3}{e^x} \sqrt[3]{1 - \frac{2}{x^3}} = 0 \cdot 1 = 0$$

MAX E MIN

$$f(x) = \begin{cases} \dfrac{\sqrt[3]{x-2}}{e^x} & x > 2 \\[2ex] \dfrac{\sqrt[3]{2-x}}{e^x} & x < 2 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{\frac{1}{3}(x-2)^{-\frac{2}{3}} e^x - \sqrt[3]{x-2}\, e^x}{e^{2x}} & x > 2 \\[2ex] \dfrac{-\frac{1}{3}(2-x)^{-\frac{2}{3}} e^{x} - \sqrt[3]{2-x}\, e^x}{e^{2x}} & x < 2 \end{cases}$$

$$f'(x)=\begin{cases} \dfrac{\dfrac{1}{3}\dfrac{e^x}{\sqrt[3]{(x-2)^2}} - e^x\sqrt[3]{x-2}}{e^{2x}} & x>2 \\[2ex] -\dfrac{\dfrac{e^x}{3\sqrt[3]{(2-x)^2}} - \sqrt[3]{2-x}\,e^x}{e^{2x}} & x<2 \end{cases}$$

$$f'(x)=\begin{cases} \dfrac{\cancel{e}\;1-3\sqrt[3]{(x-2)^3}}{3e^{\cancel{2}x}\sqrt[3]{(x-2)^2}} & x>2 \\[2ex] \dfrac{-1+3\sqrt[3]{(x-2)^3}}{3e^x\sqrt[3]{(x-2)^2}} & x<2 \end{cases}$$

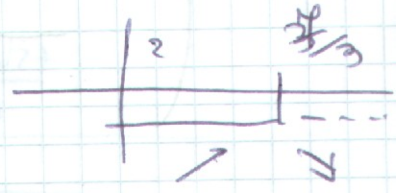
$$f'(x)=\begin{cases} \dfrac{1-3x+6}{3e^x\sqrt[3]{(x-2)^2}} & x>2 \\[2ex] \dfrac{-1+3x-6}{3e^x\sqrt[3]{(x-2)^2}} & x<2 \end{cases}$$

$$f'(x)=\begin{cases} \dfrac{7-3x}{3e^x\sqrt[3]{(x-2)^2}} & x>2 \\[2ex] \dfrac{3x-7}{3e^x\sqrt[3]{(x-2)^2}} & x<2 \end{cases}$$

$\lim_{x\to 2^+} f'(x) = +\infty$

$\lim_{x\to 2^-} f'(x) = -\infty$

per $x>2$ $\quad f'(x)>0$ per $7-3x>0$ $\quad x<\frac{7}{3}$



$x=\frac{7}{3}$ max

per ... $f'(x)>0$ per $3x-7>0$ ... $x>\frac{7}{3}$

$$\left\{ -\frac{\dfrac{e^{x}}{3\sqrt[3]{(2-x)^2}} - \sqrt[3]{2-x}\,e^{x}}{e^{2x}} \qquad x<2 \right.$$

$$f'(x)=\begin{cases} \dfrac{\cancel{e}\,1 - 3\sqrt[3]{(x-2)^3}}{3e^{\cancel{2}x}\sqrt[3]{(x-2)^2}} & x>2 \\[2ex] \dfrac{-1+3\sqrt[3]{(x-2)^3}}{3e^{x}\sqrt[3]{(x-2)^2}} & x<2 \end{cases}$$

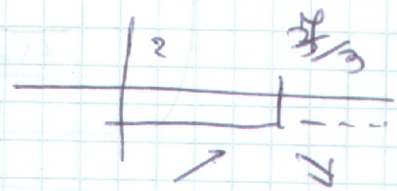
$$f'(x)=\begin{cases} \dfrac{1-3x+6}{3e^{x}\sqrt[3]{(x-2)^2}} & x>2 \\[2ex] \dfrac{-1+3x-6}{3e^{x}\sqrt[3]{(x-2)^2}} & x<2 \end{cases}$$

$$f'(x)=\begin{cases} \dfrac{7-3x}{3e^{x}\sqrt[3]{(x-2)^2}} & x>2 \\[2ex] \dfrac{3x-7}{3e^{x}\sqrt[3]{(x-2)^2}} & x<2 \end{cases}$$

$\lim_{x\to 2^+} f'(x) = +\infty$

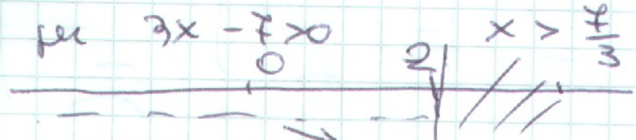
$\lim_{x\to 2^-} f'(x) = -\infty$

per $x>2$ $\quad f'(x)>0$ per $7-3x>0$ $\quad x<\frac{7}{3}$



$x=\frac{7}{3}$ max

per $x<2$ $\quad f'(x)>0$ per $3x-7>0$ $\quad x>\frac{7}{3}$



quindi la $f(x)$ è decrescente per $x \in ]-\infty, 2[ \cup ]\frac{7}{3}, +\infty[$

e crescente per $x \in ]2, \frac{7}{3}[$

CONCAVITÀ E CONVESSITÀ

$$f''(x) = \begin{cases} \dfrac{-3\left(3e^x \sqrt[3]{(x-2)^2}\right) + 3(7-3x)\left[e^x \sqrt[3]{(x-2)^2} + \frac{2}{3}e^x \frac{1}{\sqrt[3]{x-2}}\right]}{9e^{2x}\sqrt[3]{(x-2)^4}} & x>2 \\ \dfrac{3\left(3e^x \sqrt[3]{(x-2)^2}\right) + 3(3x-7)\left[e^x \sqrt[3]{(x-2)^2} + \frac{2}{3}e^x \frac{1}{\sqrt[3]{x-2}}\right]}{9e^{2x}\sqrt[3]{(x-2)^4}} & x<2 \end{cases}$$

$$f''(x) = \begin{cases} -\dfrac{3\sqrt[3]{(x-2)^2} + (7-3x)\left[\sqrt[3]{(x-2)^2} + \frac{2}{3}\frac{1}{\sqrt[3]{x-2}}\right]}{\cancel{9}\,3e^x \sqrt[3]{(x-2)^4}} & x>2 \\ \dfrac{3\sqrt[3]{(x-2)^2} + (7-3x)\left[\sqrt[3]{(x-2)^2} + \frac{2}{3}\frac{1}{\sqrt[3]{x-2}}\right]}{3e^x \sqrt[3]{(x-2)^4}} & x<2 \end{cases}$$

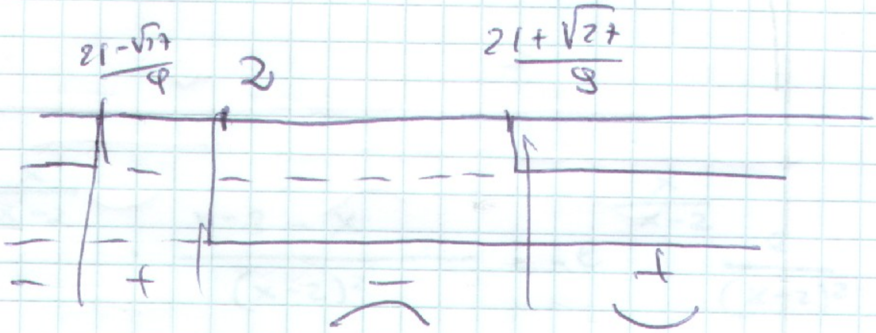
$$f''(x) = \begin{cases} -\dfrac{9(x-2) + (7-3x)\left[3(x-2) + 2\right]}{9e^x \sqrt[3]{x-2}\sqrt[3]{(x-2)^4}} & x>2 \\ \dfrac{9(x-2) + (7-3x)\left[3(x-2) + 2\right]}{9e^x \sqrt[3]{x-2}\sqrt[3]{(x-2)^4}} & x<2 \end{cases}$$

$$f''(x)=\begin{cases} -\dfrac{9x-18+(7-3x)(3x-4)}{9e^{x}\sqrt[3]{x-2}\,\sqrt[3]{(x-2)^4}} & x>2 \\[2ex] \dfrac{9x-18+21x-28-9x^2+12x}{9e^{x}\sqrt[3]{x-2}\,\sqrt[3]{(x-2)^4}} & x<2 \end{cases}$$

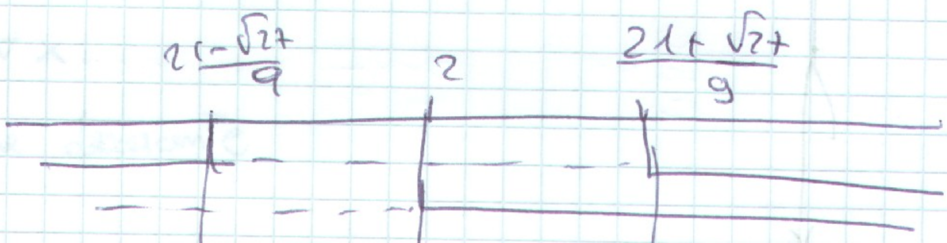
$$f''(x)=\begin{cases} +\dfrac{+9x^2+42x+46}{9e^{x}\sqrt[3]{x-2}\,\sqrt[3]{(x-2)^4}} & x\not>2 \\[2ex] -\dfrac{9x^2-42x+46}{9e^{x}\sqrt[3]{x-2}\,\sqrt[3]{(x-2)^4}} & x\not<2 \end{cases}$$

$$x=\frac{21\pm\sqrt{441-414}}{\cancel{18}\,9}=\frac{21\pm\sqrt{27}}{\cancel{18}\,9}$$

$f''(x)$ per $x>2$




per $x<2$

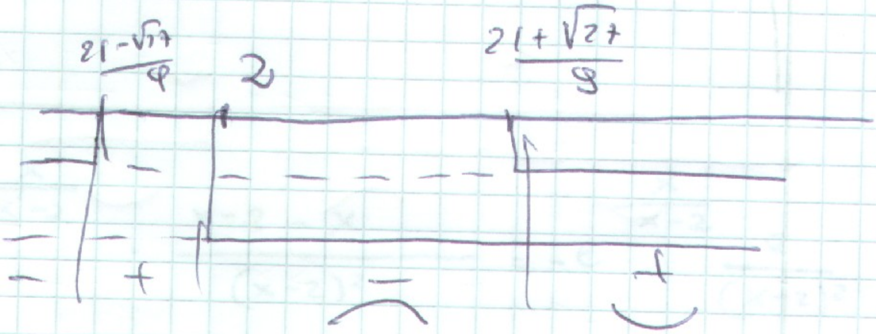


$$f''(x)=\begin{cases} -\dfrac{9x-18+(7-3x)(3x-4)}{9e^x\sqrt[3]{x-2}\,\sqrt[3]{(x-2)^4}} & x>2 \\[2ex] \dfrac{9x-18+21x-28-9x^2+12x}{9e^x\sqrt[3]{x-2}\,\sqrt[3]{(x-2)^4}} & x<2 \end{cases}$$

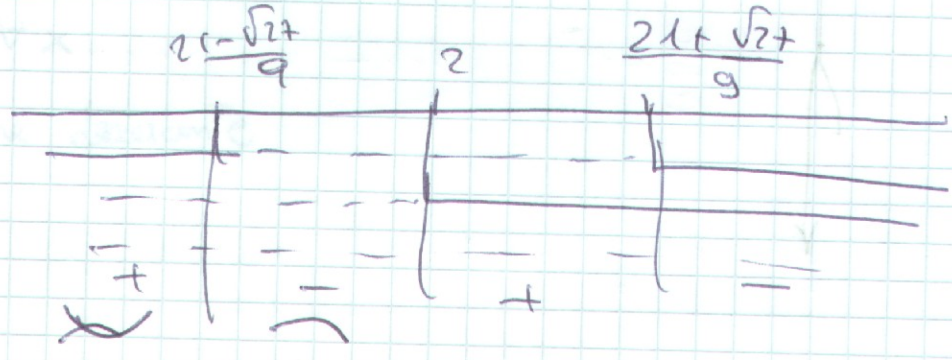
$$f''(x)=\begin{cases} +\dfrac{+9x^2-42x+46}{9e^x\sqrt[3]{x-2}\,\sqrt[3]{(x-2)^4}} & x>2 \\[2ex] -\dfrac{9x^2-42x+46}{9e^x\sqrt[3]{x-2}\,\sqrt[3]{(x-2)^4}} & x<2 \end{cases}$$

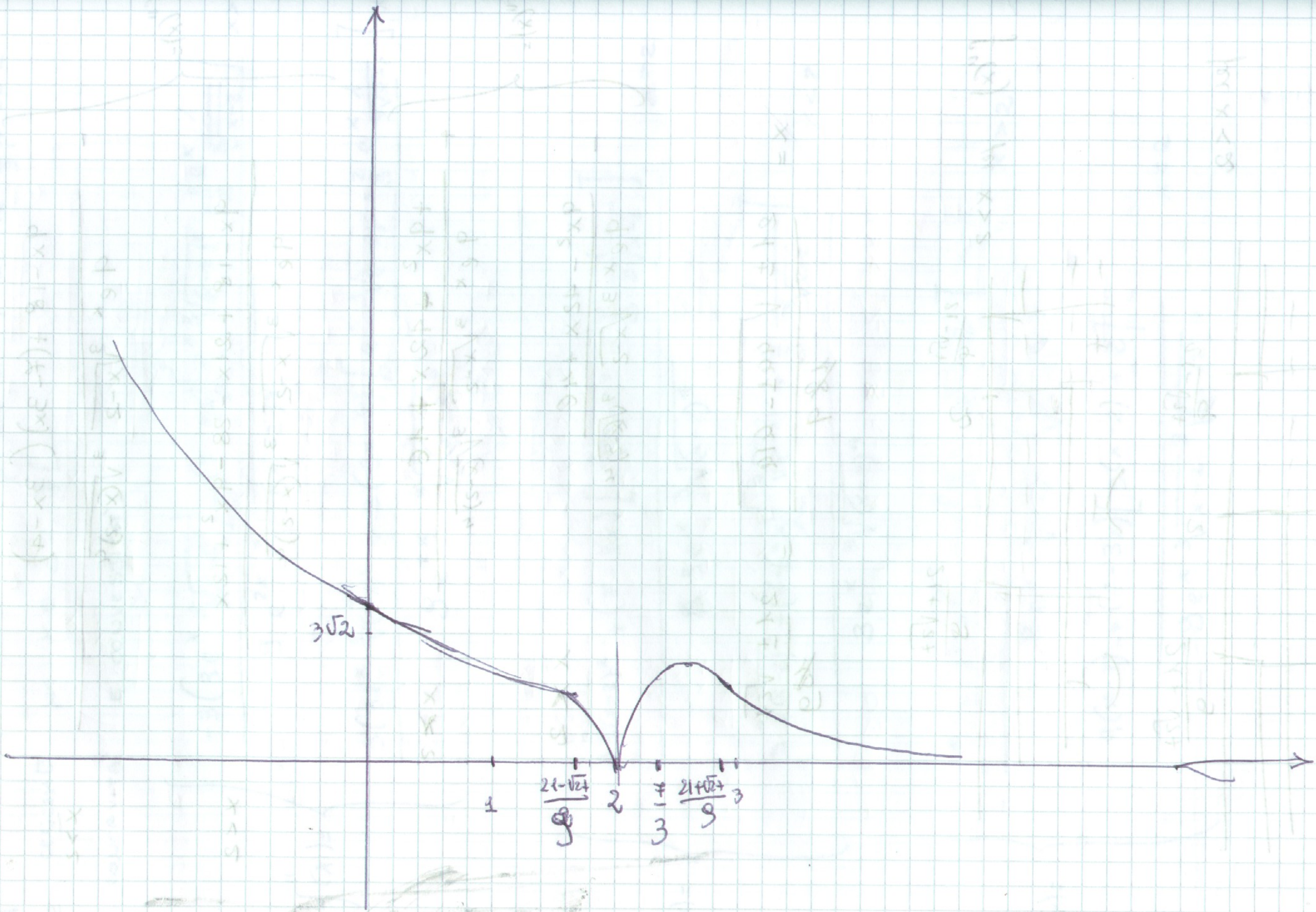
$$x=\frac{21\pm\sqrt{441-414}}{\cancel{18}\,9}=\frac{21\pm\sqrt{27}}{\cancel{18}\,9}$$

$f''(x)$ per $x>2$




per $x<2$





3√2
1
21-√27 / 9
2
7/3
21+√27 / 9
3

Giugno 1993 Del Prete

$$f(x) = e^{\frac{x}{x-2}}$$

CAMPO DI ESISTENZA.

$\cancel{\lim_{x-}}$ $x-2 \neq 0$ $x \neq 2$

CE : $\mathbb{R} - \{2\}$

POSITIVITÀ $f(x) > 0$ $\forall x \in CE$

INTERSEZIONI

con l'asse y : $x = 0$ $y = e^{\frac{0}{-2}} = 1$

Con l'asse x : nessuna

ASINTOTI

$$\lim_{x \to 2^-} e^{\frac{x}{x-2}} = e^{-\infty} = 0$$

$$\lim_{x \to 2^+} e^{\frac{x}{x-2}} = e^{+\infty} = +\infty$$

$$\lim_{x \to +\infty} e^{\frac{x}{x-2}} = e^{1} = e$$

$$\lim_{x \to -\infty} e^{\frac{x}{x-2}} = e$$

MAX E MIN

$$f'(x) = e^{\frac{x}{x-2}} \cdot \frac{x-2-x}{(x-2)^2} = -e^{\frac{x}{x-2}} \frac{2}{(x-2)^2}$$

$f'(x) < 0$ $\forall x$

$f(x)$ sempre decrescente

CONCAVITÀ E CONVESSITÀ

$$f''(x) = -\left( \frac{-2e^{\frac{x}{x-2}}}{(x-2)^2} + 2e^{\frac{x}{x-2}} \frac{2(x-2)}{(x-2)^4} \right) =$$

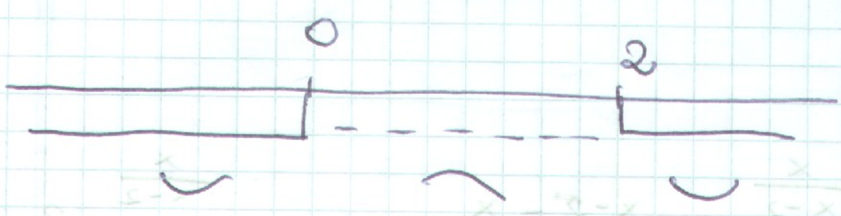
$$= 2e^{\frac{x}{x-2}} \left( \frac{1}{(x-2)^2} + 2\frac{x-2}{(x-2)^4} \right) =$$

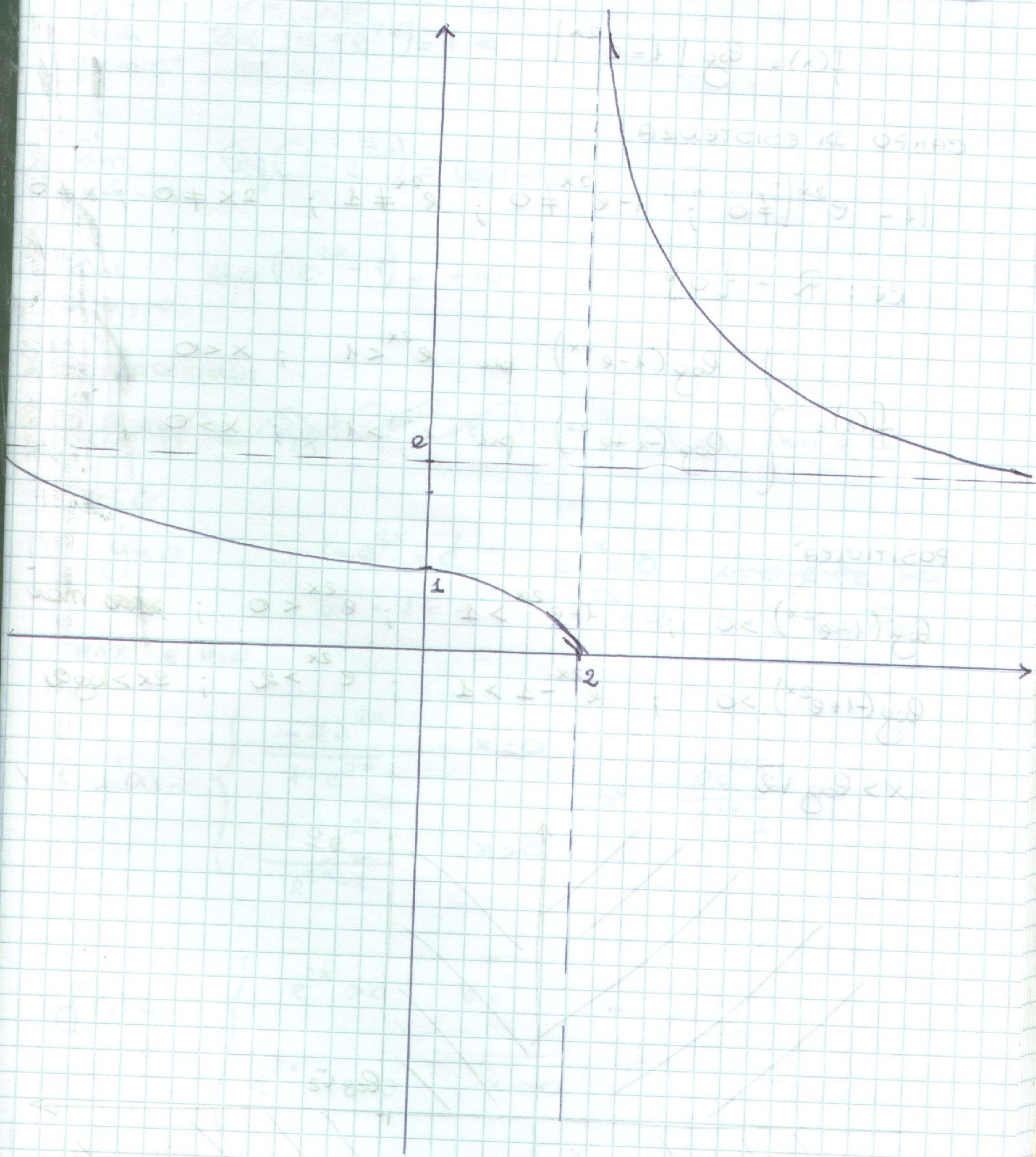
$$= \frac{2e^{\frac{x}{x-2}}}{(x-2)^2} \left( 1 + \frac{2x-4}{(x-2)^2} \right) =$$

$$= 2 \frac{e^{\frac{x}{x-2}}}{(x-2)^2} \frac{x^2 + \cancel{4} - 4x + 2x - \cancel{4}}{(x-2)^2} =$$

$$= 2 \frac{e^{\frac{x}{x-2}}}{(x-2)^2} \frac{x^2 - 2x \cancel{+4}}{(x-2)^2}$$

$f''(x) \geq 0$ per $x(x-2) \geq 0 \rightarrow x \leq 0, x \geq 2$




e
1
2

Del Prete - Febbraio 1994

$$f(x) = \log|1-e^{2x}|$$

CAMPO DI ESISTENZA

$|1-e^{2x}| \neq 0$ ; $1-e^{2x} \neq 0$ ; $e^{2x} \neq 1$ ; $2x \neq 0$ ; $x \neq 0$

CE : $\mathbb{R} - \{0\}$

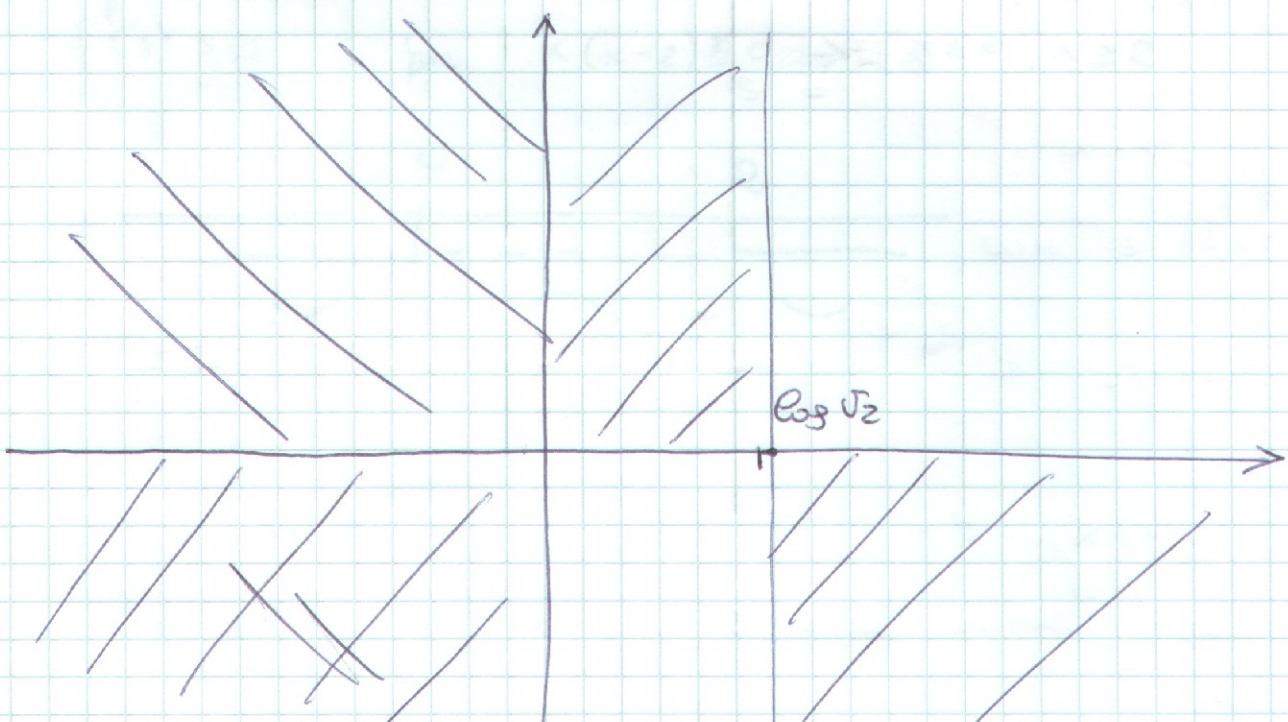
$$f(x) = \begin{cases} \log(1-e^{2x}) & \text{per } e^{2x} < 1 \ ; \ x<0 \\ \log(-1+e^{2x}) & \text{per } e^{2x} > 1 \ ; \ x>0 \end{cases}$$

POSITIVITÀ

$\log(1-e^{2x}) > 0$ ; $1-e^{2x} > 1$ ; $e^{2x} < 0$ ; mai

$\log(-1+e^{2x}) > 0$ ; $e^{2x}-1 > 1$ ; $e^{2x} > 2$ ; $2x > \log 2$

$x > \log\sqrt{2}$



$$\lim_{x \to 0^-} \log(1-e^{2x}) = -\infty$$

$$\lim_{x \to 0^+} \log(-1+e^{2x}) = +\infty$$

$$\lim_{x \to -\infty} \log(1-e^{2x}) = \log 1 = 0$$

$$\lim_{x \to +\infty} \log(e^{2x}-1) = +\infty$$

INTERSEZIONI

$\cancel{y=0 \quad \log|1-e^{2x}|=0} \quad |1-e^{2x}|=1$ non $\in$ CE

$x \neq 0$

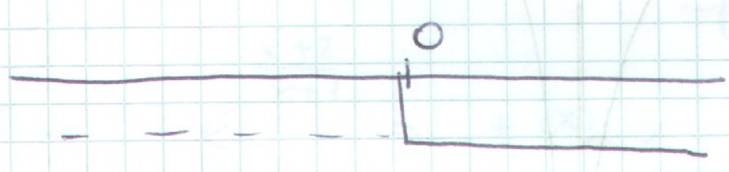
$y=0 \qquad 1-e^{2x}=1 \qquad e^{2x}=0 \qquad \cancel{x=0 \notin CE}$ mai

$\qquad\qquad e^{2x}-1=1 \qquad e^{2x}=2 \qquad x=\log\sqrt{2}$

MAX E MIN

$$f'(x)=\begin{cases} \dfrac{-2e^{2x}}{1-e^{2x}} & x<0 \\[2ex] \dfrac{2e^{2x}}{e^{2x}-1} & x>0 \end{cases} \quad \rightarrow \quad \frac{2e^{2x}}{e^{2x}-1} \neq 0 \quad \forall x$$

$$f'(x)>0 \quad \begin{cases} e^{2x}>0 & \forall x \\[2ex] e^{2x}-1>0 & x>0 \end{cases}$$



$$\lim_{x \to 0^-} \log(1-e^{2x}) = -\infty$$

$$\lim_{x \to 0^+} \log(-1+e^{2x}) = +\infty$$

$$\lim_{x \to -\infty} \log(1-e^{2x}) = \log 1 = 0$$

$$\lim_{x \to +\infty} \log(e^{2x}-1) = +\infty$$

INTERSEZIONI

$\cancel{y=0 \quad \log|1-e^{2x}|=0} \quad |1-e^{2x}|=1$ non $\in$ CE

$\cancel{x \neq 0}$

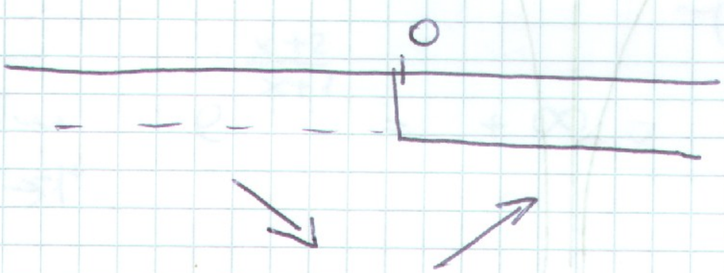
$y=0 \qquad 1-e^{2x}=1 \qquad e^{2x}=0 \qquad \cancel{x=0 \notin CE}$ mai

$\qquad e^{2x}-1=1 \qquad e^{2x}=2 \qquad x=\log\sqrt{2}$

MAX E MIN

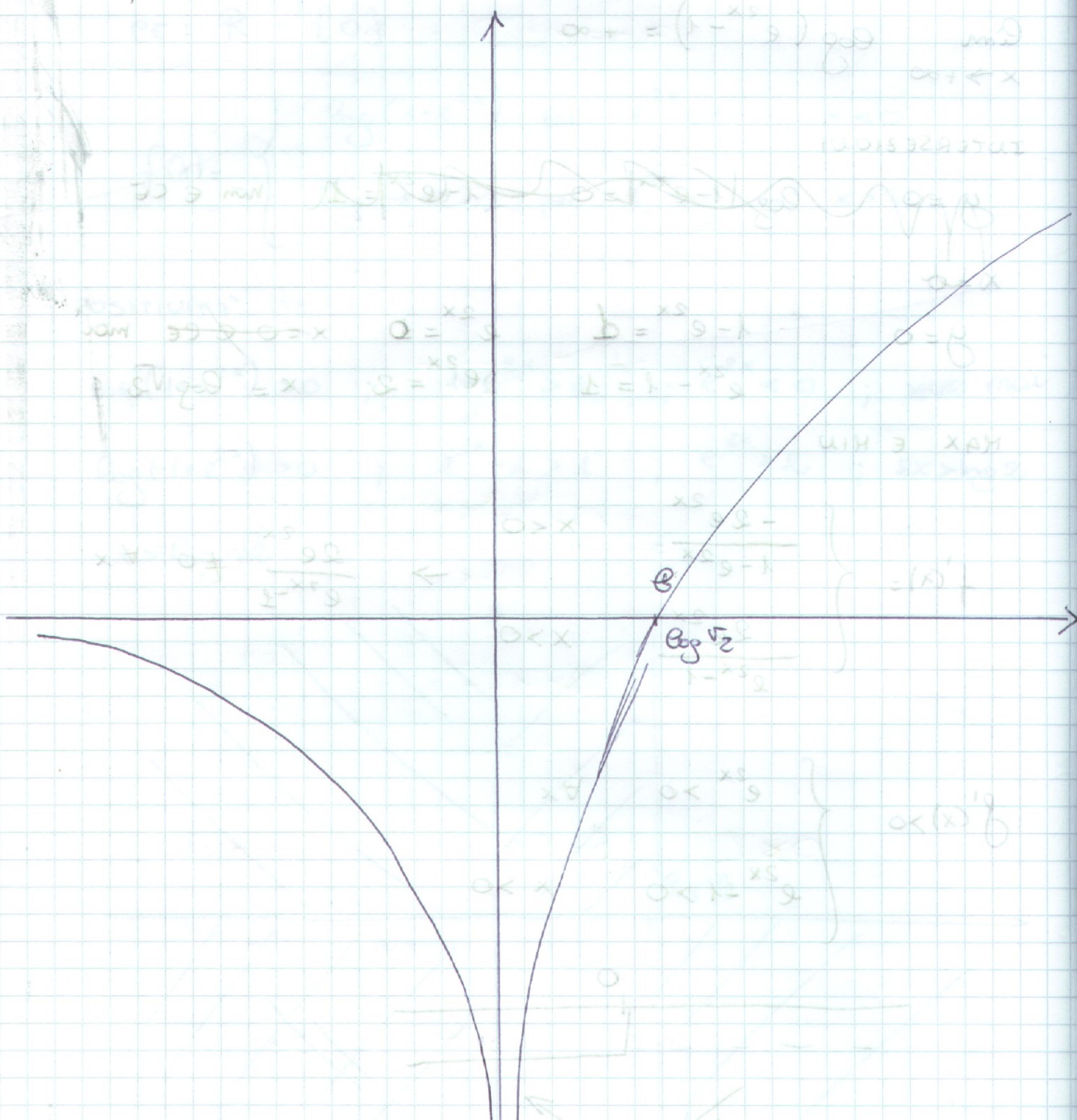
$$f'(x) = \begin{cases} \dfrac{-2e^{2x}}{1-e^{2x}} & x<0 \\[2ex] \dfrac{2e^{2x}}{e^{2x}-1} & x>0 \end{cases} \quad \rightarrow \quad \frac{2e^{2x}}{e^{2x}-1} \neq 0 \quad \forall x$$

$$f'(x)>0 \quad \begin{cases} e^{2x}>0 & \forall x \\[2ex] e^{2x}-1>0 & x>0 \end{cases}$$



$$f''(x) = \frac{4e^{2x}(e^{2x}-1) - 2e^{2x}\cdot 2e^{2x}}{(e^{2x}-1)^2} = \frac{\cancel{4e^{4x}} - 4e^{2x} - \cancel{4e^{4x}}}{(e^{2x}-1)^2}$$

$$= -\frac{4e^{2x}}{(e^{2x}-1)^2} < 0 \quad \forall x \qquad f(x) \text{ sempre concava}$$



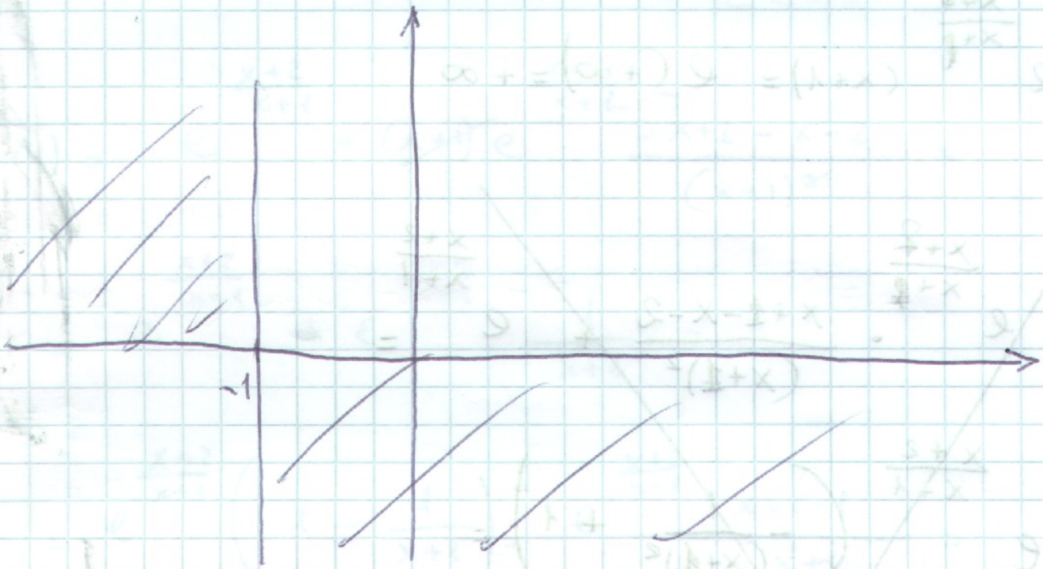
Del Prete luglio '80

$$f(x) = (x+1)\, e^{\frac{x+2}{x+1}}$$

CE: $x+1 \neq 0$ $\quad x \neq -1$ $\qquad$ CE: $\mathbb{R} - \{-1\}$

POSITIVITÀ

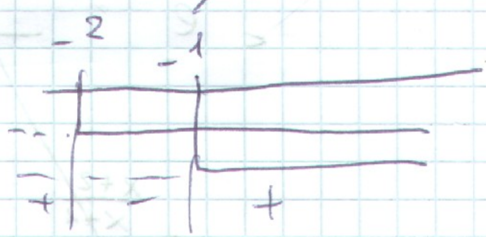
$f(x) > 0 \qquad x+1 > 0 \qquad x > -1$




Intersezione con l'asse y: $x = 0 \qquad f(0) = 1 \cdot e^2 = e^2$

con l'asse x $\qquad f(x) = 0 \qquad x+1 = 0 \qquad x = -1 \notin$ CE

$$\lim_{x \to -1^-} (x+1)\, e^{\frac{x+2}{x+1}} = 0\, e^{-\infty} = 0$$




$$\lim_{x \to -1^+} (x+1)\, e^{\frac{x+2}{x+1}} = 0\, e^{+\infty}$$

$$= \lim_{x \to -1^+} \frac{e^{\frac{x+2}{x+1}}}{\frac{1}{x+1}} = \lim_{x \to -1^+} \frac{e^{\frac{x+2}{x+1}} \cdot \frac{x+1-x-2}{(x+1)^2}}{-\frac{1}{(x+1)^2}} =$$

$$= \lim_{x \to -1^+} e^{\frac{x+2}{x+1}} = +\infty$$

$$\lim_{x \to -\infty} e^{\frac{x+2}{x+1}}(x+1) = e^{1} \cdot (-\infty) = -\infty$$

$$\cancel{= \lim_{x \to -\infty} \frac{e^{\frac{x+1}{x+2}}}{\frac{1}{x+1}} = e}$$

$$\lim_{x \to +\infty} e^{\frac{x+2}{x+1}}(x+1) = e(+\infty) = +\infty$$

$$\cancel{f'(x) = e^{\frac{x+2}{x+1}} \cdot \frac{x+1-x-2}{(x+1)^2} + e^{\frac{x+2}{x+1}} =}$$

$$\cancel{= e^{\frac{x+2}{x+1}} \left( -\frac{1}{(x+1)^2} + 1 \right) =}$$

$$\cancel{= e^{\frac{x+2}{x+1}} \frac{-1 + x^2 + 1 + 2x}{(x+1)^2} =}$$

$$\cancel{= e^{\frac{x+2}{x+1}} \frac{x^2 + 2x}{(x+2)^2}}$$

$$\cancel{f'(x) \geq 0 \quad x^2 + 2x + 3 \geq 0 \quad \Delta < 0}$$

$$f'(x) = 0 \quad \text{mai}$$

$$f'(x) > 0 \quad \forall x$$

$x(x+2)>0$

$x=-2$

$$f(x) = (x+1)\, e^{\frac{x+2}{x+1}}$$

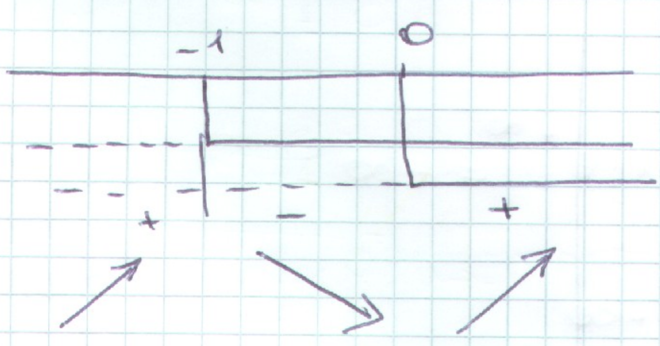
$$f'(x) = e^{\frac{x+2}{x+1}} + \cancel{(x+1)}\, e^{\frac{x+2}{x+1}} \, \frac{x+1-x-2}{(x+1)^2} =$$

$$= e^{\frac{x+2}{x+1}} - e^{\frac{x+2}{x+1}} \cdot \frac{1}{x+1} =$$

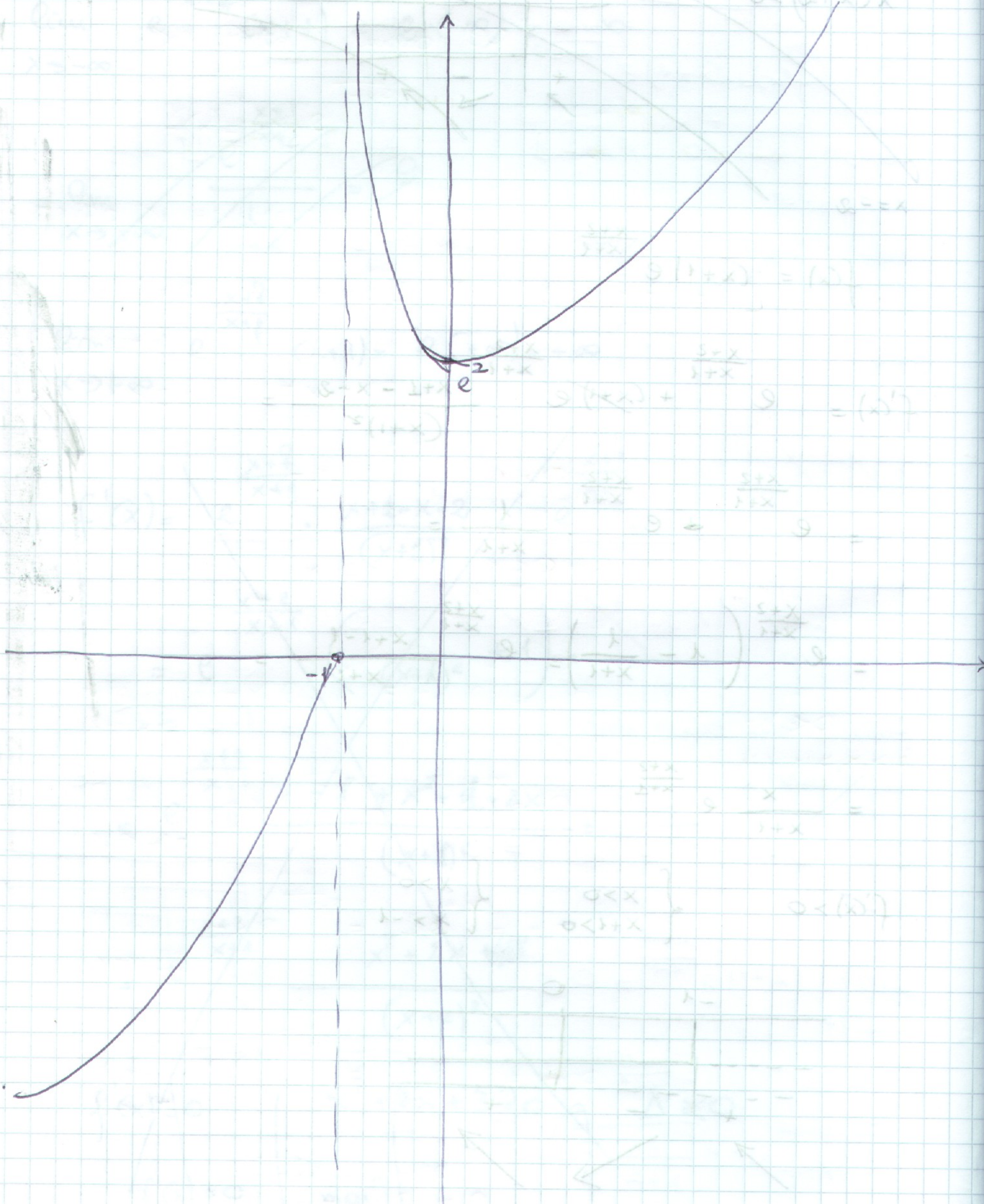
$$= e^{\frac{x+2}{x+1}} \left( 1 - \frac{1}{x+1} \right) = e^{\frac{x+2}{x+1}} \, \frac{x+1-1}{x+1} =$$

$$= \frac{x}{x+1}\, e^{\frac{x+2}{x+1}}$$

$f'(x)>0 \qquad \begin{cases} x>0 \\ x+1>0 \end{cases} \qquad \begin{cases} x>0 \\ x>-1 \end{cases}$



$x=0$ punto di min



$e^2$
$-1$

$$f''(x) = D \frac{x}{x+1} e^{\frac{x+2}{x+1}} =$$

$$= \frac{x+1-x}{(x+1)^2} e^{\frac{x+2}{x+1}} + \frac{x}{x+1} e^{\frac{x+2}{x+1}} \frac{\cancel{x}+1-\cancel{x}-2}{(x+1)^2} =$$

$$= \frac{1}{(x+1)^2} e^{\frac{x+2}{x+1}} + \frac{x}{(x+1)^3} e^{\frac{x+2}{x+1}} =$$

$$= \frac{1}{(x+1)^2} e^{\frac{x+2}{x+1}} \left( 1 - \frac{x}{x+1} \right) =$$

$$= \frac{1}{(x+1)^2} e^{\frac{x+2}{x+1}} \frac{x+1-x}{x+1} =$$

$$= \frac{1}{(x+1)^2} e^{\frac{x+2}{x+1}} > 0 \quad \forall x$$

f(x) sempre convessa

Del Prete luglio '90

$$f(x) = |x|\, e^{\frac{1}{1-x^2}}$$

CAMPO DI ESISTENZA

$1-x^2 \neq 0$ ; $x \neq \pm 1$ $CE: \mathbb{R} - \{-1, 1\}$

POSITIVITA'

$f(x) > 0 \quad \forall x \in CE$

$$f(x) = \begin{cases} -x\, e^{\frac{1}{1-x^2}} & x < 0 \\ x\, e^{\frac{1}{1-x^2}} & x > 0 \end{cases}$$

$$\lim_{x \to -\infty} -x\, e^{\frac{1}{1-x^2}} = +\infty\, e^{0} = +\infty$$

$1-x^2 > 0$
per $-1 < x < 1$

$$\lim_{x \to -1^-} -x\, e^{\frac{1}{1-x^2}} = 1 \cdot e^{-\infty} = 0$$

$$\lim_{x \to -1^+} -x\, e^{\frac{1}{1-x^2}} = 1\, e^{+\infty} = +\infty$$

$$\lim_{x \to 1^-} x\, e^{\frac{1}{1-x^2}} = 1 \cdot e^{+\infty} = +\infty$$

$$\lim_{x \to 1^+} x\, e^{\frac{1}{1-x^2}} = 1\, e^{-\infty} = 0$$

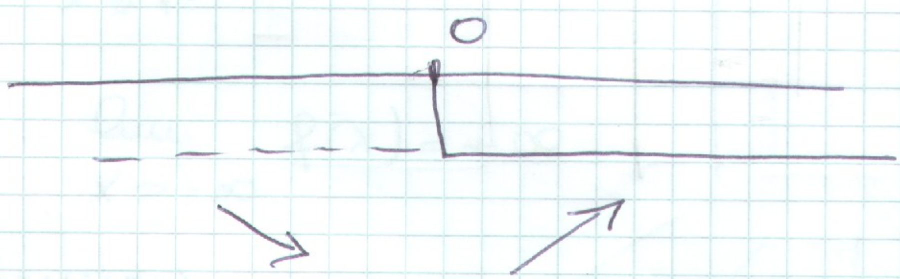
MAX E MIN.

$$f'(x) = \begin{cases} -e^{\frac{1}{1-x^2}} - x\, e^{\frac{1}{1-x^2}} \cdot \dfrac{2x}{(1-x^2)^2} & x<0 \\[2ex] e^{\frac{1}{1-x^2}} + x\, e^{\frac{1}{1-x^2}} \dfrac{2x}{(1-x^2)^2} & x>0 \end{cases}$$

$$f'(x) = \begin{cases} -e^{\frac{1}{1-x^2}} \left(1 + \dfrac{2x^2}{(1-x^2)^2}\right) & x<0 \\[2ex] e^{\frac{1}{1-x^2}} \left(1 + \dfrac{2x^2}{(1-x^2)^2}\right) & x>0 \end{cases}$$

$$f'(x) = \begin{cases} -e^{\frac{1}{1-x^2}} \dfrac{1+x^4}{(1-x^2)^2} & x<0 \\[2ex] e^{\frac{1}{1-x^2}} \dfrac{1+x^4}{(1-x^2)^2} & x>0 \end{cases}$$

$f'(x) > 0$ per $x > 0$




$x = 0$ punto di min

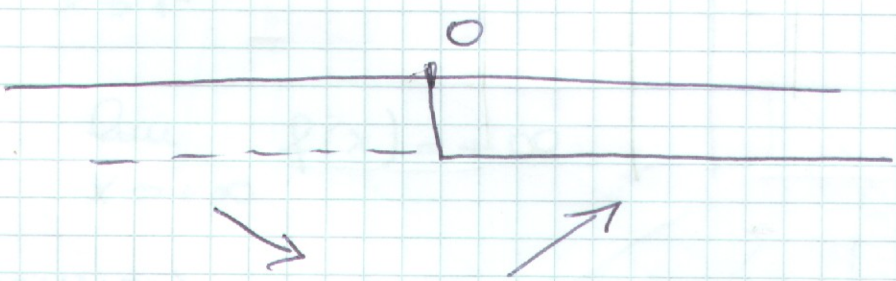
$f(0) = 0$

$$f'(x) = \begin{cases} -e \quad -x e \cdot \dfrac{\quad}{(1-x^2)^2} & x<0 \\ \\ e^{\frac{1}{1-x^2}} + x e^{\frac{1}{1-x^2}} \dfrac{2x}{(1-x^2)^2} & x>0 \end{cases}$$

$$f'(x) = \begin{cases} -e^{\frac{1}{1-x^2}} \left(1 + \dfrac{2x^2}{(1-x^2)^2}\right) & x<0 \\ \\ e^{\frac{1}{1-x^2}} \left(1 + \dfrac{2x^2}{(1-x^2)^2}\right) & x>0 \end{cases}$$

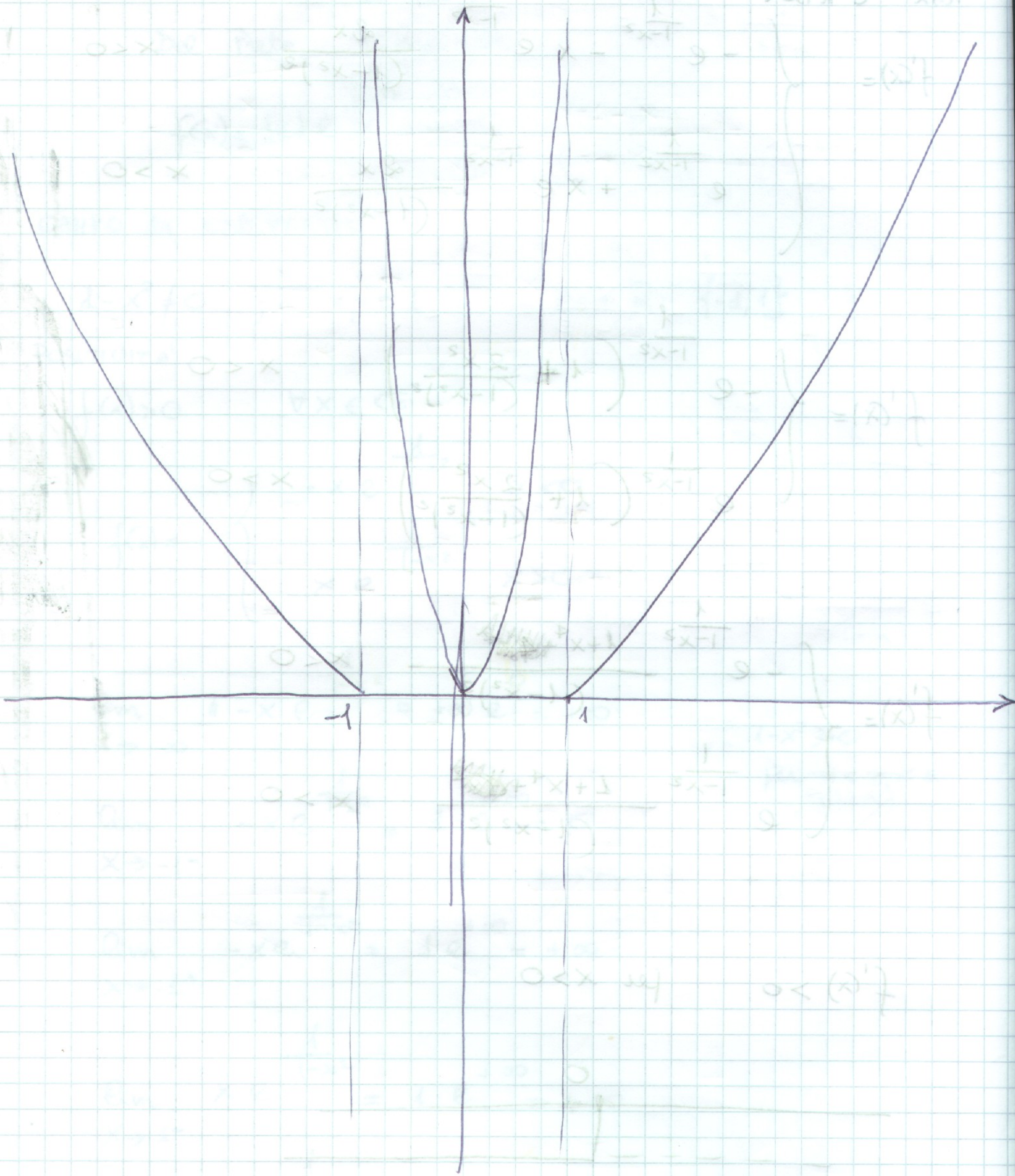
$$f'(x) = \begin{cases} -e^{\frac{1}{1-x^2}} \dfrac{1+x^4+}{(1-x^2)^2} & x<0 \\ \\ e^{\frac{1}{1-x^2}} \dfrac{1+x^4+}{(1-x^2)^2} & x>0 \end{cases}$$

$f'(x) > 0$ per $x>0$




$x=0$ punto di min

$f(0)=0$

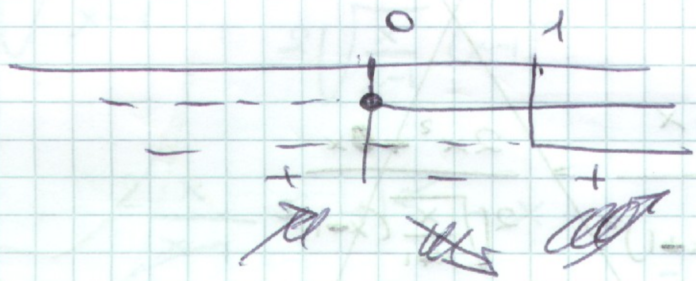

-1
1

Del Prete

$$f(x) = x\sqrt{\frac{x}{x-1}}$$

CAMPO DI ESISTENZA

$$\frac{x}{x-1} \geq 0 \qquad \begin{cases} x \geq 0 \\ x-1>0 \end{cases} \qquad \begin{cases} x \geq 0 \\ x>1 \end{cases}$$



CE: $]-\infty, 0] \cup ]1, +\infty[$

$$\lim_{x \to -\infty} f(x) = -\infty \cdot 1 = -\infty$$

$$\lim_{x \to 0^-} f(x) = f(0) = 0$$

$$\lim_{x \to 1^+} f(x) = +\infty$$

$$\lim_{x \to +\infty} f(x) = +\infty$$

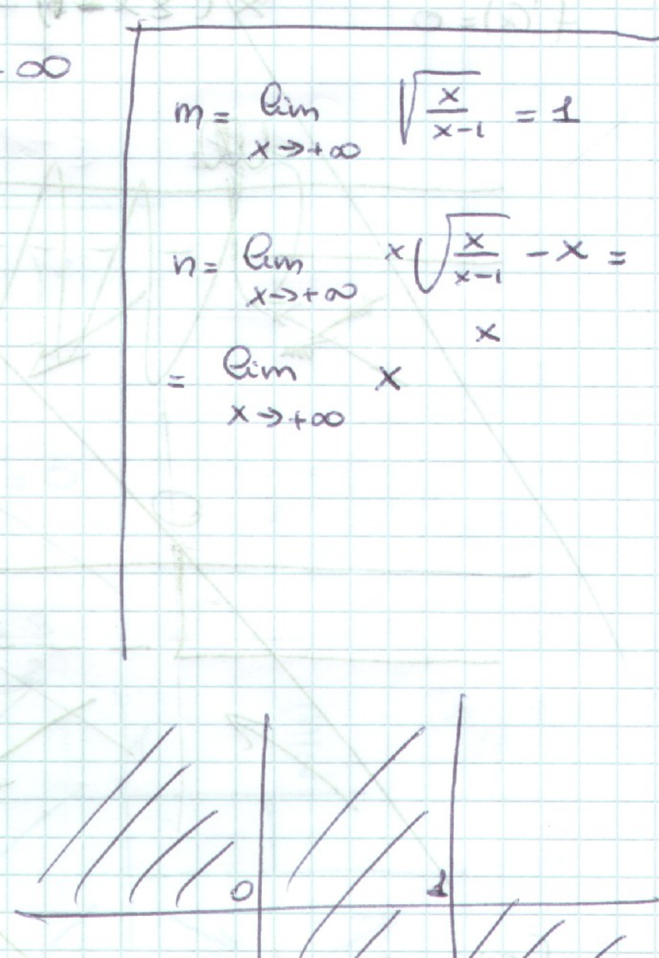
$$m = \lim_{x \to +\infty} \sqrt{\frac{x}{x-1}} = 1$$

$$n = \lim_{x \to +\infty} x\sqrt{\frac{x}{x-1}} - x =$$

$$= \lim_{x \to +\infty} \frac{x}{x}$$

POSITIVITA'

$f(x) > 0$ per $x > 0$



MAX E MIN

$$f(x) = x\sqrt{\frac{x}{x-1}}$$

$$f'(x) = \sqrt{\frac{x}{x-1}} + x\,\frac{1}{2\sqrt{\frac{x}{x-1}}}\cdot\frac{\cancel{x}-1-\cancel{x}}{(x-1)^2}$$

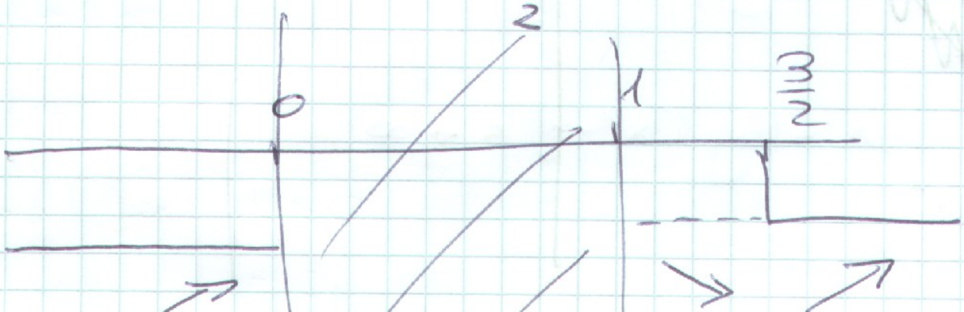
$$= \sqrt{\frac{x}{x-1}} - \frac{x}{2\sqrt{\frac{x}{x-1}}\,(x-1)^2} =$$

$$= \frac{2\frac{x}{\cancel{x-1}}(x-1)^{\cancel{2}} - x}{2\sqrt{\frac{x}{x-1}}\,(x-1)^2} =$$

$$= \frac{2x^2-3x}{2\sqrt{\frac{x}{x-1}}\,(x-1)^2}$$

$f'(x)=0 \qquad x(2x-3)=0$

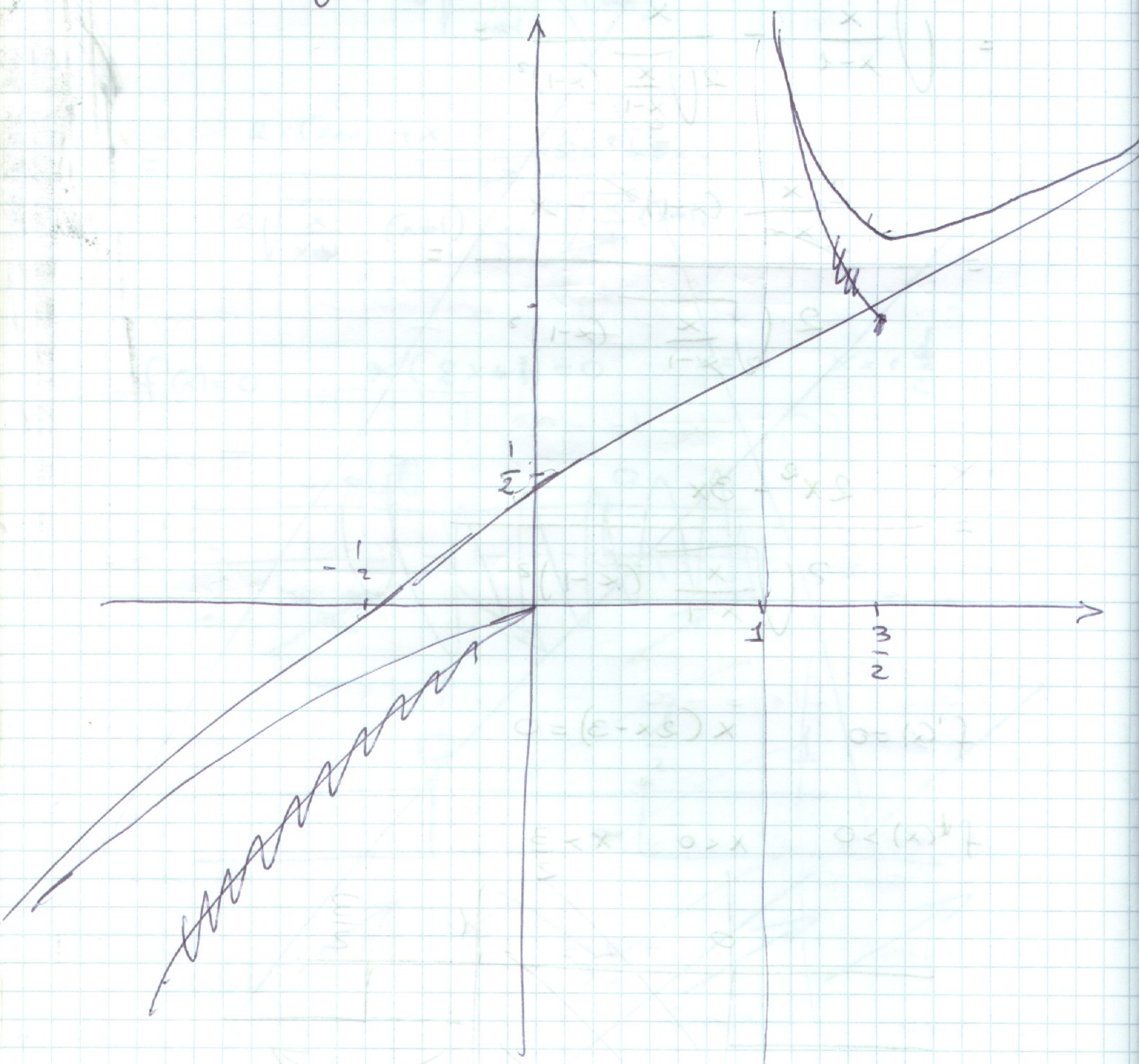
$f'(x)>0 \qquad x<0 \qquad x>\frac{3}{2}$



$x = \frac{3}{2}$ punto di minimo

$$f\left(\frac{3}{2}\right) = \frac{3}{2}\sqrt{\frac{\frac{3}{2}}{\frac{3}{2}-1}} = \frac{3}{2}\sqrt{\frac{\frac{3}{2}}{\frac{1}{2}}} =$$

$$= \frac{3}{2}\sqrt{3}$$



Asintoti obliqui

$$m = \lim_{x \to +\infty} \sqrt{\frac{x}{x-1}} = 1$$

$$n = \lim_{x \to +\infty} x\sqrt{\frac{x}{x-1}} - x =$$

$$= \lim_{x \to +\infty} x\left(\sqrt{\frac{x}{x-1}} - 1\right) = \lim_{x \to +\infty} x \, \frac{\frac{x}{x-1} - 1}{\sqrt{\frac{x}{x-1}} + 1} =$$

$$= \lim_{x \to +\infty} x \, \frac{\frac{\cancel{x} - \cancel{x} + 1}{x-1}}{\sqrt{\frac{x}{x-1}} + 1} = \lim_{x \to +\infty} \frac{\frac{x}{x-1}}{\sqrt{\frac{x}{x-1}} + 1} = \frac{1}{2}$$

$$y = x + \frac{1}{2}$$

$$m = \lim_{x \to -\infty} \cancel{x}\sqrt{\frac{x}{x-1}} = 1$$

$$n = \lim_{x \to -\infty} x\sqrt{\frac{x}{x-1}} - x = \lim_{x \to -\infty} x\left(\sqrt{\frac{x}{x-1}} - 1\right)$$

$$= \frac{1}{2}$$

$$f(x) = \frac{2\log x - 1}{\log x - 1}$$

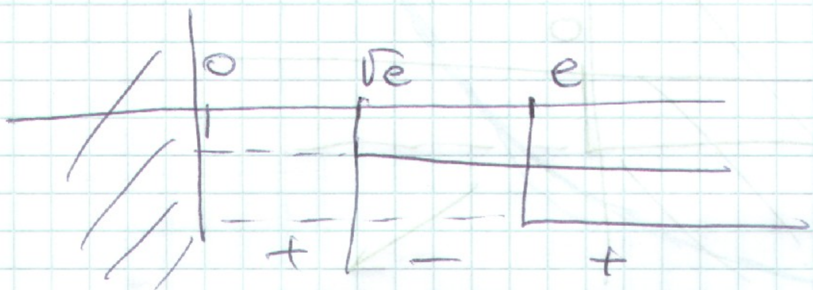
CAMPO DI ESISTENZA

$$\begin{cases} \log x - 1 \neq 0 \\ x > 0 \end{cases} \qquad \begin{cases} x \neq e \\ x > 0 \end{cases}$$

CE: $]0, e[ \cup ]e, +\infty[$

POSITIVITÀ

$$f(x) > 0 \qquad \begin{cases} 2\log x - 1 > 0 \\ \log x - 1 > 0 \end{cases} \qquad \begin{cases} \log x > \frac{1}{2} \\ \log x > 1 \end{cases} \qquad \begin{cases} x > \sqrt{e} \\ x > e \end{cases}$$




Intersezione con l'asse x per $x = \sqrt{e}$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{\log x}{\log x} \frac{2 - \frac{1}{\log x}}{1 - \frac{1}{\log x}} = 2$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\log x}{\log x} \frac{2 - \frac{1}{\log x}}{1 - \frac{1}{\log x}} = 2$$

$$\lim_{x \to e^-} f(x) = -\infty$$

$$\lim_{x \to e^+} f(x) = +\infty$$

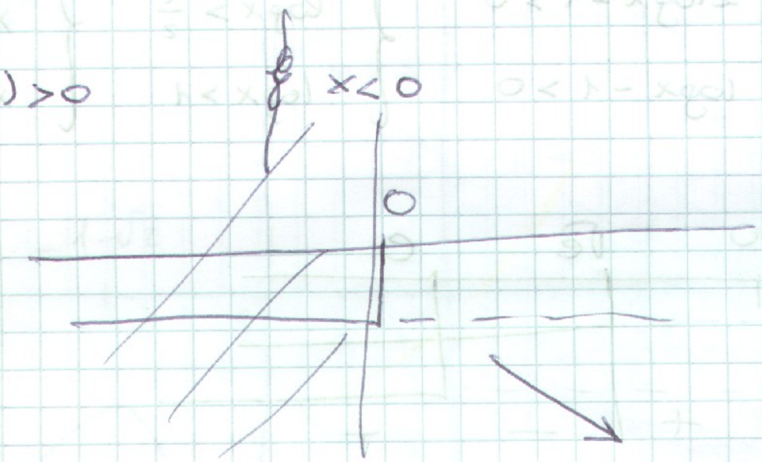
MAX E MIN

$$f'(x) = \frac{\frac{2}{x}(\log x - 1) - (2\log x - 1)\frac{1}{x}}{(\log x - 1)^2} =$$

$$= \frac{2\log x - 2 - 2\log x + 1}{x(\log x - 1)^2}$$

$$= -\frac{1}{x(\log x - 1)^2}$$

$f'(x) > 0$




$$f''(x) = + \frac{(\log x - 1)^2 + 2\cancel{x}(\log x - 1)\cdot\frac{1}{\cancel{x}}}{x^2(\log x - 1)^2}$$

$$= \frac{\log^2 x + 1 - 2\log x + 2\log x - 2}{x^2(\log x - 1)^2} =$$

$$= \frac{\log^2 x - 1}{x^2(\log x - 1)^2} \longrightarrow \log^2 x = \pm 1$$

$$x = e^{-1} \qquad x = e$$

$f''(x) > 0 \quad x < e^{-1} \; , x > e$