$$f(x) = \sqrt[5]{(x^2-1)^2}$$

C.E. : ~~$\mathbb{R}$~~ $\mathbb{R}$

~~$(x^2-1)^2 \nexists$~~

POSITIVITÀ

$$\sqrt[5]{(x^2-1)^2} \geq 0 \qquad \forall \mathbb{R}$$

INTERSEZIONE CON GLI ASSI

Con l'asse x

$$\begin{cases} y = 0 \\ y = \sqrt[5]{(x^2-1)^2} = 0 \end{cases} \Rightarrow \begin{matrix} x = -1 \\ x = +1 \end{matrix} \qquad A(-1,0) \quad B \equiv (1,0)$$

Con l'asse y

$$\begin{cases} x = 0 \\ y = \sqrt[5]{(x^2-1)^2} \end{cases} \Rightarrow \quad y = \sqrt[5]{(-1)^2} = 1 \qquad C \equiv (0,1)$$

ASINTOTI

Non vi sono asintoti verticali

A.O.

$$\lim_{x \to +\infty} \sqrt[5]{(x^2-1)^2} = \infty$$

A. OBLIQUI $\quad y = mx + n$

$$m = \lim_{x \to +\infty} \frac{\sqrt[5]{(x^2-1)^2}}{x} = \lim_{x \to +\infty} \sqrt[5]{\frac{(x^2-1)^2}{x^5}} = 0$$

non vi sono as. orizzontali od obliqui

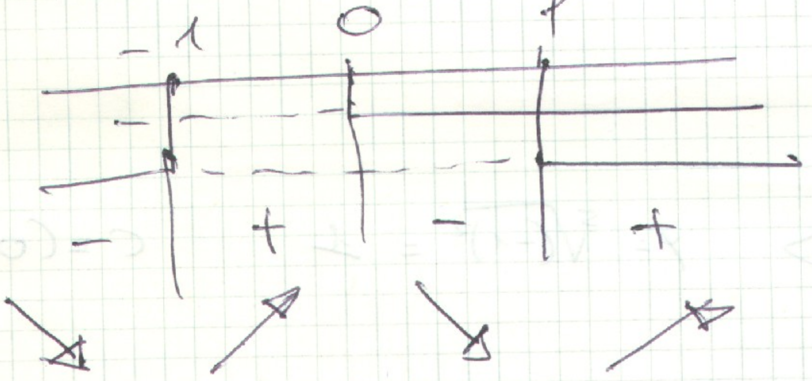
MAX e MIN.

$$Df(x) = D\ (x^2-1)^{\frac{2}{5}} = \frac{2}{5}(x^2-1)^{\frac{2}{5}-1} \cdot 2x =$$

$$= \frac{2}{5}(x^2-1)^{-\frac{3}{5}}\ 2x = \frac{4x}{5(x^2-1)^{3/5}}$$

$$f'(x)\ \frac{4x}{5(x^2-1)^{3/5}} = 0 \Rightarrow x = 0$$

$$\frac{4x}{5(x^2-1)^{3/5}} \geq 0 \Rightarrow \begin{cases} 4x \geq 0 \\ x^2-1 \geq 0 \end{cases} \quad \begin{cases} x \geq 0 \\ x < -1,\ x > 1 \end{cases}$$



$x=0$ è un punto di max

$$f''(x) = D\ \frac{4x}{5(x^2-1)^{3/5}} = \frac{4}{5}\ D\ x\,(x^2-1)^{-3/5} =$$

$$= \frac{4}{5}\Big[(x^2-1)^{-3/5} + \frac{3}{5}x\,(x^2-1)^{-\frac{3}{5}-1}\ 2x\Big] =$$

$$\Big[\ 1 \qquad 3x\ (x^2-1)^{-8/5}\ \Big] =$$

$$= \frac{4}{5}\left[\frac{1}{\sqrt[5]{(x^2-1)^3}} - \frac{6}{5}\frac{x}{\sqrt[5]{(x^2-1)^8}}\right] =$$

$$= \frac{4}{5}\left[\frac{5x^2-5 \quad -6x}{5\sqrt[5]{(x^2-1)^8}}\right] = \frac{4}{5}\frac{\qquad}{25\sqrt[5]{(x^2-1)^8}}$$

$$f''(x) = D\frac{4x}{5(x^2-1)^{3/5}} = \frac{4}{5}D\frac{x}{(x^2-1)^{3/5}}$$

$$= \frac{4}{5}\frac{(x^2-1)^{3/5} + x\,\frac{3}{5}(x^2-1)^{\frac{3}{5}-1}\cdot 2x}{(x^2-1)^{6/5}} =$$

$$= \frac{4}{5}\frac{\sqrt[5]{(x^2-1)^3} - \frac{6}{5}x^2(x^2-1)^{-\frac{2}{5}}}{\sqrt[5]{(x^2-1)^6}} =$$

$$= \frac{4}{5}\frac{\sqrt[5]{(x^2-1)^3} - \frac{6}{5}\cdot\frac{x^2}{\sqrt[5]{(x^2-1)^2}}}{\sqrt[5]{(x^2-1)^6}} =$$

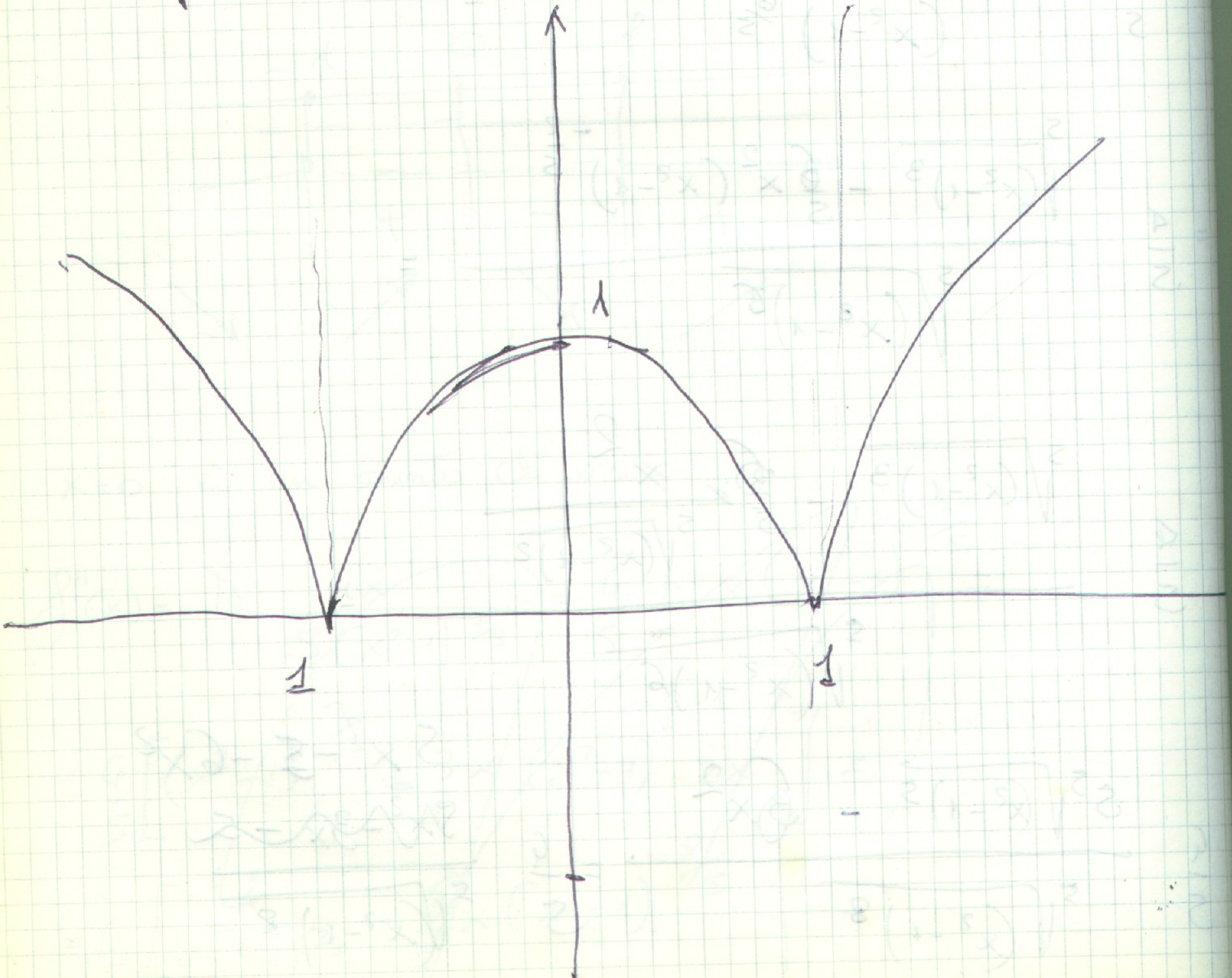
$$= \frac{4}{5}\frac{5\sqrt[5]{(x^2-1)^5} - 6x^2}{\sqrt[5]{(x^2-1)^8}} = \frac{4}{5}\frac{\overset{5x^2-5-6x^2}{\cancel{5x^2-3x-5}}}{\sqrt[5]{(x^2-1)^8}}$$

~~$f''(x)=0 \Rightarrow 5x^2-3x-5=0$~~

~~$x = \frac{3 \pm \sqrt{9+100}}{10} =$~~

$-x^2-5=0 \Rightarrow -(x^2+5)=0 \quad x^2=-5$

la derivata 2ª non si annulla mai
il numeratore è sempre negativo
e il denominatore è // positivo
per cui la funzione non ha flessi ed
sempre concava



$f(x) = \sqrt[5]{x^2-1}$

CE : $\mathbb{R}$

Positività

$\sqrt[5]{x^2-1} > 0 \Longrightarrow x^2-1>0 \Rightarrow x<-1 , x>1$

Intersezione con gli assi

Asse x $\begin{cases} y=0 \\ \sqrt[5]{x^2-1}=0 \end{cases} \Longrightarrow x=-1 , x=1$

Asse y $x=0$

$y = \sqrt[5]{-1} = -1$

Asintoti verticali: No

As. orizzontali

$\lim_{x\to\pm\infty} \sqrt[5]{x^2-1} = \infty$ No

As. obliqui

$m = \lim_{x\to\pm\infty} \frac{\sqrt[5]{x^2-1}}{x} = \sqrt[5]{\frac{x^2-1}{x^5}} = 0$ No

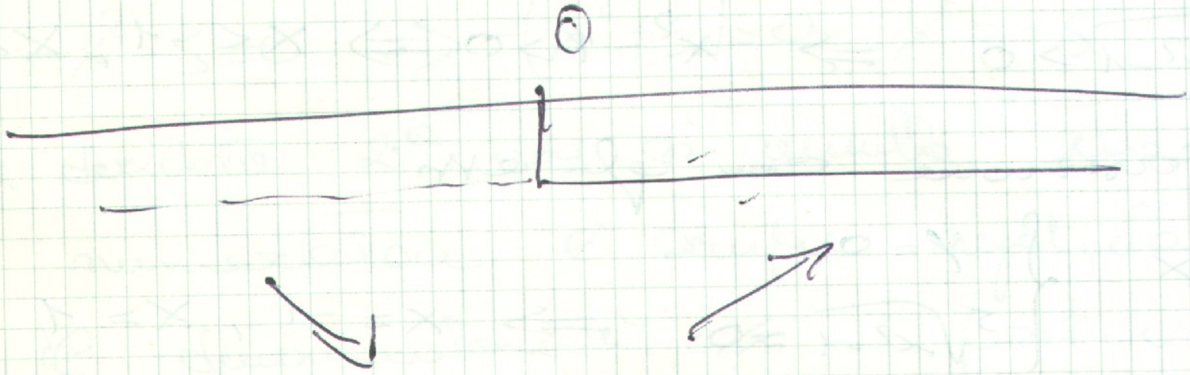
MAX e MIN

$Df(x) = \frac{1}{5}(x^2-1)^{\frac{1}{5}-1} 2x = \frac{1}{5}(x^2-1)^{-\frac{4}{5}} 2x =$

$= \frac{1}{5} \frac{2x}{\sqrt[5]{(x^2-1)^4}}$

$Df(x) = 0$ per $x=0$

$$Df(x) \geq 0 \quad \left\{ \begin{array}{l} x > 0 \\ \sqrt[5]{(x^2-1)^4} \geq 0 \end{array} \right\} \begin{array}{l} x > 0 \\ \forall x \end{array}$$



$$D^2 f(x) = \frac{2}{5} D \frac{x}{\sqrt[5]{(x^2-1)^4}} =$$

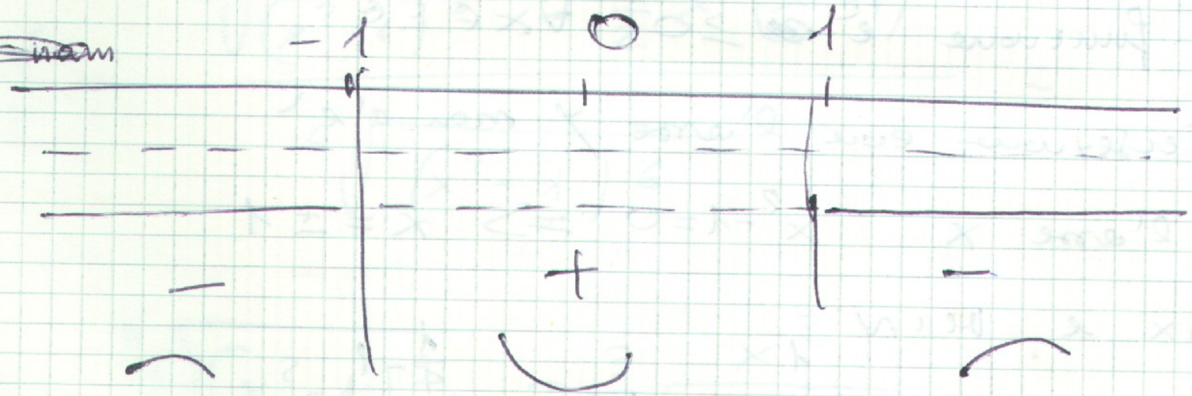
$$= \frac{2}{5} \frac{\sqrt[5]{(x^2-1)^4} - x \frac{4}{5} \cdot (x^2-1)^{\frac{4}{5}-1} \cdot 2x}{\sqrt[5]{(x^2-1)^8}} =$$

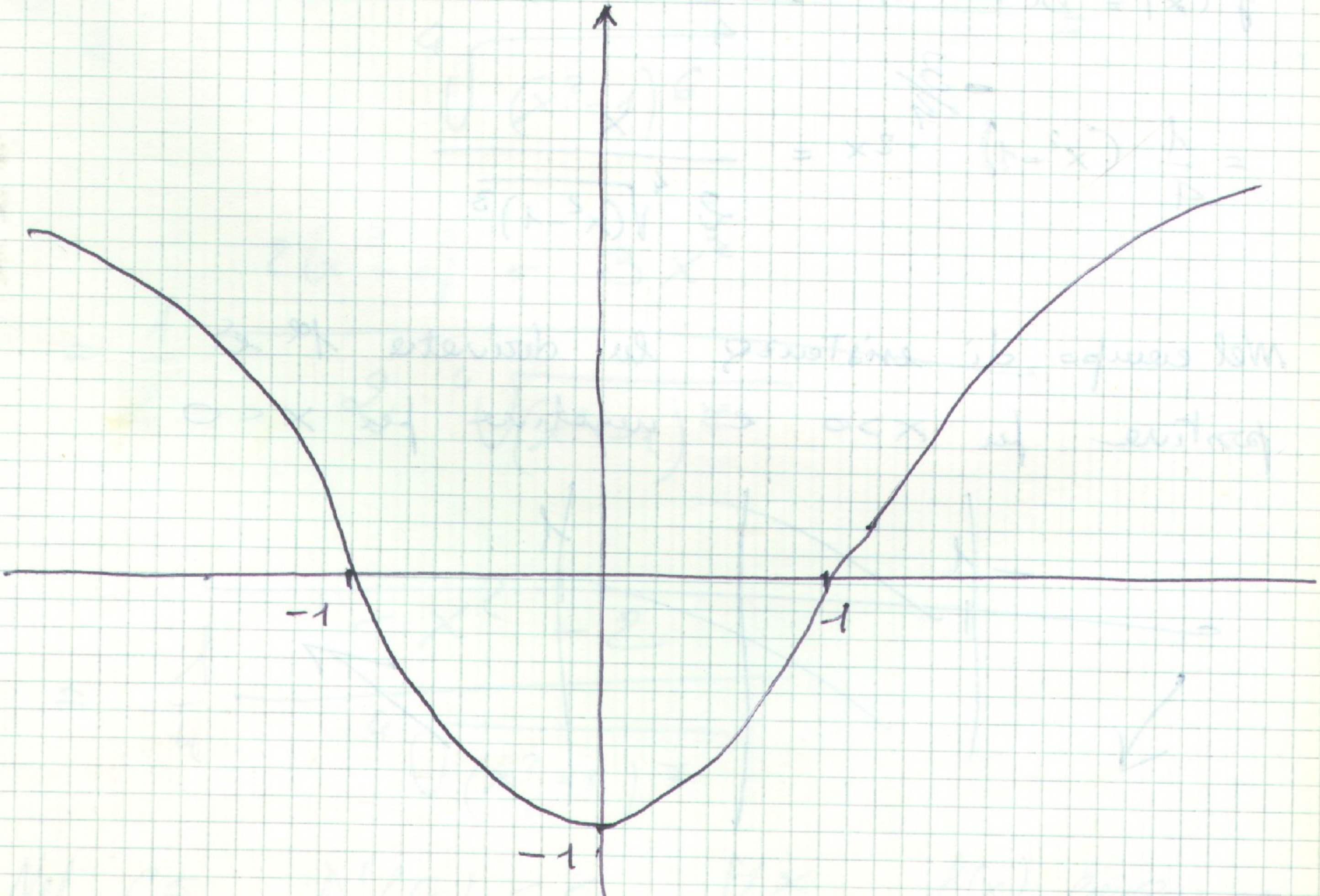
$$= \frac{2}{5} \frac{\sqrt[5]{(x^2-1)^4} - \frac{8}{5} x^2 \frac{1}{\sqrt{x^2-1}}}{\sqrt[5]{(x^2-1)^8}} =$$

$$= \frac{2}{5} \frac{\frac{5(x^2-1) - 8x^2}{5\sqrt{x^2-1}}}{\sqrt[5]{(x^2-1)^8}} =$$

$$= \frac{2}{5}\,\frac{5x^2 - 5 - 8x^2}{\sqrt[5]{(x^2-1)^9}} =$$

$$= \frac{2}{5}\,\frac{-3x^2 - 5}{\sqrt[5]{(x^2-1)^9}}$$





3

- $f(x) = \sqrt[4]{x^2-1}$

CE : $x^2-1 \geq 0$ $\qquad x \leq -1 \, , x \geq 1$

$]-\infty, -1[\cup]1, +\infty[$

Positività

la funzione è $\geq 0 \quad \forall x \in CE$

Intersezione con l'asse y non c'è

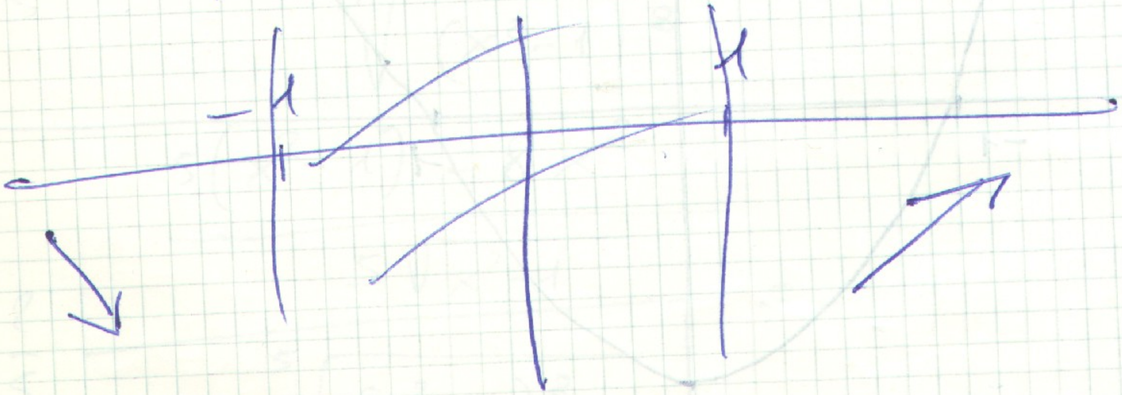
con l'asse x $\qquad x^2-1=0 \Rightarrow x = \pm 1$

MAX e MIN

$$f'(x) = D(x^2-1)^{\frac{1}{4}} = \frac{1}{4}(x^2-1)^{\frac{1}{4}-1} \cdot 2x =$$

$$= \frac{1}{4}(x^2-1)^{-\frac{3}{4}} \cdot 2x = \frac{x}{2\sqrt[4]{(x^2-1)^3}}$$

Nel campo di esistenza la derivata 1ª è positiva per $x>0$ e negativa per $x<0$



$$D^2 f(x) = \frac{1}{2} D \frac{x}{\sqrt[4]{(x^2-1)^3}} =$$

$$= \frac{1}{2} \frac{\sqrt[4]{(x^2-1)^3} - x \cdot \frac{3}{4}(x^2-1)^{\frac{3}{4}-1} 2x}{\sqrt[4]{(x^2-1)^6}} =$$

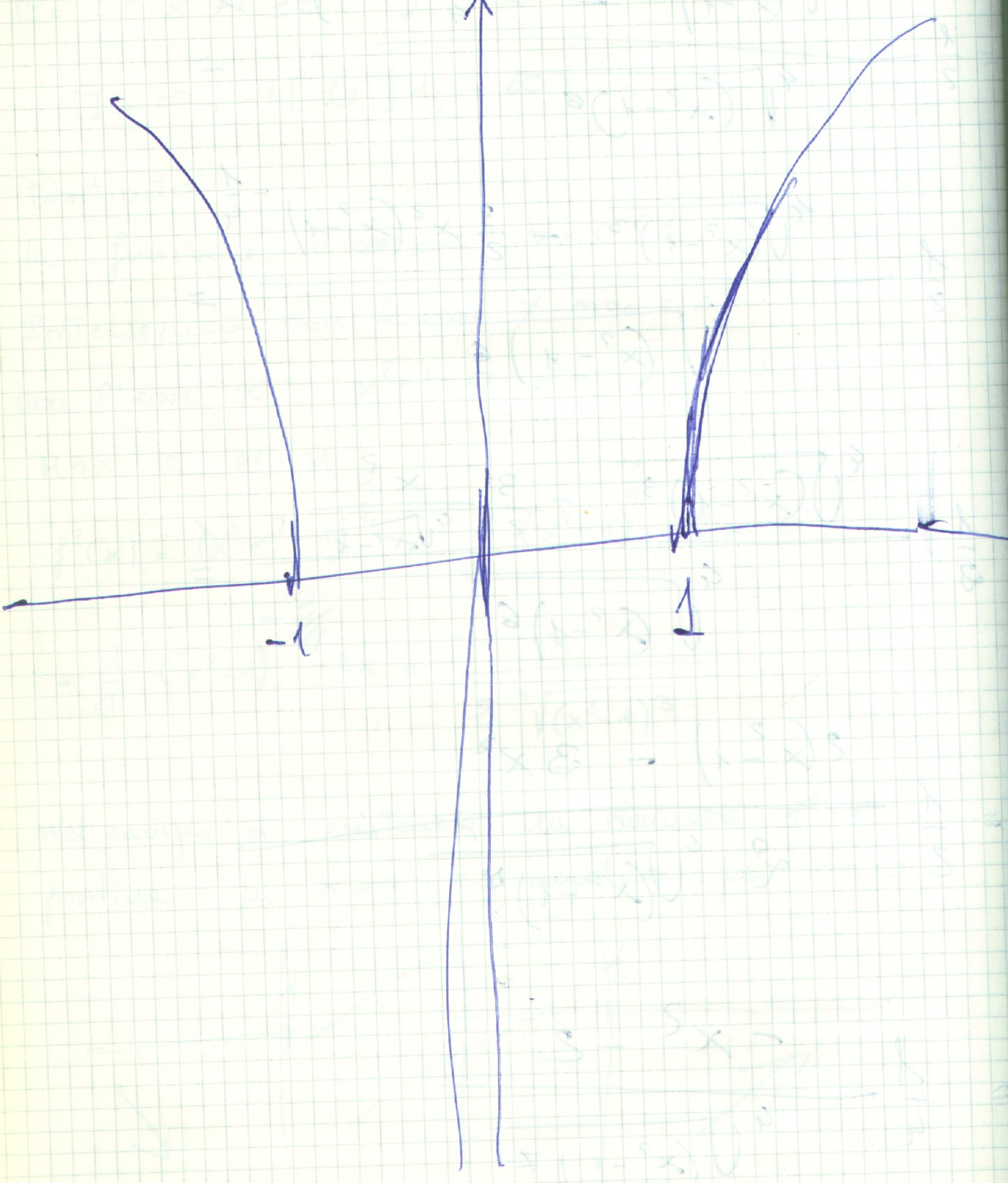
$$= \frac{1}{2} \frac{\sqrt[4]{(x^2-1)^3} - \frac{3}{2}x^2(x^2-1)^{-\frac{1}{4}}}{\sqrt[4]{(x^2-1)^6}} =$$

$$= \frac{1}{2} \frac{\sqrt[4]{(x^2-1)^3} - \frac{3}{2}\frac{x^2}{\sqrt[4]{x^2-1}}}{\sqrt[4]{(x^2-1)^6}} =$$

$$= \frac{1}{2} \frac{2(x^2-1) - 3x^2}{2\sqrt[4]{(x^2-1)^7}}$$

$$= \frac{1}{4} \frac{-x^2-2}{\sqrt[4]{(x^2-1)^7}}$$

Nel CE $D^2 f(x) < 0$ $\forall x$ $f(x)$ concava


-1
1

$f(x) = \sqrt[5]{(x^2-1)^4} = (x^2-1)^{4/5}$

CE : $\mathbb{R}$

Positività $f(x) \geq 0 \quad \forall x \in \mathbb{R}$

Intersezione con gli assi

$x = 0$

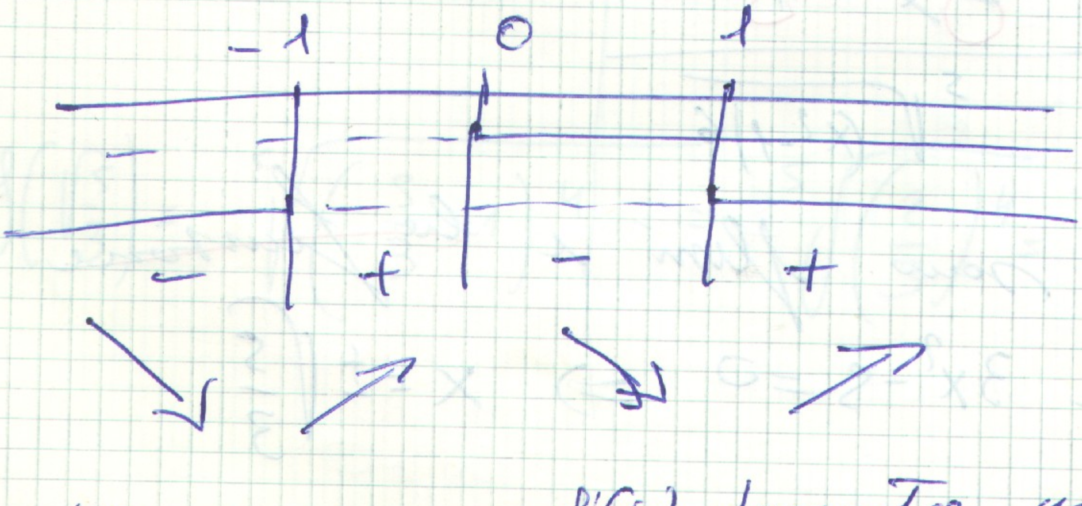
$\sqrt[5]{(x^2-1)^4} = 1$

$y = 0 \qquad \sqrt[5]{(x^2-1)^4} = 0 \Rightarrow x = \pm 1$

MAX e MIN

$D f(x) = \frac{4}{5}(x^2-1)^{\frac{4}{5}-1} 2x =$

$= \frac{8}{5}\frac{\cancel{x}\, x}{\sqrt[5]{x^2-1}}$ $\qquad D f(x) = 0$ per $x = 0$



per $x = 0$ $\quad f'(x)$ presenta un MAX

$$D^2 f(x) = \frac{8}{5} D \frac{x}{\sqrt[5]{x^2-1}} =$$

$$= \frac{8}{5} \frac{\sqrt[5]{x^2-1} - x \frac{1}{5}(x^2-1)^{\frac{1}{5}-1} \cdot 2x}{\sqrt[5]{(x^2-1)^2}} =$$

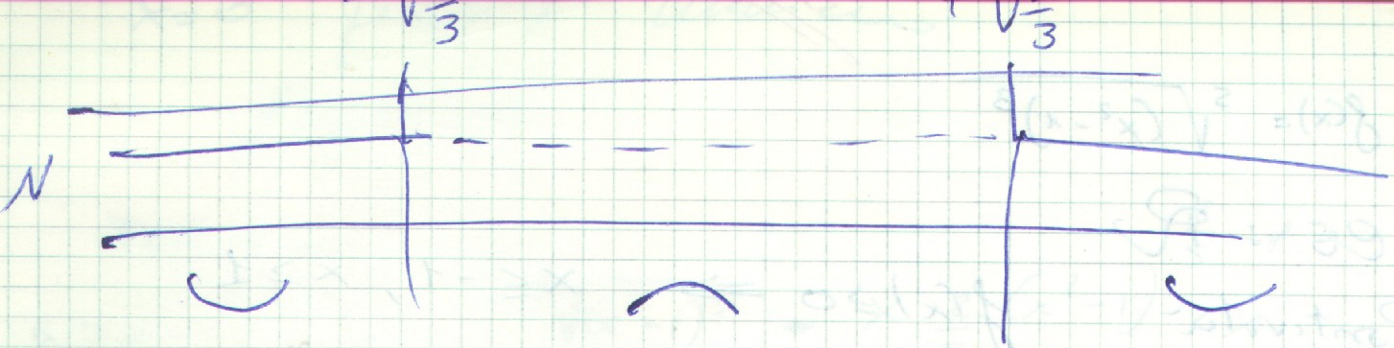
$$= \frac{8}{5} \frac{\sqrt[5]{x^2-1} \quad \frac{-2x^2}{5\sqrt[5]{(x^2-1)^4}}}{\sqrt[5]{(x^2-1)^2}} =$$

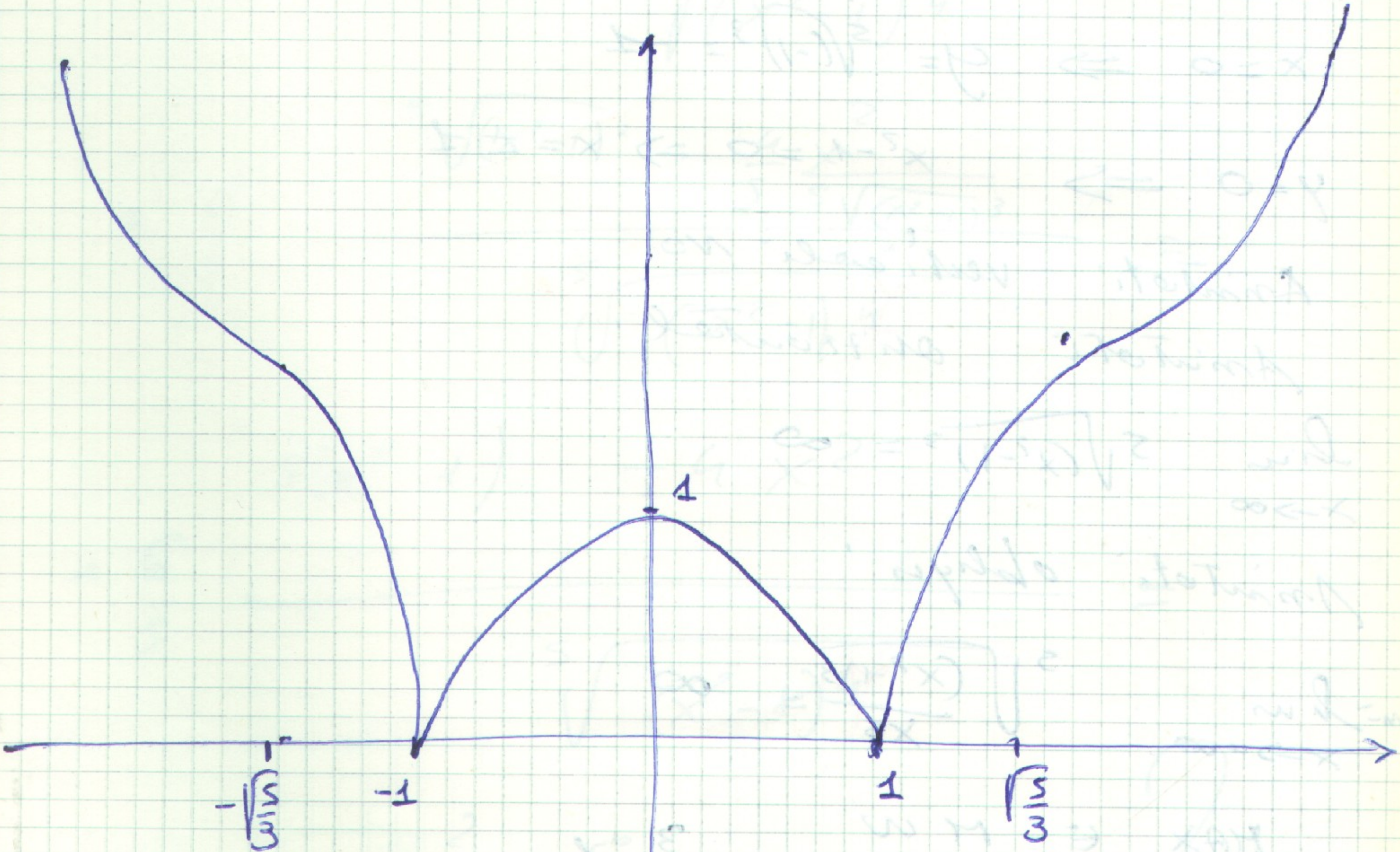
$$= \frac{8}{5} \frac{5x^2-5 - 2x^2}{5\sqrt[5]{(x^2-1)^6}}$$

$$= \frac{8}{5} \frac{3x^2-5}{\sqrt[5]{(x^2-1)^6}}$$

~~non~~ ci sono flessi e ~~la funzione~~

in per $3x^2-5=0 \Rightarrow x = \pm\sqrt{\frac{5}{3}}$





$$f\left(\sqrt{\frac{5}{3}}\right) = \sqrt[5]{\left(\frac{5}{3}-1\right)^4} = \sqrt[5]{\left(\frac{2}{3}\right)^4} =$$

$f(x) = \sqrt[5]{(x^2-1)^3}$

CE : $\mathbb{R}$

Positività $f(x) \geq 0 \Rightarrow x \leq -1,\ x \geq 1$

Intersezione con gli assi

$x = 0 \Rightarrow y = \sqrt[5]{(-1)^3} = -1$

$y = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1$

Asintoti verticali NO

Asintoti orizzontali

$$\lim_{x \to \infty} \sqrt[5]{(x^2-1)^3} = \infty$$

Asintoti obliqui

$$m = \lim_{x \to +\infty} \sqrt[5]{\frac{(x^2-1)^3}{x^5}} = \infty$$

MAX e MIN

$$Df(x) = \frac{3}{5}(x^2-1)^{\frac{3}{5}-1} 2x =$$

$$= \frac{6}{5} \frac{x}{\sqrt[5]{(x^2-1)^2}}$$

$f(x)$ è decrescente per $x < 0$

è crescente per $x > 0$

$x=0$ punto di minimo

$f(0)=-1$

FLESSI:

$$D^2f(x) = \frac{6}{5}\,\frac{\sqrt[5]{(x^2-1)^2} - x\,\frac{2}{5}(x^2-1)^{\frac{2}{5}-1}\,2x}{\sqrt[5]{(x^2-1)^4}} =$$

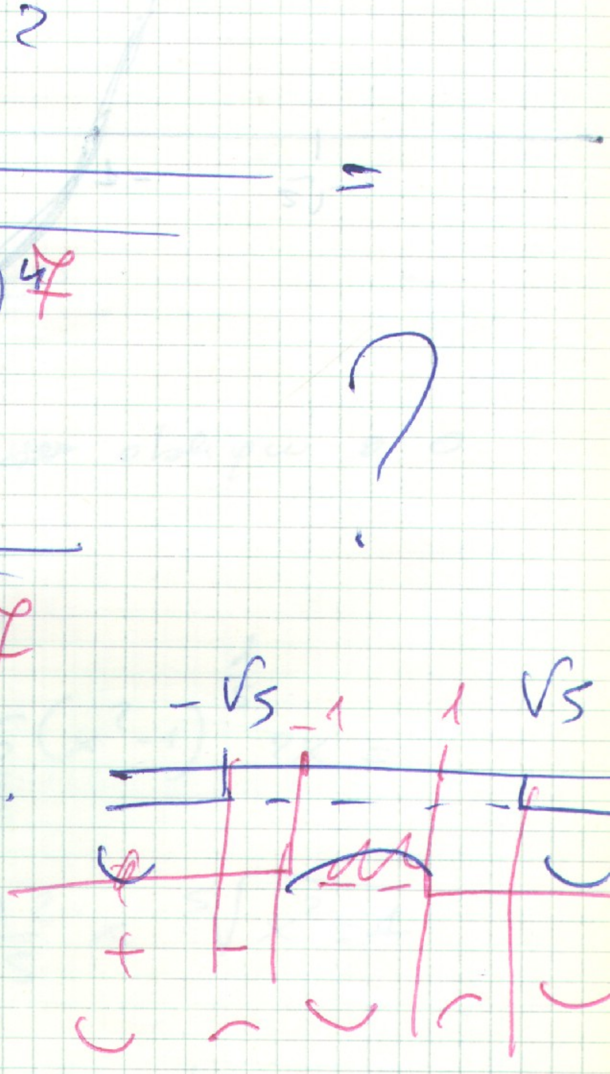
$$= \frac{6}{5}\,\frac{\sqrt[5]{(x^2-1)^2} - \frac{4}{5}\frac{x^2}{\sqrt[5]{(x^2-1)^3}}}{\sqrt[5]{(x^2-1)^7}} =$$

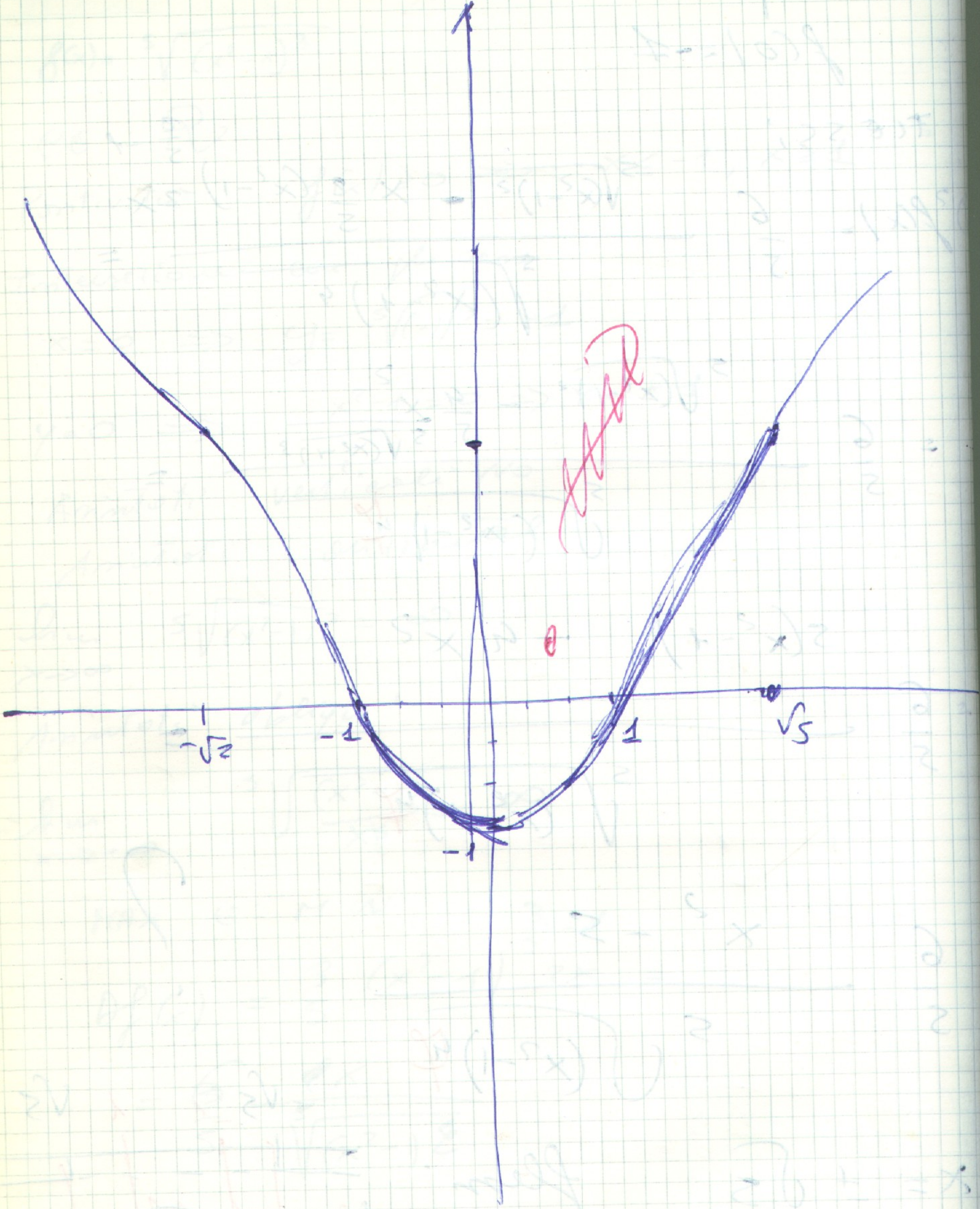
$$= \frac{6}{5}\,\frac{5(x^2-1) \quad -4x^2}{\sqrt[5]{(x^2-1)^7}} =$$

$$= \frac{6}{5}\,\frac{x^2-5}{\sqrt[5]{(x^2-1)^7}} \quad ?$$

$x = \pm\sqrt{5}$ flessi

$f(\pm\sqrt{5}) = \sqrt[5]{4^3} = 2,3$




-√2
-1
1
√5
-1

$y = \sqrt[5]{(x^2-1)^6}$

CE : R

Positività $f(x) \geq 0 \quad \forall x \in CE$

Intersezione con gli assi

$x = 0 \qquad y = 1$

$y = 0 \qquad x = \pm 1$

Non vi sono asintoti verticali

$$\lim_{x \to \infty} \sqrt[5]{(x^2-1)^6} = \infty$$

$$m = \lim_{x \to \infty} \sqrt[5]{\frac{(x^2-1)^6}{x^5}} = \infty$$

non vi sono asintoti ~~ve~~ obliqui o orizzontali

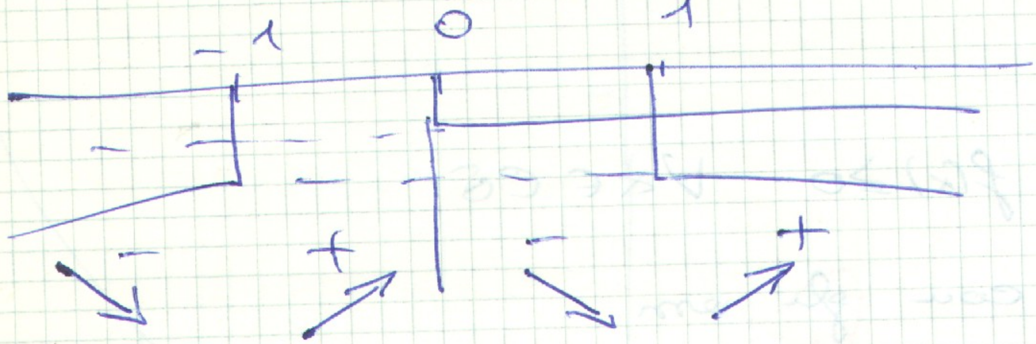
MAX e MIN

$$Df(x) = \frac{6}{5}(x^2-1)^{\frac{6}{5}-1} 2x = \frac{6}{5}(x^2-1)^{\frac{1}{5}} 2x =$$

$$= \frac{6}{5}\sqrt[5]{x^2-1} \cdot 2x = \frac{12}{5} x \sqrt[5]{x^2-1}$$

$Df(x) = 0$ per $x = 0, -1, +1$

$x\sqrt[5]{x^2-1} \geq 0$ $\quad \begin{cases} \\ x \leq -1,\ x \geq 1 \\ \end{cases}$



x = -1 punto di min

x = 0 " di max

x = 1 " di min

Flessi

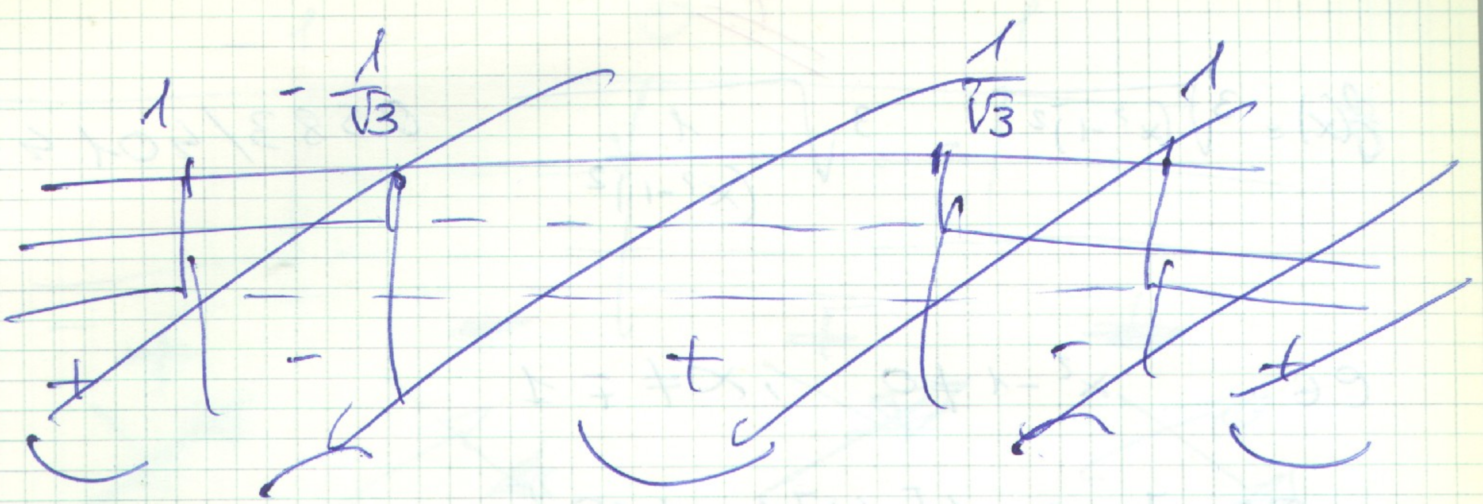
$\frac{12}{5} D\ x\sqrt[5]{x^2-1} =$

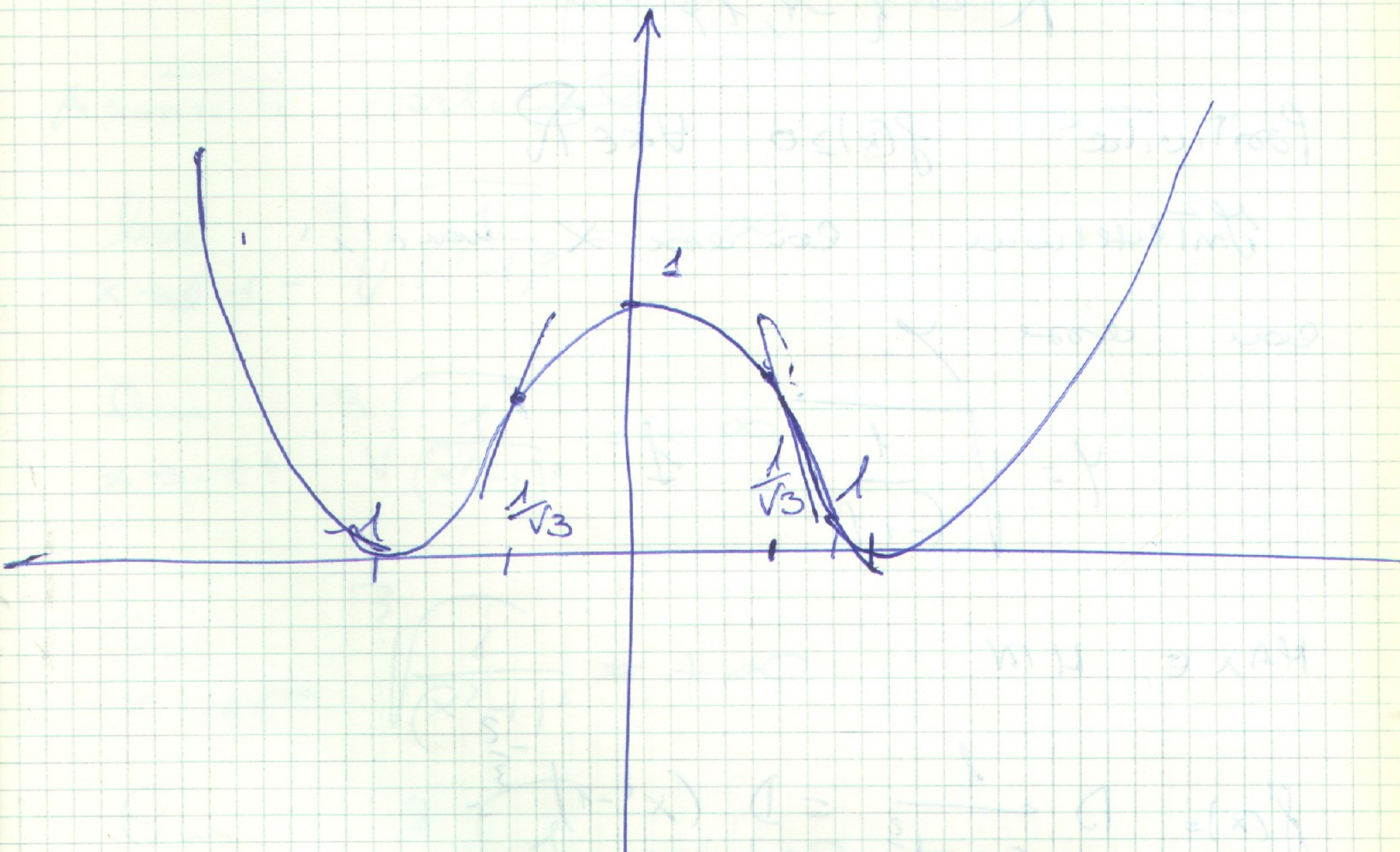
$= \frac{12}{5}\left[\sqrt[5]{x^2-1} + \frac{x}{5}(x^2-1)^{\frac{1}{5}-1} \cdot 2x \right] =$

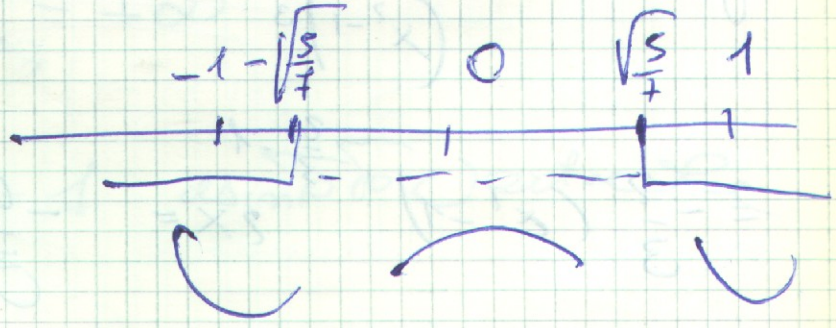
$= \frac{12}{5}\left[\sqrt[5]{x^2-1} + \frac{2}{5}x^2 (x^2-1)^{-\frac{4}{5}} \right] =$

$= \frac{12}{5}\left[\sqrt[5]{x^2-1} + \frac{2}{5}\frac{x^2}{\sqrt[5]{(x^2-1)^4}} \right] =$

$= \frac{12}{5}\frac{5x^2-5+2x^2}{5\sqrt[5]{(x^2-1)^4}} = \frac{12}{5}\frac{7x^2-}{5\sqrt[5]{(x^2-1)}}$


1
$-\frac{1}{\sqrt{3}}$
$\frac{1}{\sqrt{3}}$
1
+
-
+
-
-
+


1
-1
$\frac{1}{\sqrt{3}}$
$\frac{1}{\sqrt{3}}$
1


-1
$-\sqrt{\frac{5}{7}}$
0
$\sqrt{\frac{5}{7}}$
1

$f(x) = \sqrt[3]{(x^2-1)^2} = \sqrt[3]{\frac{1}{(x^2-1)^2}}$ 0583/401

$CE: \quad x^2-1 \neq 0 \quad , x \neq \mp 1$

$CE:$ ~~$]-\infty, -1[\cup]1, +\infty[$~~

$R - \{-1, 1\}$

Positività $f(x) \geq 0 \quad \forall x \in R$

Intersezione con asse X non c'è

con asse Y

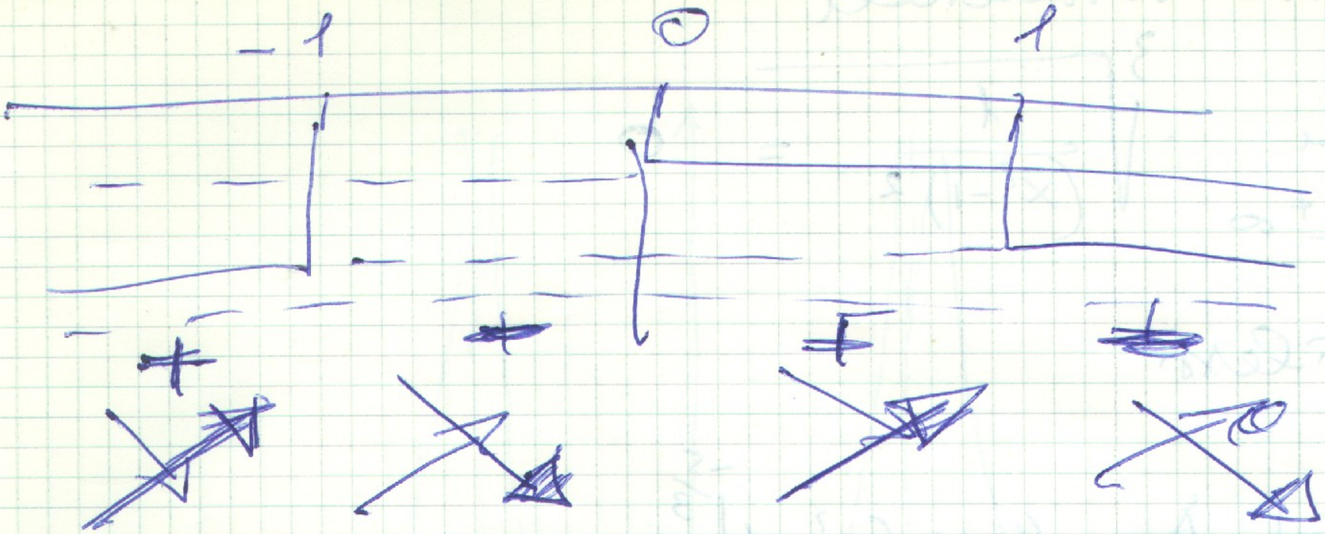
$$Y = \sqrt{\frac{1}{(-1)^2}} = 1$$

MAX E MIN

$$f'(x) = D \frac{1}{(x^2-1)^{\frac{2}{3}}} = D \, (x^2-1)^{-\frac{2}{3}} =$$

$$= -\frac{2}{3} (x^2-1)^{-\frac{2}{3}-1} \, 2x = -\frac{4}{3} \frac{x}{\sqrt[3]{(x^2-1)^5}}$$

$x = 0$



$x = 0$ punto di minimo

Asintoti verticali

$$\lim_{x \to -1^-} \sqrt[3]{\frac{1}{(x^2-1)^2}} = +\infty$$

$$\lim_{x \to -1^+} \sqrt[3]{\frac{1}{(x^2-1)^2}} = +\infty$$

$$\lim_{x \to +1^-} \sqrt[3]{\frac{1}{(x^2-1)^2}} = +\infty$$

$$\lim_{x \to +1^+} \sqrt[3]{\frac{1}{(x^2-1)^2}} = +\infty$$

$x = -1$ e $x = 1$ asintoto verticale

Asintoto orizzontale

$$\lim_{x \to \pm\infty} \sqrt[3]{\frac{1}{(x^2-1)^2}} = 0$$

Flessi

$$f''(x) = D \; -\frac{4}{3} x (x^2-1)^{-\frac{5}{3}} =$$

$$= -\frac{4}{3} (x^2-1)^{-\frac{5}{3}} + \frac{4}{3} x \left(-\frac{5}{3}\right) (x^2-1)^{-\frac{5}{3}-1} \cdot 2x =$$

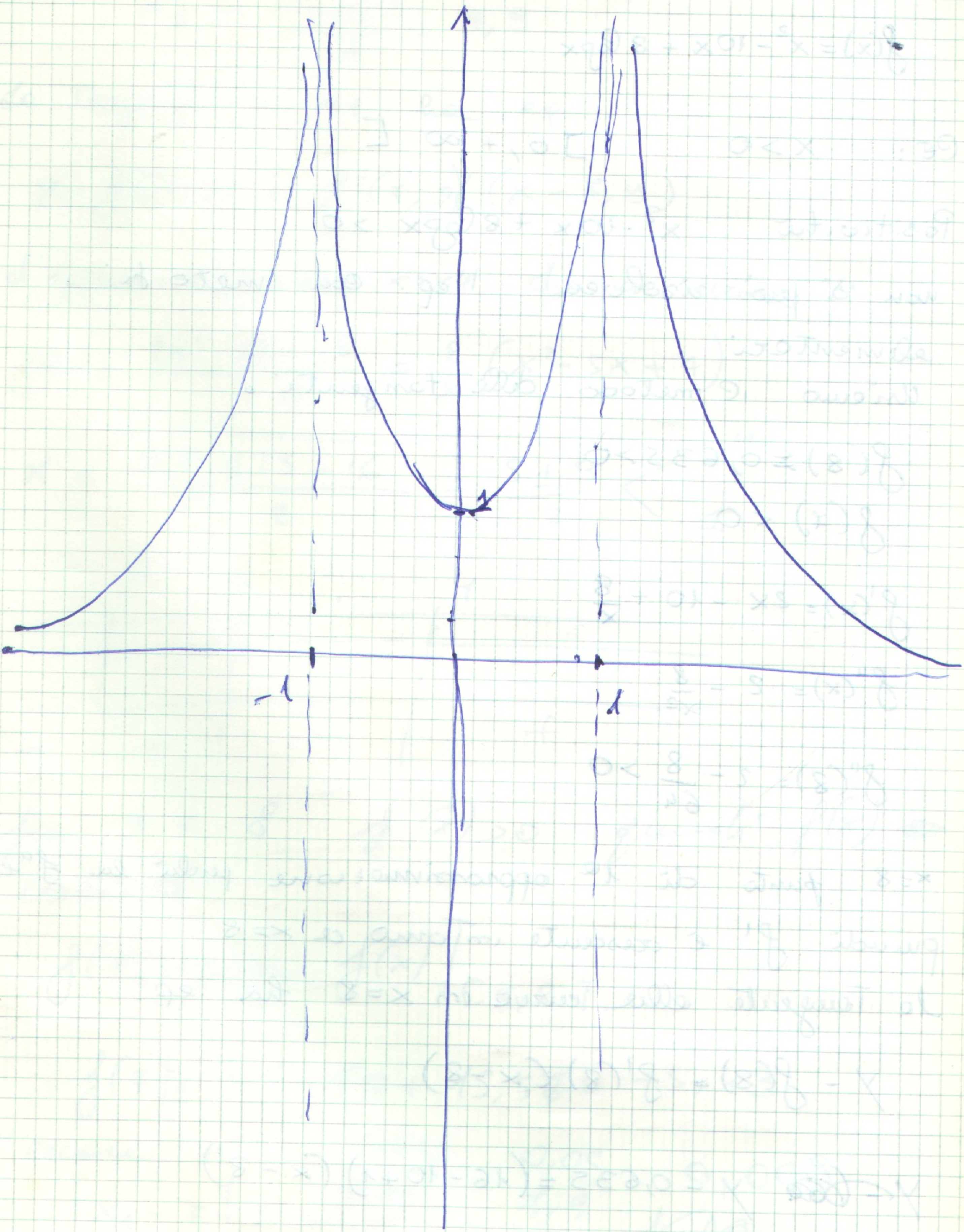
$$= -\frac{4}{3}\left(\frac{1}{\sqrt[3]{(x^2-1)^5}} + \frac{10}{3} \frac{x^2}{\sqrt[3]{(x^2-1)^8}}\right) =$$

$$= -\frac{4}{3} \frac{3(x^2-1) - 10x^2}{3\sqrt[3]{(x^2-1)^8}} =$$

$$= \frac{4}{3} \frac{7x^2+3}{3\sqrt[3]{(x^2-1)^8}}$$

non si annulla mai

è sempre positiva e la funzione


1
-1
1

$= x^2 - 10x + 8 \log x$

C.E.: $x > 0$ $]0, +\infty[$

Positività $x^2 - 10x + 8 \log x > 0$

non si può risolvere l'eq. con metodi elementari

Usiamo il metodo delle tangenti.

$f(8) \simeq 0,635 > 0$

$f(7) < 0$

$f'(x) = 2x - 10 + \frac{8}{x}$

$f''(x) = 2 - \frac{8}{x^2}$

$f''(8) = 2 - \frac{8}{64} > 0$

$x = 8$ punto di 1ª approssimazione perché la f quindi f' è crescente intorno a $x = 8$

la Tangente alla curva in $x = 8$ ha eq.

$y - f(8) = f'(8)(x - 8)$

~~$y - 66$~~ $y - 0,635 = (16 - 10 + 1)(x - 8)$

$y = 7x - 55,365$

Interseca l'asse x nel punto

$$x_1 = \frac{55,365}{7} = 7,9$$

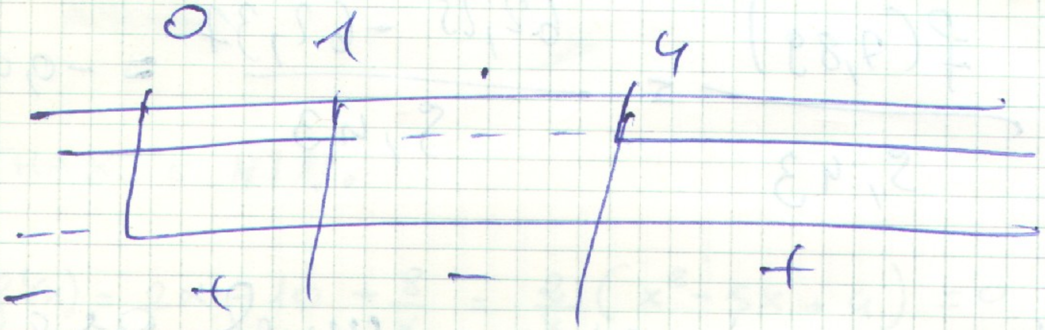
La tangente in x_1 ha eq.

$$y - f(7,9) = f'(7,9)(x - 7,9)$$

L'approssimazione $x = x_1$ dà un errore:

$$f'(x) = \frac{2x^2 - 10x + 8}{x} = \frac{2}{x}(x^2 - 5x + 4)$$

$$x = \frac{5 \pm \sqrt{25 - 16}}{2} = \frac{5 \pm 3}{2} < \begin{matrix} 4 \\ 1 \end{matrix}$$



fra 7 e 8 $f'(x) > 0$ quindi $f(x)$ crescente

$$f(7) = \min f(x)\Big|_{[7,8]}$$

$$f(7) = 49 - 70 + 15,57 = -5,43$$

l'errore è

$$|x_1 - \xi| \le \frac{\overset{0,05}{\cancel{f(x_1)}}}{5,43} = \frac{\overset{0,05}{\cancel{7,9}}}{5,43} = \overset{0,009}{\cancel{1,45}}$$

$y - [(7,9)^2 - 79 + 8 \log 7,9] =$

$= \left(2 \times 7,9 - 10 + \frac{8}{7,9}\right)(x - 7,9) =$

$y - 0,05 = 6,81x - 53,82$

$x_2 = \frac{53,77}{6,81} = 7,89$

$|x_2 - \xi| = \frac{f(7,89)}{5,43} = \frac{62,25 - 62,37}{5,43} = -$

$x_1 \overset{7,9}{=}$ è un'approssimazione miglio

~~tutto~~ Queste conclusioni va fatta alla fine

quando si è sicuri che il

diagramma taglia l'asse del

in un punto!

Asintoti verticali

$$\lim_{x\to 0^+} x^2-10x+8\log x = 0-\infty = -\infty$$ AS. VERT.

Asintoti orizzontali

$$\lim_{x\to +\infty} x^2-10x+8\log x =$$

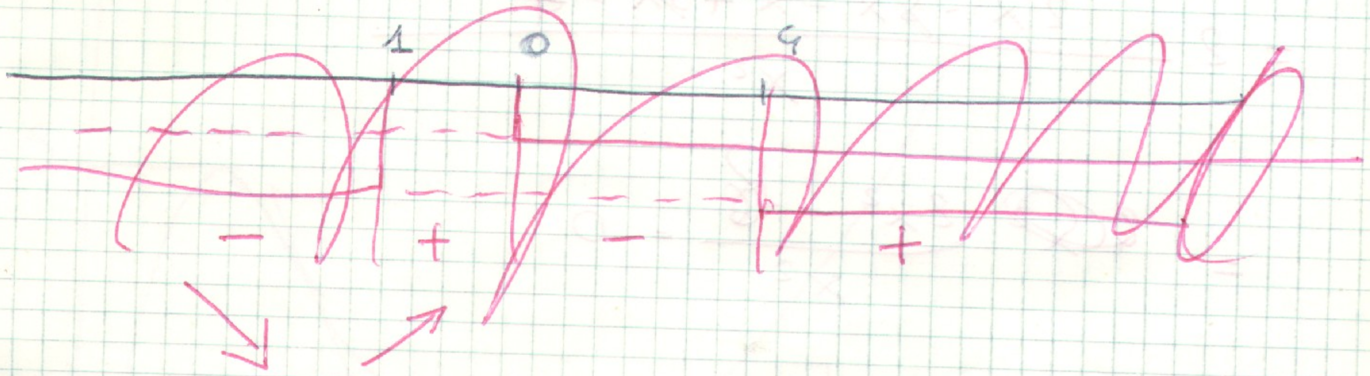
$$= \lim_{x\to +\infty} x^2\left(1-\frac{10}{x}+\frac{8\log x}{x^2}\right) = \infty\left(1-0+0\right) = \infty$$

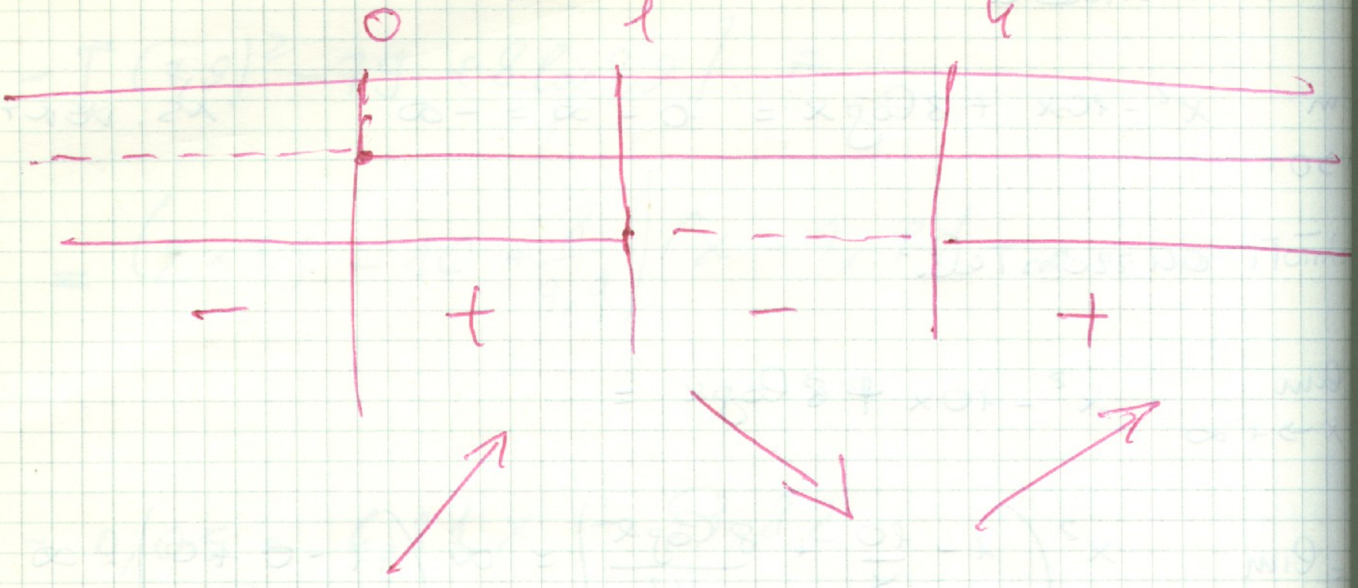
As. obliquo

$$m = \lim_{x\to +\infty} \frac{x^2\left(1-\frac{10}{x}+8\,\frac{\log x}{x^2}\right)}{\cancel{x}} = +\infty$$

MAX E MIN.

$$f'(x) = 2x-10+\frac{8}{x} = \frac{2}{x}\left(x^2-5x+4\right) = 0 \Rightarrow \begin{matrix} x=4 \\ x=1 \end{matrix}$$





x=1 massimo

x=4 max

flessi

$$f''(x)=2\,\frac{(2x-5)\,x-(x^2-5x+4)}{x^2}=$$

$$=2\,\frac{2x^2-\cancel{5x}-x^2+\cancel{5x}-4}{x^2}$$

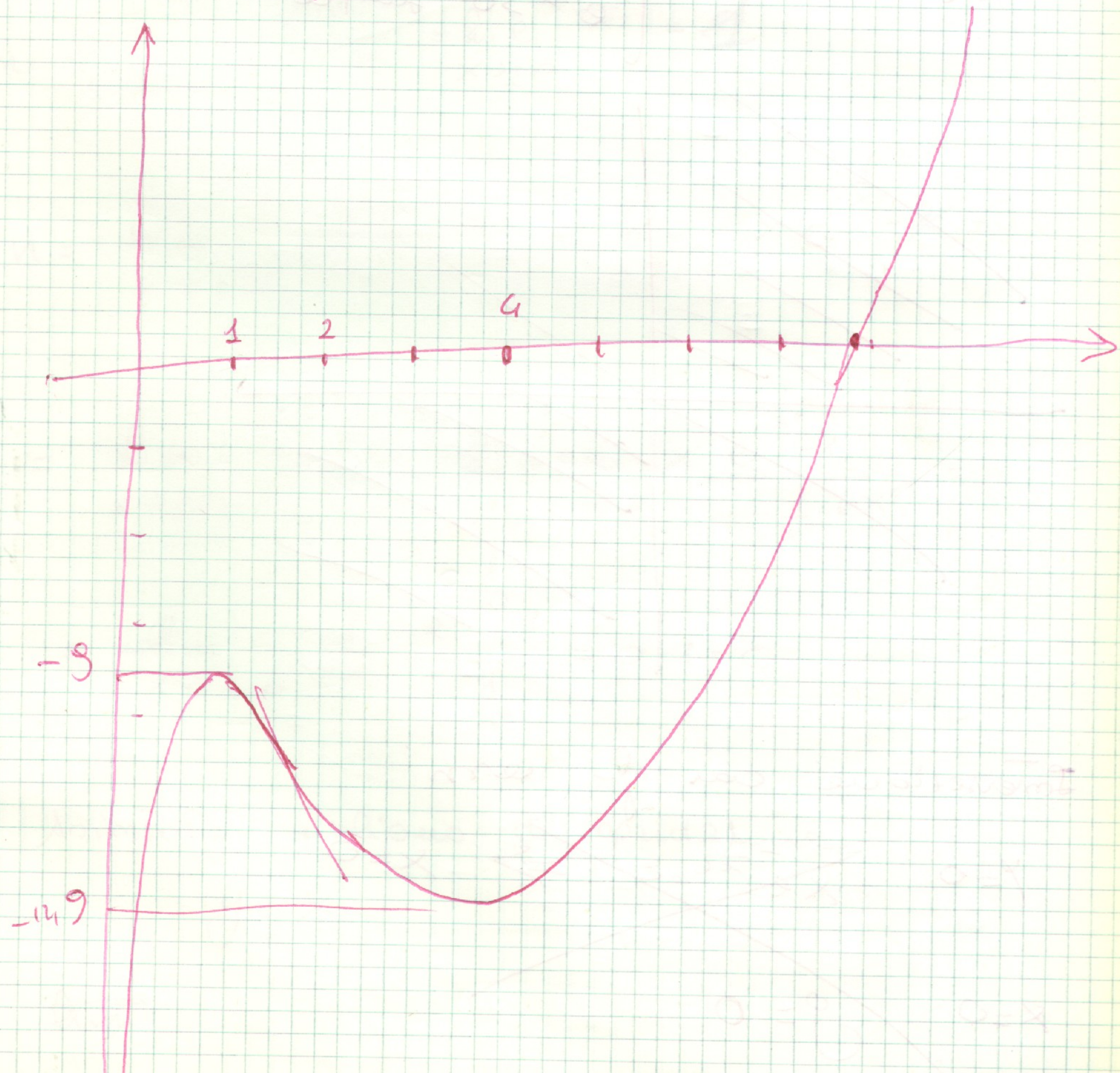
$$=\frac{2x^2-8}{x^2}=0$$

$$x=\pm 2\sqrt{2}$$

↑ $f(1) = 1 - 10 + 0 = -9$

$f(4) = -12,9$

Il diagramma attraversa allora l'asse x solo in un punto che abbiamo calcolato prima con approssimazione.

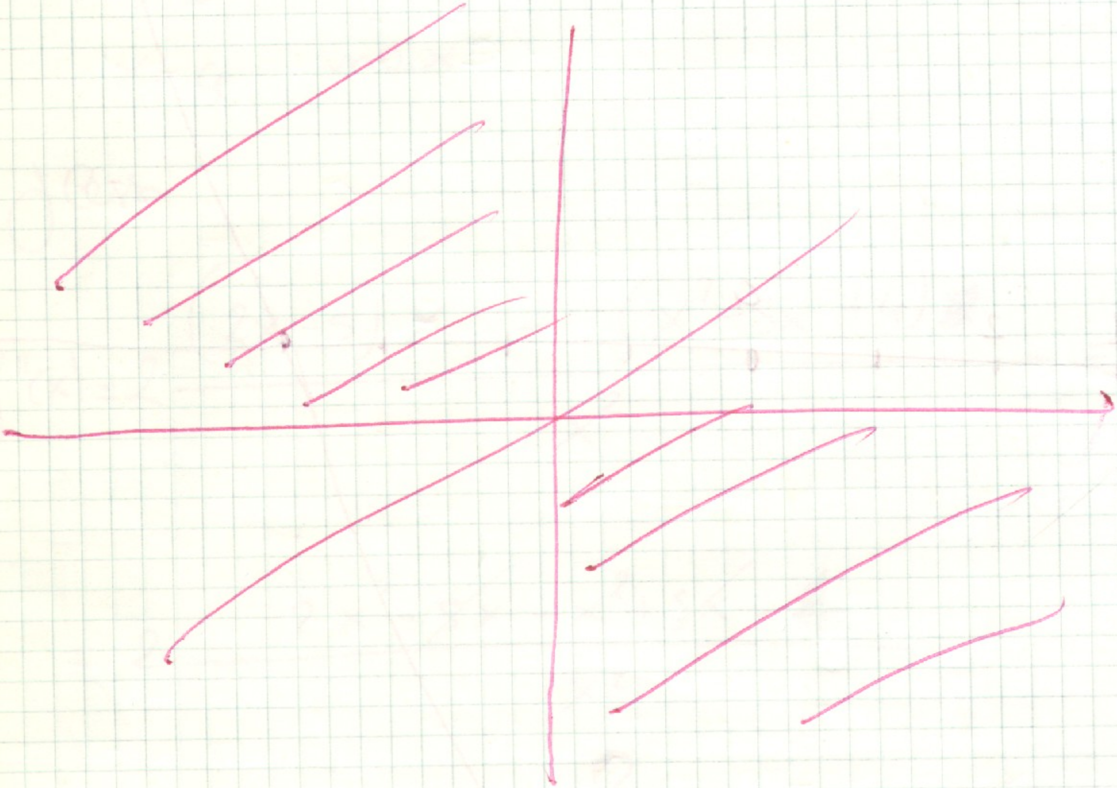


$f(x)=x e^{-\frac{1}{x^2}}$

CE : $\mathbb{R}-\{0\}$

Positività

$x e^{-\frac{1}{x^2}} \geq 0 \Rightarrow \begin{cases} x \geq 0 \\ e^{-\frac{1}{x^2}} > 0 \text{ sempre} \end{cases}$



Intersezione con gli assi

$y=0$ ~~$x e^{-\frac{1}{x^2}}=0$~~ $\Rightarrow x=0$

$x=0$ $y=0$

passa per l'origine $\lim_{x\to 0} xe^{-\frac{1}{x^2}} = 0\cdot 1 = 0$

Non vi sono asintoti verticali

$$\lim_{x\to+\infty} xe^{-\frac{1}{x^2}} = +\infty$$

$$\lim_{x\to-\infty} xe^{-\frac{1}{x^2}} = -\infty$$

$$m = \lim_{x\to\pm\infty} \frac{xe^{-\frac{1}{x^2}}}{x} = \lim_{x\to\pm\infty} e^{-\frac{1}{x^2}} = 1$$

$$n = \lim_{x\to\pm\infty} f(x) - mx = \lim_{x\to\pm\infty} xe^{-\frac{1}{x^2}} - x =$$

$$= \lim_{x\to\pm\infty} x(e^{-\frac{1}{x^2}} - 1) = \lim_{x\to\pm\infty} x\frac{e^{-\frac{1}{x^2}} - 1}{-\frac{1}{x^2}(-x^2)} =$$

$$= -\lim_{x\to\pm\infty} \frac{1}{-x}\frac{e^{-\frac{1}{x^2}} - 1}{-\frac{1}{x^2}} = 0\cdot 1 = 0$$

Asintoto obliquo di equazione

$$y = x$$

~~lim~~

~~$D x e^{-\frac{1}{x^2}} = e^{-\frac{1}{x^2}} + 2 e^{-\frac{1}{x^2}} D(-x^{-2}) =$~~

~~$= e^{-\frac{1}{x^2}} + e^{-\frac{1}{x^2}} \cdot 2x^{-3} = e^{-\frac{1}{x^2}}(1 + 2 e^{-\frac{1}{x^2}}$~~

~~$= e^{-\frac{1}{x^2}} + 2\frac{e^{-\frac{1}{x^2}}}{x^3}$~~

~~$f''(x) = D e^{-\frac{1}{x^2}} + 2(D e^{-\frac{1}{x^2}}) 2x^{-3} + 2 e^{-\frac{1}{x^2}} D(x^{-3}) =$~~

- $D x e^{-\frac{1}{x^2}} = e^{-\frac{1}{x^2}} + x e^{-\frac{1}{x^2}} D(-x^{-2}) =$

$= e^{-\frac{1}{x^2}} + 2 \cancel{x} e^{-\frac{1}{x^2}} \frac{1}{x^{\cancel{3}2}} = e^{-\frac{1}{x^2}} \left(\frac{x^2+2}{x^2}\right) \geq 0 \ \forall x$

la funzione è sempre crescente

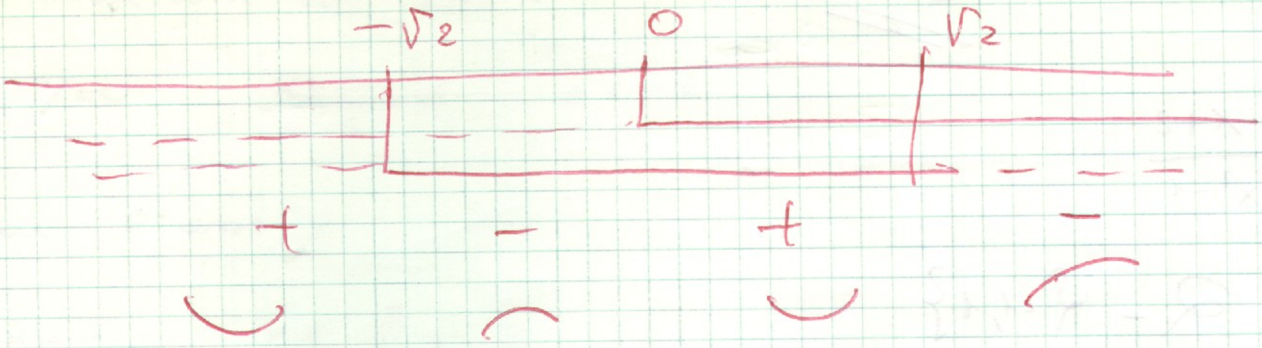
$D e^{-\frac{1}{x^2}}(1 + 2x^{-2}) =$

$= e^{-\frac{1}{x^2}}(2x^{-3})(1 + 2x^{-2}) + e^{-\frac{1}{x^2}}(-4x^{-3}) =$

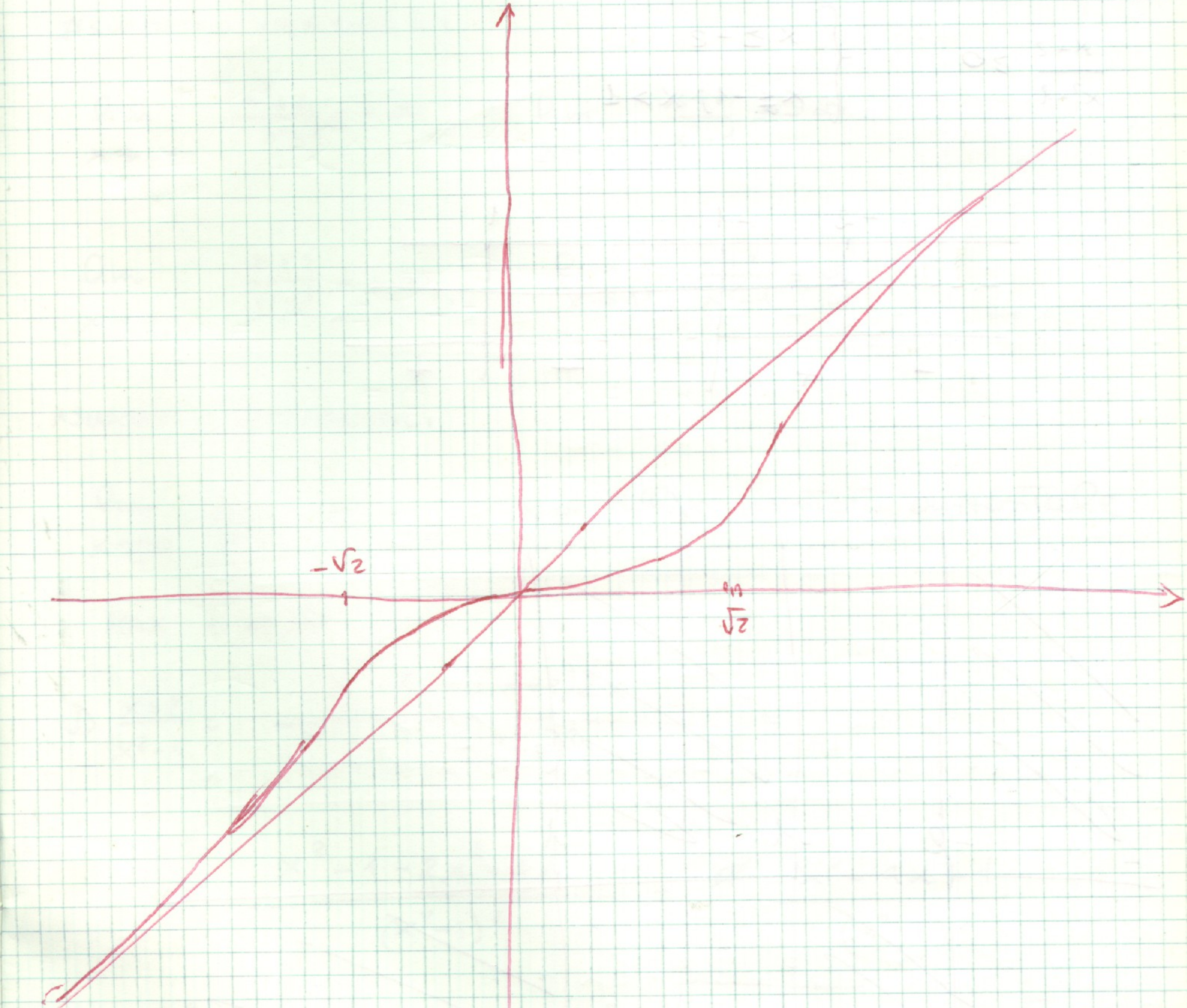
$= e^{-\frac{1}{x^2}}\left(\frac{2}{x^3} + \frac{4}{x^5} - \frac{4}{x^3}\right) = e^{-\frac{1}{x^2}}\left(\frac{4}{x^5} - \frac{2}{x^3}\right) =$

$= e^{-\frac{1}{x^2}} \frac{4 - 2x^2}{x^5}$

$4 - 2x^2 = 0$ $x^2 = 2$ $x = \pm\sqrt{2}$ punti di fle



−√2
0
√2
+
−
+
−



−√2
√2

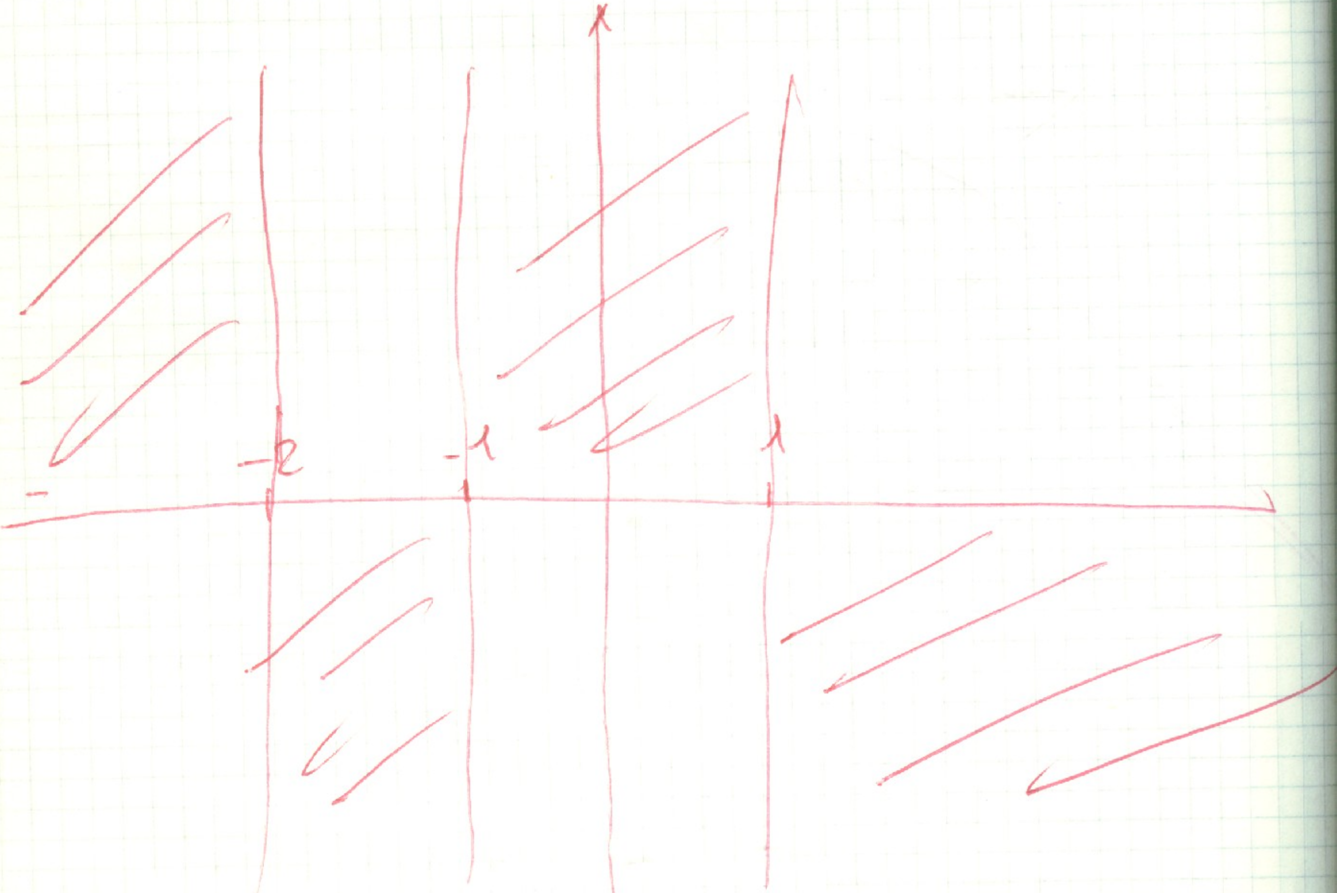
10

$$y = \frac{x+2}{x^2-1}$$

CE: $\mathbb{R} - \{-1, 1\}$

Positività

$$\frac{x+2}{x^2-1} \geq 0 \qquad \begin{cases} x \geq -2 \\ x \leq -1 \,;\; x > 1 \end{cases}$$



Intersezione con asse x

$y=0$ $\frac{x+2}{x^2-1}=0$ $x=-2$

con l'asse y

$x=0$ $y=\frac{2}{-1}=-2$

ASINTOTI VERTICALI

$\lim_{x\to -1^-}\frac{x^2+2}{x^2-1}=+\infty$ $\lim_{x\to -1^+}\frac{x+2}{x^2-1}=-\infty$

$\lim_{x\to 1^-}\frac{x+2}{x^2-1}=-\infty$ $\lim_{x\to 1^+}\frac{x+2}{x^2-1}=\infty$

ASINTOTI ORIZZONTALI

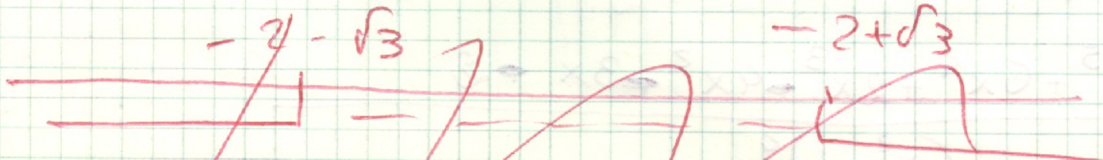
$\lim_{x\to\pm\infty}\frac{x^2+2}{x^2-1}=0$ asse delle x asintoto orizzontale

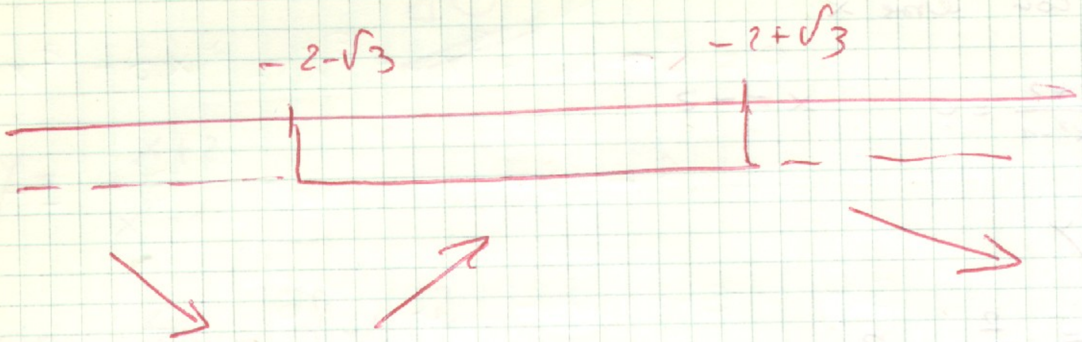
MAX E MIN

$$D\,\frac{x+2}{x^2-1}=\frac{x^2-1-(x+2)2x}{(x^2-1)^2}=$$

$$=\frac{x^2-1-2x^2-4x}{(x^2-1)^2}=\frac{\cancel{x^2}-x^2-4x-1}{(x^2-1)^2}$$

$x^2+4x+1=0$ $x=\frac{-2\pm\sqrt{4-1}}{\cancel{1}}=-2\pm\sqrt{3}$





Concavità e convessità

$$-D\ \frac{x^2+4x+1}{(x^2-1)^2} =$$

$$= -\frac{(2x+4)(x^2-1)^2-(x^2+4x+1)2(x^2-1)2x}{(x^2-1)^4} =$$

$$= -\frac{(2x+4)(x^4+1-2x^2)-(x^2+4x+1)(4x^3-4x)}{(x^2-1)^4} =$$

$$= -\frac{\underset{\sim}{2x^5}+\underline{2x}\overset{/}{-4x^3}\overset{\sim}{+4x^4}+4\overset{o}{-8x^2}-\underline{4x^5}\overset{\sim}{-16x^4}\overset{/}{-4x^3}\overset{/}{+4x^3}\overset{o}{+16x^2}+\underline{4x}}{(x^2-1)^4} =$$

$$= -\frac{-2x^5-12x^4-4x^3+8x^2+6x+4}{(x^2-1)^4}$$

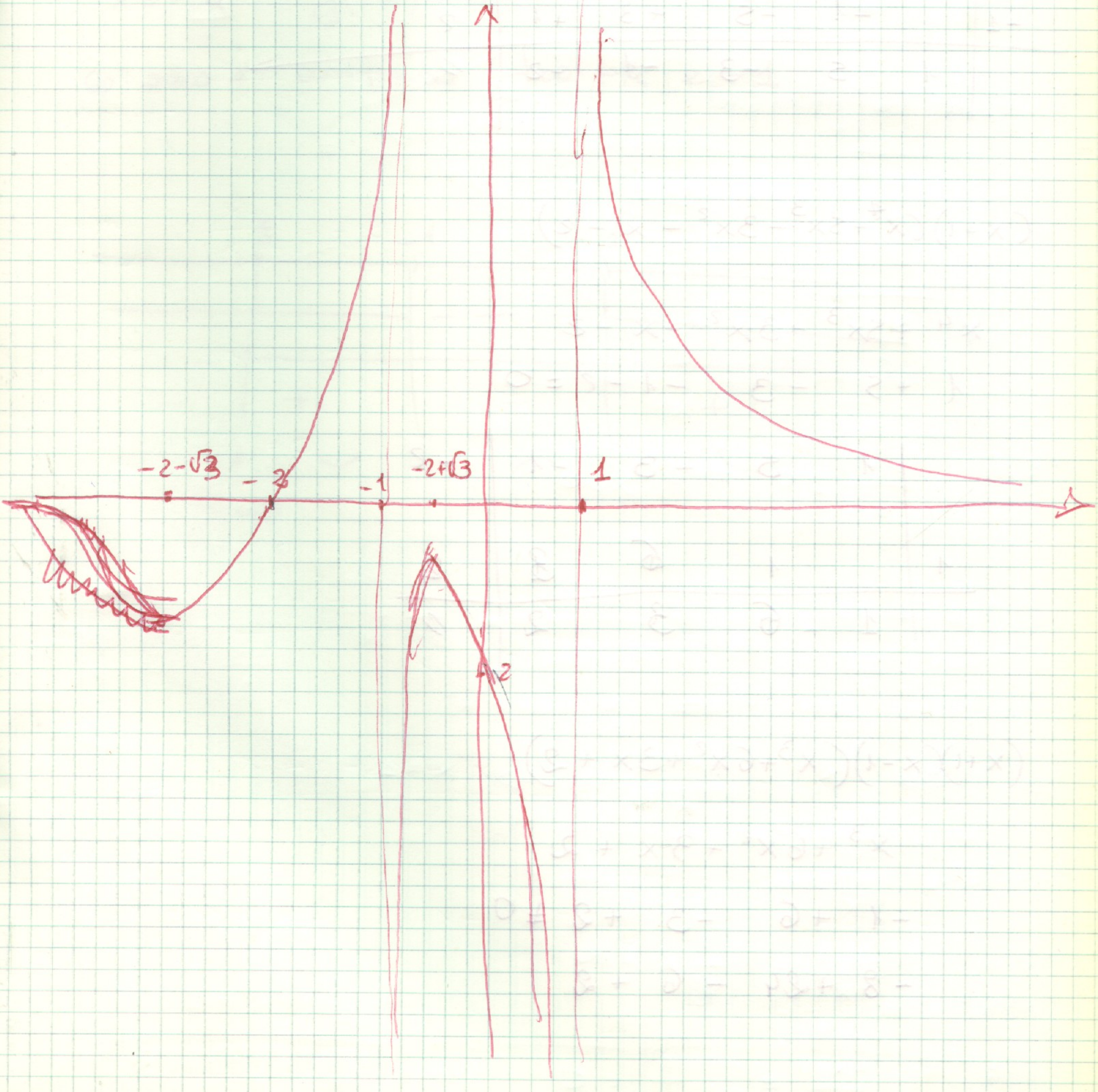
$$= 2\ \frac{x^5+6x^4+2x^3-4x^2-3x-2}{}$$

$x^5 + 6x^4 + 2x^3 - 4x^2 + 3x + 2$

$+1 \quad +6 \quad -2 \quad -4 \quad +3 \quad +2$

$-32 \quad +96 \quad -16 \quad -16 \quad -6 \quad +2 \neq 0$

non vi sono radici sempre positivo i sempre convesso



$x^5 + 6x^4 + 2x^3 - 4x^2 - 3x - 2 = 0$

$-1 + 6 - 2 - 4 + 3 - 2 = 0$

	1	6	2	-4	-3	-2
-1		-1	-5	+3	+1	2
	1	5	-3	-1	-2	//

$(x+1)(x^4 + 5x^3 - 3x^2 - x - 2)$

$x^4 + 5x^3 - 3x^2 - x - 2$

$1 + 5 - 3 - 1 - 2 = 0$

	1	5	-3	-1	-2
1		1	6	3	2
	1	6	3	2	//

$(x+1)(x-1)(x^3 + 6x^2 + 3x + 2)$

$x^3 + 6x^2 + 3x + 2$

$-1 + 6 - 3 + 2 \neq 0$

$-8 + 24 - 6 + 2$

x^3+6x^2+3x+2

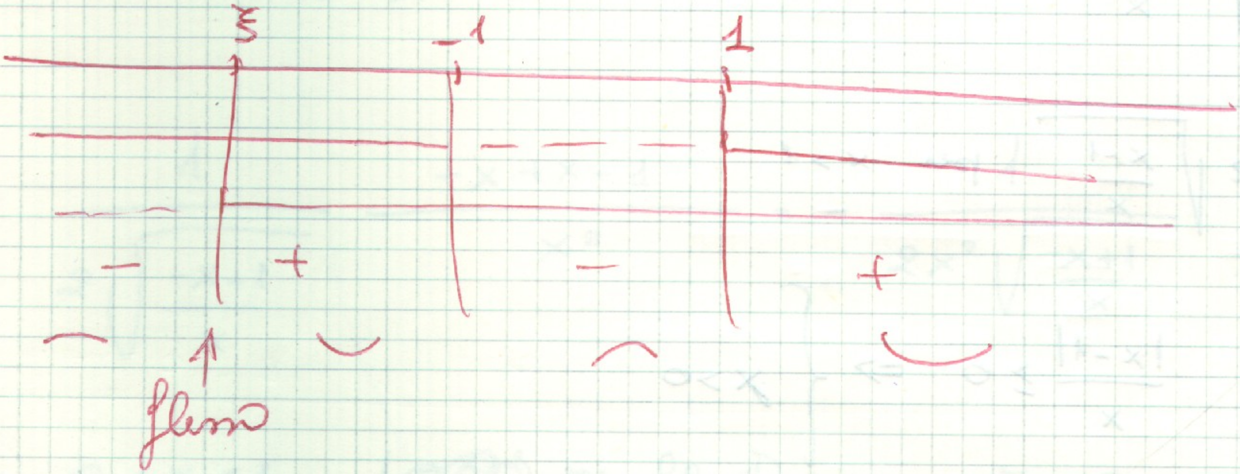
$x=-1 \Rightarrow -1+6-3+2>0$

$x=-\infty \qquad \lim_{x\to-\infty} x^3\left(1+\frac{6}{x}+\frac{3}{x^2}+\frac{2}{x^2}\right)=-\infty$

c'è un'altro zero dopo $x<-1$

$x=-4 \qquad f(-4)<0$

lo zero è compreso fra -4 e -1



$$\sqrt{\frac{|x-1|}{x}}$$

$$|x-1| = \begin{cases} -x+1 & \text{per } x-1<0 \text{ cioè } x<1 \\ x-1 & \text{per } x-1>0 \Rightarrow x>1 \end{cases}$$

Studiamo la funzione

$$\sqrt{\frac{-x+1}{x}} \quad \text{per} \quad x<1$$

$$\text{e} \sqrt{\frac{x-1}{x}} \quad \text{per} \quad x>1$$

C E: $\frac{|x-1|}{x} \geq 0 \Rightarrow \left\{ x>0 \right.$

$] 0, +\infty [$

Intersezione con l'asse y non c'è

Con l'asse x . $\frac{|x-1|}{x} = 0 \Rightarrow x = 1$

Asintoti verticali

$$\lim_{x \to 0^+} \sqrt{\frac{-x+1}{x}} = \sqrt{\frac{1}{0^+}} = +\infty$$

$$\lim \sqrt{\frac{-x+1}{}} = \sqrt{\frac{0}{}} = 0$$

$$\lim_{x \to 1^+} \sqrt{\frac{x-1}{x}} = \sqrt{\frac{1-1}{1}} = 0$$

l Asimt. orizzontali

$$\lim_{x \to +\infty} \sqrt{\frac{x-1}{x}} = 1$$

MAX e MIN

$$D\sqrt{\frac{-x+1}{x}} = \frac{1}{2\sqrt{\frac{-x+1}{x}}} \; \frac{-1 \cdot x - (-x+1)\cancel{x}}{x^2} =$$

$$= \frac{1}{2\sqrt{\frac{-x+1}{x}}} \; \frac{-x+x-1}{x^2} = -\frac{1}{2x^2\sqrt{\frac{-x+1}{x}}}$$

per $0<x<1$ ~~$\left(\frac{-x+1}{x}\right) \geq 0$~~ la D prima è <0

quindi la funzione è decrescente

$$D\left[-\frac{1}{2x^2\sqrt{\frac{-x+1}{l}}}\right] = -\frac{1}{2}\left[-\frac{2}{x^3\sqrt{\frac{-x+1}{x}}} + \frac{1}{2}\,\frac{1}{2x^2\sqrt{\left(\frac{-x+1}{x}\right)^3}}\left(-\frac{1}{x^2}\right)\right] =$$

$$= \frac{1}{x^3\sqrt{\frac{-x+1}{x}}} - \frac{1}{4x^4\sqrt{\left(\frac{-x+1}{x}\right)^3}} = \frac{4\cancel{x}\,\frac{-x+1}{\cancel{x}} - 1}{4x^4\sqrt{\left(\frac{-x+1}{x}\right)^3}} =$$

$$-4x + 4 - 1$$

$x \to 1^+$

e Asint. orizzontali

$$\lim_{x \to +\infty} \sqrt{\frac{x-1}{x}} = 1$$

MAX e MIN

$$D\sqrt{\frac{-x+1}{x}} = \frac{1}{2\sqrt{\frac{-x+1}{x}}} \cdot \frac{-1 \cdot x - (-x+1)\cancel{x}}{x^2} =$$

$$= \frac{1}{2\sqrt{\frac{-x+1}{x}}} \cdot \frac{-x+x-1}{x^2} = -\frac{1}{2x^2\sqrt{\frac{-x+1}{x}}}$$

per $0 < x < 1$ la D prima è <0

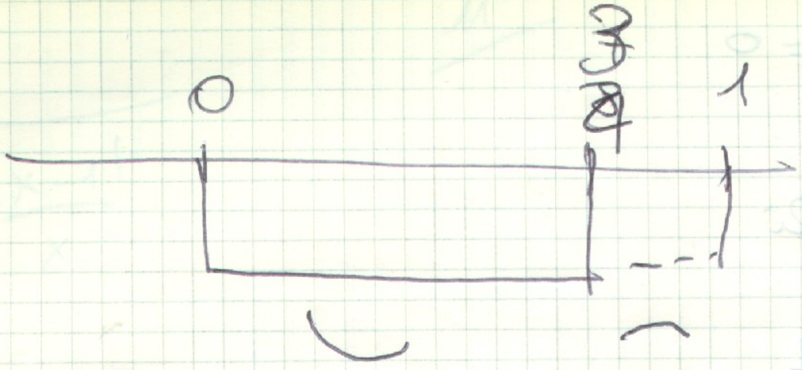
quindi la funzione è decrescente

$$D\left[-\frac{1}{2x^2\sqrt{\frac{-x+1}{x}}}\right] = -\frac{1}{2}\left[-\frac{2}{x^3\sqrt{\frac{-x+1}{x}}} + \frac{1}{2}\frac{1}{2x^2\sqrt{\left(\frac{-x+1}{x}\right)^3}}\left(-\frac{1}{x^2}\right)\right] =$$

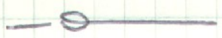
$$= \frac{1}{x^3\sqrt{\frac{-x+1}{x}}} - \frac{1}{4x^4\sqrt{\left(\frac{-x+1}{x}\right)^3}} = \frac{4\cancel{x}\frac{-x+1}{\cancel{x}} - 1}{4x^4\sqrt{\left(\frac{-x+1}{x}\right)^3}} =$$

$$= \frac{-4x+4-1}{4x^4\sqrt{\left(\frac{-x+1}{x}\right)^3}}$$

$$-4x+3=0 \Rightarrow x = \frac{3}{4}$$



$$-4x+3 \geq 0 \Rightarrow 4x-3 \leq 0 \Rightarrow x \leq \frac{3}{4}$$



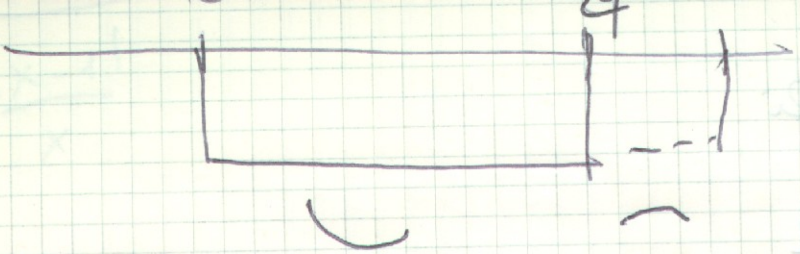
$$D\sqrt{\frac{x-1}{x}} = \frac{1}{2\sqrt{\frac{x-1}{x}}} \cdot \frac{x-(x-1)}{x^2} =$$

$$= \frac{1}{2\sqrt{\frac{x-1}{x}}} \quad \frac{1}{x^2}$$

$$D\frac{1}{2x^2\sqrt{\frac{x-1}{x}}} = \frac{1}{2}\Bigg[-\frac{2}{x^3\sqrt{\frac{x-1}{x}}} + \frac{1}{x^2}\cdot\frac{1}{2}\,\frac{1}{\sqrt{\left(\frac{x-1}{x}\right)^3}}\,\frac{x-x}{x^2}$$

$$= -\frac{1}{x^3\sqrt{\frac{x-1}{x}}} - \frac{1}{2x^4\sqrt{\left(\frac{x-1}{x}\right)^3}} =$$

$$= -\frac{2x\,\frac{x-1}{x} + 1}{2x^4\sqrt{\left(\frac{x-1}{x}\right)^3}} = -\frac{2x-2+1}{2x^4\sqrt{\left(\frac{x-1}{x}\right)^3}}$$



$$-4x+3 \geq 0 \Rightarrow 4x-3 \leq 0 \Rightarrow x \leq \frac{3}{4}$$

$$D\sqrt{\frac{x-1}{x}} = \frac{1}{2\sqrt{\frac{x-1}{x}}} \cdot \frac{x-(x-1)}{x^2} =$$

$$= \frac{1}{2\sqrt{\frac{x-1}{x}}} \frac{1}{x^2}$$

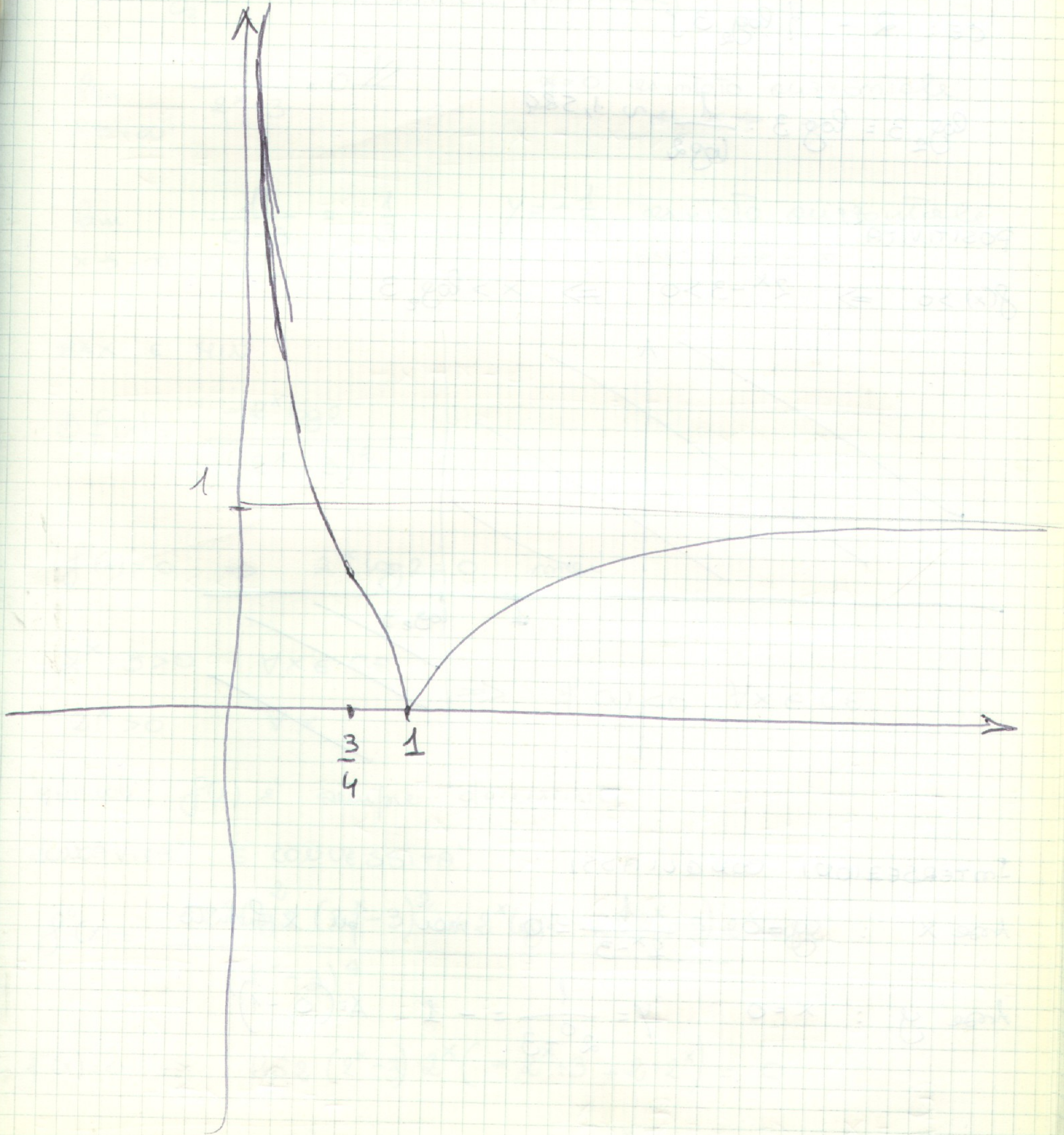
$$D\frac{1}{2x^2\sqrt{\frac{x-1}{x}}} = \frac{1}{2}\left[-\frac{2}{x^3\sqrt{\frac{x-1}{x}}} + \frac{1}{x^2} \cdot \frac{1}{2} \frac{1}{\sqrt{\left(\frac{x-1}{x}\right)^3}} \frac{x-x}{x^2}\right.$$

$$= -\frac{1}{x^3\sqrt{\frac{x-1}{x}}} - \frac{1}{2x^4\sqrt{\left(\frac{x-1}{x}\right)^3}} =$$

$$= -\frac{2x\frac{x-1}{x} + 1}{2x^4\sqrt{\left(\frac{x-1}{x}\right)^3}} = -\frac{2x-2+1}{2x^4\sqrt{\left(\frac{x-1}{x}\right)^3}}$$

$$= - \frac{2x-1}{2x^4 \sqrt{\left(\frac{x-1}{x}\right)^3}} \qquad 2x-1 \geq 0 \Rightarrow x \geq \frac{1}{2}$$

quindi per $x > 1$ la D seconda è sempre negativa



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$$y = \frac{1}{2^x - 3}$$

CAMPO DI ESISTENZA

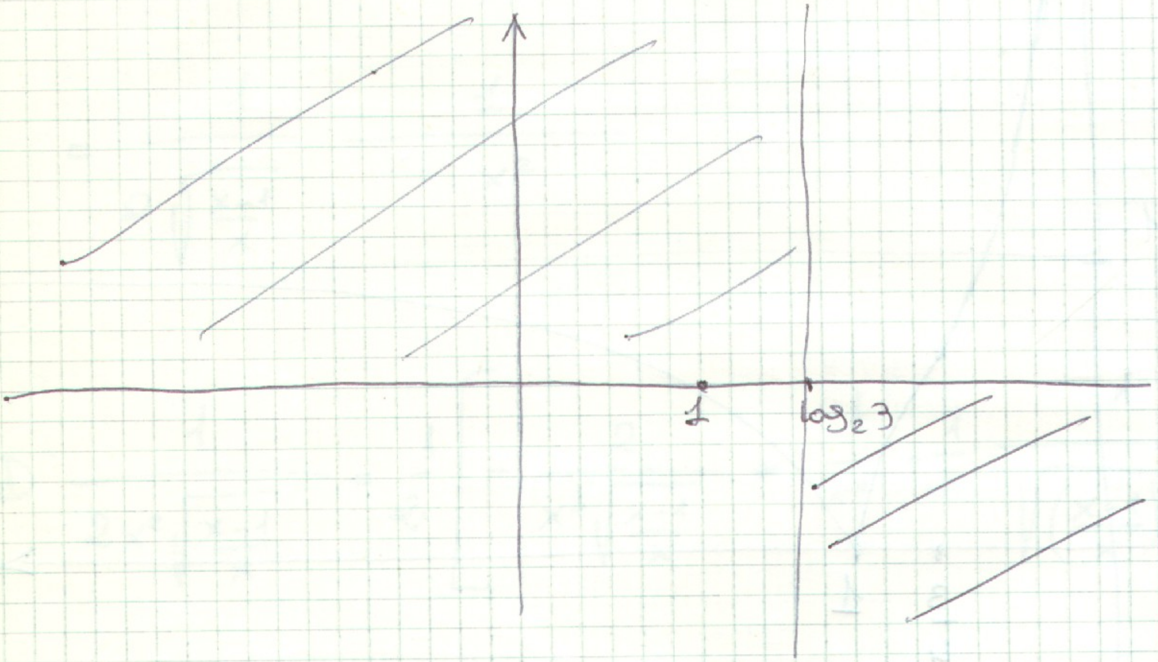
$2^x - 3 \neq 0$; $2^x \neq 3$; $x = \log_2 3$ ~~x~~ [illegible]

CE: $\mathbb{R} - \{\log_2 3\}$

$\log_2 3 = \log 3 \cdot \frac{1}{\log 2} \sim 1,584$

POSITIVITÀ

$f(x) > 0 \Rightarrow 2^x - 3 > 0 \Rightarrow x > \log_2 3$



INTERSEZIONI CON GLI ASSI

Asse x : $y = 0$ $\quad \frac{1}{2^x - 3} = 0$ mai per x finito

Asse y : $x = 0$ $\quad y = \frac{1}{2^0 - 3} = -1$ $\quad A = (0, -1)$

ASINTOTI

$$\lim_{x \to \log_2 3^-} \frac{1}{2^x - 3} = -\infty$$

$$\lim_{x \to \log_2 3^+} \frac{1}{2^x-3} = +\infty$$

$x = \log_2 3$ è un asintoto verticale

$$\lim_{x \to +\infty} \frac{1}{2^x-3} = 0 \cancel{\frac{1}{\ }}$$

$y = 0$ asintoto orizzontale per $x \to +\infty$

$$\lim_{x \to -\infty} \frac{1}{0-3} = -\frac{1}{3}$$

$y = -\frac{1}{3}$ asintoto orizzontale per $x \to -\infty$

MAX E MIN

$$D f(x) = \frac{-2^x \log 2}{(2^x-3)^2}$$

$f'(x) = 0 \Rightarrow 2^x \log 2 = 0$ mai

$2^x - 3 > 0 \quad \forall x \in CE$

$2^x > 0 \quad \forall x$

$\Rightarrow f'(x) < 0 \quad \forall x \in CE$

quindi $f(x)$ è sempre decrescente

CONCAVITA' E CONVESSITA'

$$f''(x) = \frac{-2^x \log^2 2\,(2^x-3)^2 + 2^x \log 2 \cdot 2(2^x-3)\,2^x \log 2}{(2^x-3)^4}$$

$$f''(x) = 0 \Rightarrow \log^2 2\,(2^x-3) \cdot 2^x\left(-2^x+3+2\cdot 2^x\right) = 0$$

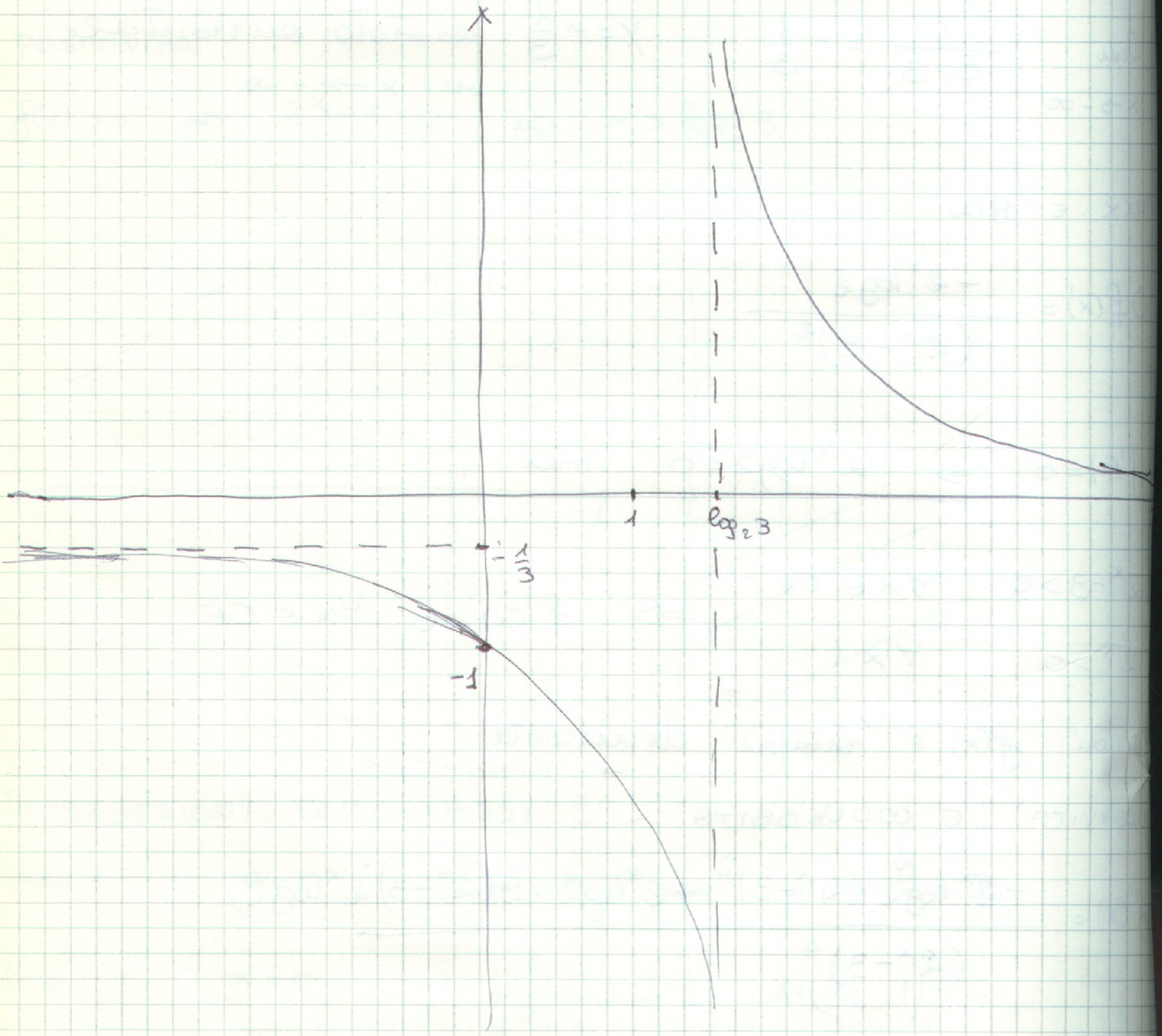
$2^x - 3 = 0 \Rightarrow x = \log_2 3 \notin CE$ quindi non è un flesso

$2^x + 3 = 0$ mai

non ci sono flessi

$$f''(x) > 0 \Rightarrow \begin{cases} 2^x - 3 > 0 \Rightarrow x > \log_2 3 \\ 3 + 2^x > 0 \end{cases}$$

la funzione è concava per $x < \log_2 3$ e convessa dop



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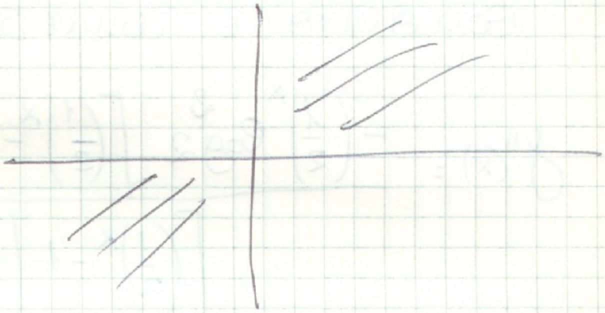
$$\frac{1}{\left(\frac{1}{2}\right)^x - 1}$$

CE: $\left(\frac{1}{2}\right)^x - 1 \neq 0$ $\quad \left(\frac{1}{2}\right)^x \neq 1$ $\quad ; x \neq \log_{\frac{1}{2}} 1 = \cancel{\log_e} 0$

CE : $\mathbb{R} - \{0\}$



Positività

$\left(\frac{1}{2}\right)^x - 1 > 0$ $\qquad x < 0$

Intersezioni con gli assi

asse y (x=0) non ~~fa~~ appartiene al campo d'esistenza

asse x (y=0) $\frac{1}{\left(\frac{1}{2}\right)^x - 1} = 0$ mai

Asintoti

$$\lim_{x \to 0^-} \frac{1}{\left(\frac{1}{2}\right)^x - 1} = +\infty$$ x=0 asintoto verticale

$$\lim_{x \to 0^+} \frac{1}{\left(\frac{1}{2}\right)^x - 1} = -\infty$$

$$\lim_{x \to -\infty} \frac{1}{\left(\frac{1}{2}\right)^x - 1} = 0$$ y=0 asintoto orizzontale per $x \to -\infty$

$$\lim_{x \to +\infty} \frac{1}{\left(\frac{1}{2}\right)^x - 1} = -1$$ y=-1 asintoto orizzontale per $x \to +\infty$

MAX E MIN

$$f'(x) = \frac{-\left(\frac{1}{2}\right)^x \log\frac{1}{2}}{\left[\left(\frac{1}{2}\right)^x - 1\right]^2} = \frac{\left(\frac{1}{2}\right)^x \log 2}{\left[\left(\frac{1}{2}\right)^x - 1\right]^2} > 0 \quad \forall x \in C$$

la funzione è sempre crescente

CONCAVITA' E CONVESSITA'

$$f''(x) = \frac{-\left(\frac{1}{2}\right)^x \log^2 2 \left[\left(\frac{1}{2}\right)^x - 1\right]^2 + \left(\frac{1}{2}\right)^x \log 2 \cdot 2\left[\left(\frac{1}{2}\right)^x - 1\right]\left(\frac{1}{2}\right)^x \log 2}{\left[\left(\frac{1}{2}\right)^x - 1\right]^4} =$$

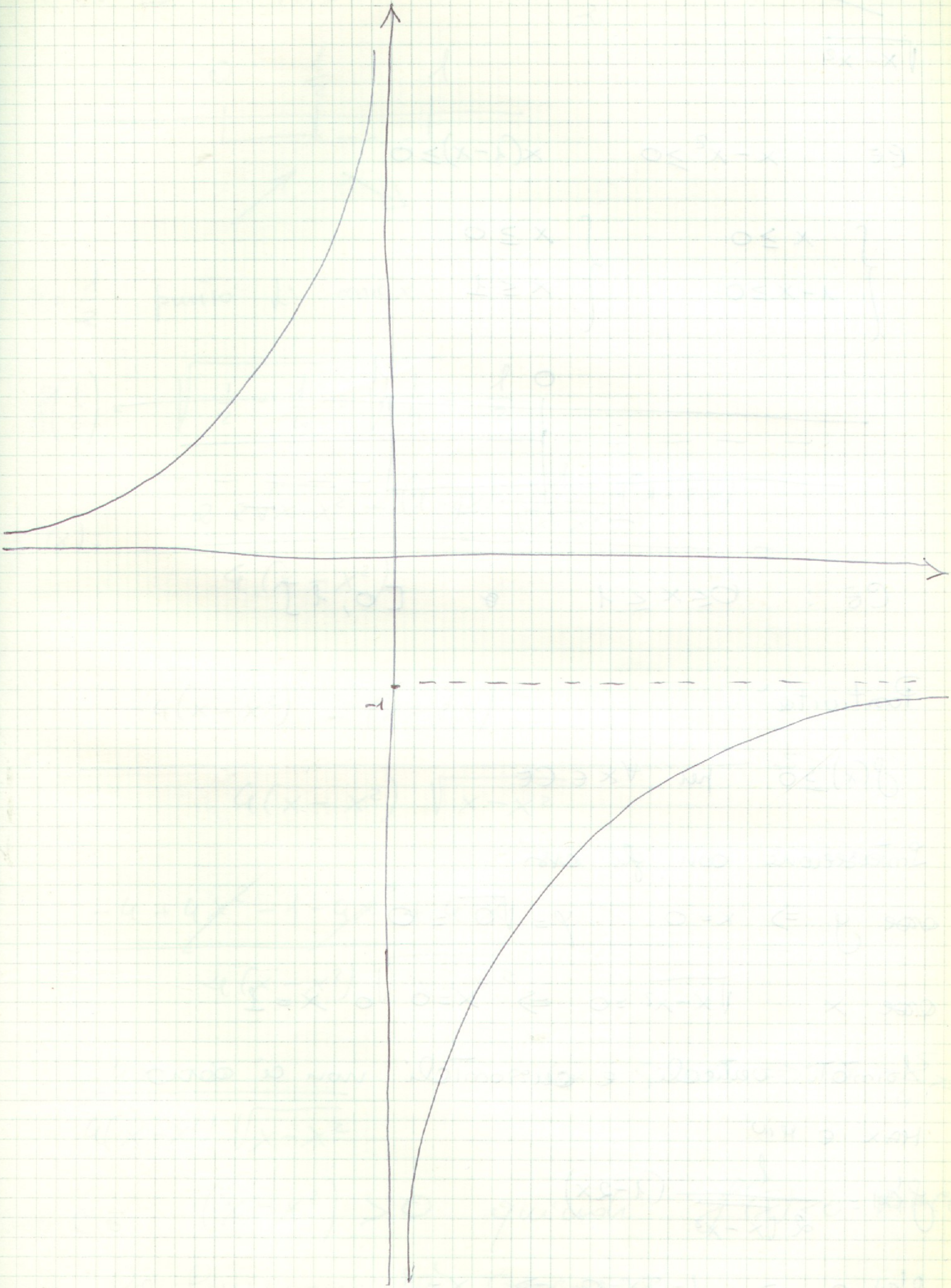
$$\Rightarrow \left(\frac{1}{2}\right)^x \log^2 2\left[\left(\frac{1}{2}\right)^x - 1\right]\left[-\left(\frac{1}{2}\right)^x + 1 + 2\left(\frac{1}{2}\right)^x\right] = 0$$

$$\Rightarrow \begin{cases} \left(\frac{1}{2}\right)^x - 1 = 0 & x = 0 \quad \text{che } \notin CE \Rightarrow \text{non ci sono fl} \\ \left(\frac{1}{2}\right)^x + 1 = 0 & \text{mai} \end{cases}$$

$$f''(x) > 0 \Rightarrow \cancel{x > 0} \left(\frac{1}{2}\right)^x - 1 > 0 \Rightarrow x < 0$$

per $x<0$ la $f(x)$ è convessa

per $x>0$ è concava

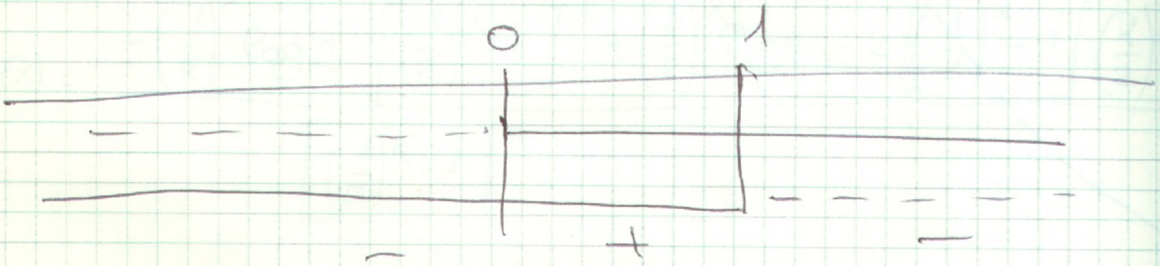

-1

14

$\sqrt{x-x^2}$

CE $\quad x-x^2\geq 0 \qquad x(1-x)\geq 0$

$$\begin{cases} x\geq 0 \\ 1-x\geq 0 \end{cases} \qquad \begin{cases} x\geq 0 \\ x\leq 1 \end{cases}$$



CE : $\quad 0\leq x\leq 1 \quad$ o $\quad [0,1]$

Positività

$f(x)\geq 0$ per $\forall x\in CE$

Intersezione con gli assi

asse y $\Rightarrow$ $x=0 \qquad y=\sqrt{0}=0$

asse x $\quad \sqrt{x-x^2}=0 \Rightarrow x=0$ o $x=1$

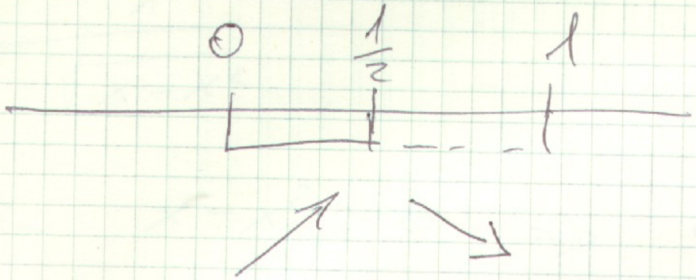
Asintoti verticali e orizzontali non ci sono

MAX E MIN

$$f'(x)=\frac{1}{2\sqrt{x-x^2}}(1-2x)$$

$f'(x)=0 \Rightarrow 1-2x=0 \Rightarrow x=\frac{1}{2}$

$f'(x) > 0 \Rightarrow 1-2x > 0 \Rightarrow x < \frac{1}{2}$



$x = \frac{1}{2}$ punto di max

$$f\left(\frac{1}{2}\right) = \sqrt{\frac{1}{2} - \frac{1}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$f''(x) = \frac{-2 \cdot 2\sqrt{x-x^2} - (1-2x)\frac{1}{\sqrt{x-x^2}}(1-2x)}{4(x-x^2)} =$$

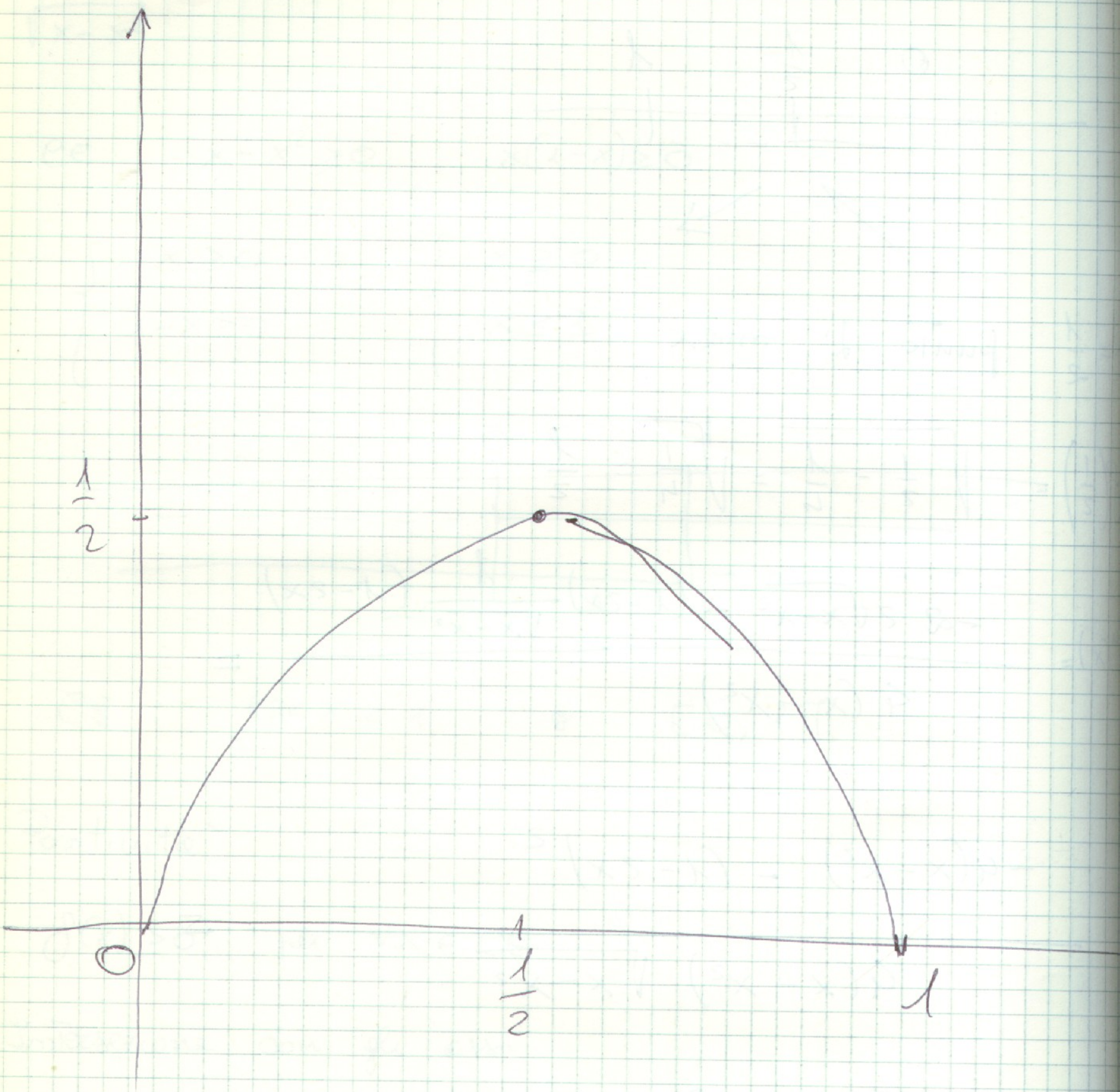
$$= \frac{-4(x-x^2) - (1-2x)^2}{4(x-x^2)\sqrt{x-x^2}} =$$

$$= \frac{-4 + \cancel{4x^2} - 1 - \cancel{4x^2} + 4x}{4(x-x^2)\sqrt{x-x^2}}$$

$$= \frac{4x-5}{4(x-x^2)\sqrt{x-x^2}} \qquad 4x-5 > 0 \Rightarrow x > \frac{5}{4} > 1$$

$x \in CE$ $(x-x^2) > 0$ quindi $f''(x) < 0$ $\forall x \in CE$

cioè la funzione è concava



1/2
0
1/2
1

15

$$\frac{1}{\sqrt{x-x^2}}$$

CE: $x - x^2 > 0 \Rightarrow 0 < x < 1$

$]0,1[$

Positività $f(x) > 0 \quad \forall x \in CE$

Intersezione con

asse y ~~$x=0$~~ non $\notin$ al CE

asse x $\frac{1}{\sqrt{x-x^2}} = 0$ mai

~~[illegible]~~ Asintoti

$$\lim_{x\to 0^+} \frac{1}{\sqrt{x-x^2}} = +\infty$$

$$\lim_{x\to 1^-} \frac{1}{\sqrt{x-x^2}} = +\infty$$

MAX e MIN

$$f'(x) = \frac{-\frac{1}{2\sqrt{x-x^2}}(1-2x)}{x-x^2} = -\frac{1-2x}{2(x-x^2)\sqrt{x-x^2}}$$

$f'(x) = 0 \Rightarrow 1-2x = 0 \Rightarrow x = \frac{1}{2}$

in CE $x - x^2 > 0 \quad -1+2x > 0 \Rightarrow x > \frac{1}{}$

$$\frac{1}{\sqrt{x-x^2}}$$

CE: $x-x^2>0 \Rightarrow 0<x<1$

$]0,1[$

Positività $f(x)>0 \quad \forall x \in CE$

Intersezione con

asse y $x=0$ non $\notin$ al CE

asse x $\frac{1}{\sqrt{x-x^2}}=0$ mai

Asintoti

$$\lim_{x\to 0^+} \frac{1}{\sqrt{x-x^2}} = +\infty$$

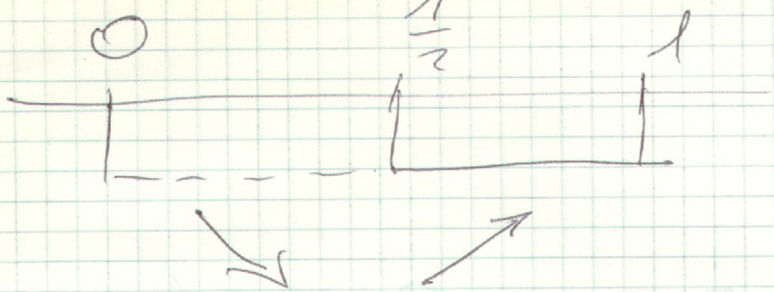
$$\lim_{x\to 1^-} \frac{1}{\sqrt{x-x^2}} = +\infty$$

MAX e MIN

$$f'(x)= \frac{-\frac{1}{2\sqrt{x-x^2}}(1-2x)}{x-x^2} = -\frac{1-2x}{2(x-x^2)\sqrt{x-x^2}}$$

$f'(x)=0 \Rightarrow 1-2x=0 \Rightarrow x=\frac{1}{2}$

in CE $x-x^2>0$ $\quad -1+2x>0 \Rightarrow x>\frac{1}{2}$



$x = \frac{1}{2}$ punto d' minimo

$$f\left(\frac{1}{2}\right) = \frac{1}{\sqrt{\frac{1}{2} - \frac{1}{4}}} = \frac{1}{\sqrt{\frac{1}{4}}} = 2$$

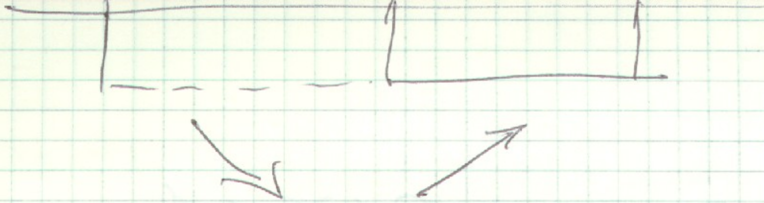
Concavitá e convessitá

$$f''(x) = \frac{2 \cdot 2(x-x^2)\sqrt{x-x^2} - (2x-1) \cdot 2\left[(1-2x)\sqrt{x-x^2} + \frac{x-x^2}{2\sqrt{x-x^2}}(1\right.}{4(x-x^2)^2(x-x^2)}$$

$$= \frac{8(x-x^2)^2 - 2(2x-1)\left[2(1-2x)(x-x^2) + (x-x^2)(1-2x)\right]}{8(x-x^2)^3\sqrt{x-x^2}}$$

$$= \frac{8(x-x^2)^2 - 2(2x-1) \cdot 3(1-2x)(x-x^2)}{8(x-x^2)^3\sqrt{x-x^2}} =$$

$4(x-x^2) +$ [illegible] $+3(1-2x)^2$



$x = \frac{1}{2}$ punto di minimo

$$f\left(\frac{1}{2}\right) = \frac{1}{\sqrt{\frac{1}{2} - \frac{1}{4}}} = \frac{1}{\sqrt{\frac{1}{4}}} = 2$$

Concavità e convessità

$$f'(x) = \frac{2\cdot 2(x-x^2)\sqrt{x-x^2} - (2x-1)\cdot 2\left[(1-2x)\sqrt{x-x^2} + \frac{x-x^2}{2\sqrt{x-x^2}}(1\right.}{4(x-x^2)^2(x-x^2)}$$

$$= \frac{8(x-x^2)^2 - 2(2x-1)\left[2(1-2x)(x-x^2) + (x-x^2)(1-2x)\right]}{8(x-x^2)^3\sqrt{x-x^2}}$$

$$= \frac{8(x-x^2)^2 - 2(2x-1)\cdot 3(1-2x)(x-x^2)}{8(x-x^2)^3\sqrt{x-x^2}} =$$

$$= \frac{4(x-x^2) + \quad +3(1-2x)^2}{4(x-x^2)^2\sqrt{x-x^2}}$$

$4x - 4x^2 + 3 + 12x^2 - 12x = 0$

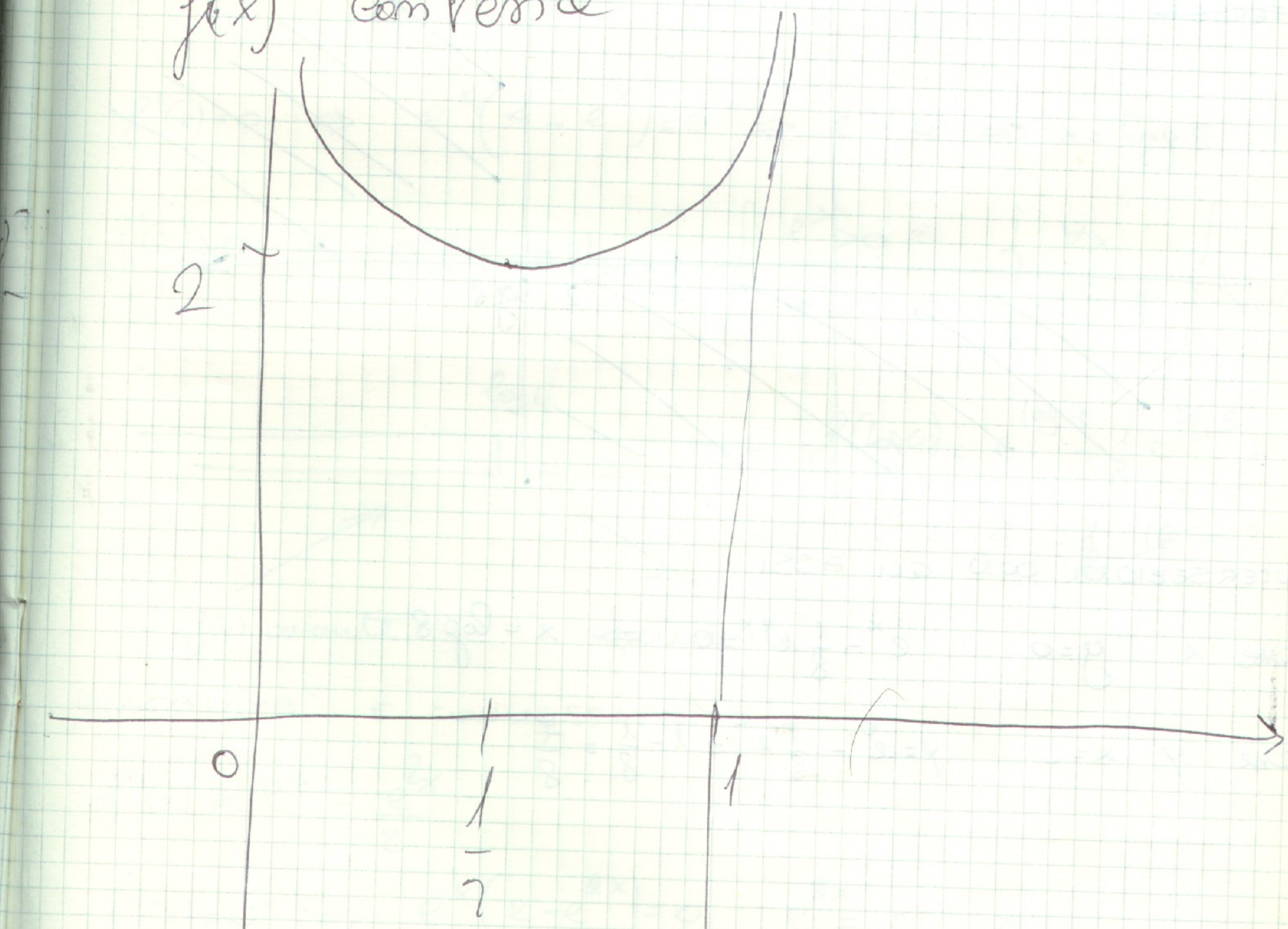
$8x^2 - 8x + 3 = 0$ $\Delta < 0$

~~$x = \cancel{8}\, 4 \pm \sqrt{16 - 24}$~~

$f''(x) = 0$ mai

$f''(x) > 0 \quad \forall x \in C\bar{E}$

$f(x)$ convessa



16

$f(x) = e^x - \frac{1}{8} e^{2x}$

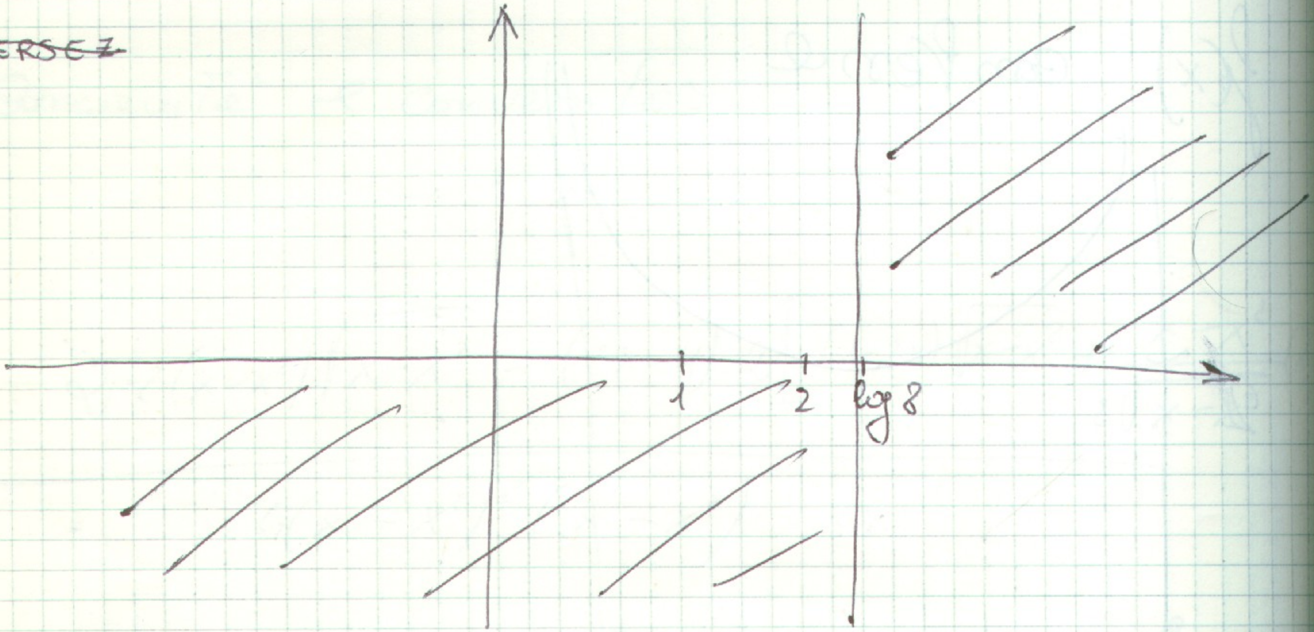
CE : $\mathbb{R}$

POSITIVITA'

$e^x - \frac{1}{8}e^{2x} > 0$; $\cancel{8}e^x(8 - e^x) > 0$ $\begin{cases} e^x > 0 \\ 8 - e^x > 0 \end{cases}$ $\begin{cases} \forall x \\ e^x < 8 \end{cases}$

$\begin{cases} \forall x \\ x < \log 8 \end{cases}$

~~INTERSEZ~~



INTERSEZIONI CON GLI ASSI

asse x $y = 0$ $e^x - \frac{1}{8}e^{2x} = 0 \Rightarrow x = \log 8$

asse y $x = 0$ $y = e^0 - \frac{1}{8}e^0 = 1 - \frac{1}{8} = \frac{7}{8}$

$$\lim_{x \to +\infty} e^x - \frac{1}{8} e^{2x} = e^x \left(1 - \frac{1}{8} e^x\right) = \infty(-\infty) = -\infty$$

$$m = \lim_{x \to +\infty} \frac{f(x)}{x} = \frac{e^x}{x}\left(1 - \frac{1}{8} e^x\right) = \infty(-\infty) = -\infty$$

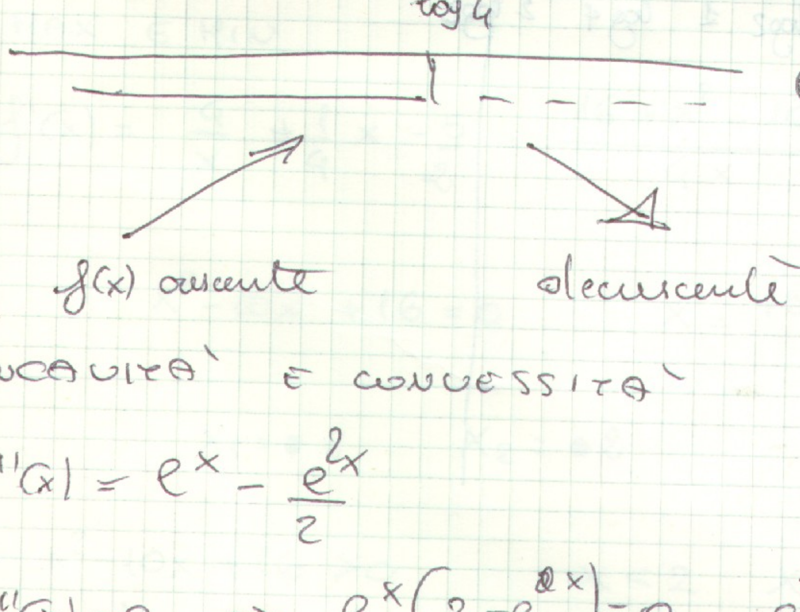
$$\lim_{x \to -\infty} e^x - \frac{1}{8} e^{2x} = 0$$ asintoto orizzontale

MAX E MIN

$$f'(x) = e^x - \frac{1}{4} e^{2x}$$

$$f'(x) = 0 \Rightarrow e^x (4 - e^x) = 0 \Rightarrow e^x = 4 \Rightarrow x = \log 4$$

$$f'(x) > 0 \Rightarrow e^x (4 - e^x) > 0 \quad \begin{cases} e^x > 0 \\ 4 - e^x > 0 \end{cases} \quad \begin{cases} \forall x \\ e^x < 4 \end{cases} \quad \begin{cases} \\ x < \log 4 \end{cases}$$



$$f(\log 4) = e^{\log 4} - \frac{1}{8} e^{\log 4^2} = 4 - \frac{1}{\cancel{8}} \cdot \cancel{16}^{2} = 2$$

CONCAVITA' E CONVESSITA'

$$f''(x) = e^x - \frac{e^{2x}}{2}$$

$$f''(x) = 0 \Rightarrow e^x\left(2 - e^{\cancel{2}x}\right) = 0 \quad e^{\cancel{2}x} = 2 \quad \cancel{x}$$

$$x = \cancel{\tfrac{1}{2}}\log 2$$

$$f(\log 2) = e^{\log 2} - 1\, e^{\log 4} = 2 - 1 = 3$$

$$m = \lim_{x \to +\infty} \frac{f(x)}{x} = \frac{e^x}{x}\left(1 - \frac{1}{8}e^x\right) = \infty(-\infty) = -\infty$$

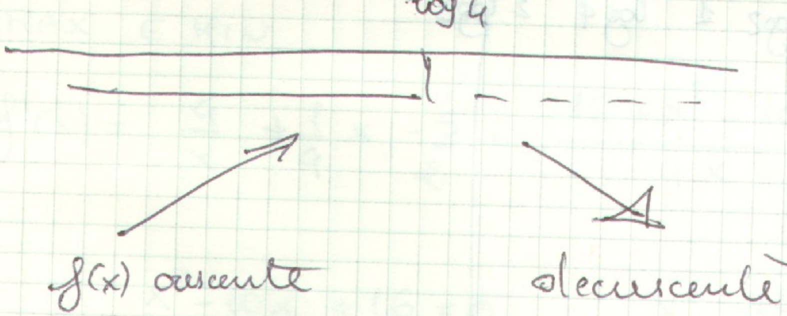
$$\lim_{x \to -\infty} e^x - \frac{1}{8}e^{2x} = 0$$ asintoto orizzontale

MAX E MIN

$$f'(x) = e^x - \frac{1}{4}e^{2x}$$

$$f'(x) = 0 \Rightarrow e^x(4 - e^x) = 0 \Rightarrow e^x = 4 \Rightarrow x = \log 4$$

$$f'(x) > 0 \Rightarrow e^x(4 - e^x) > 0 \quad \begin{cases} e^x > 0 \\ 4 - e^x > 0 \end{cases} \quad \begin{cases} \forall x \\ e^x < 4 \end{cases} \quad \begin{cases} x < \log 4 \end{cases}$$



$$f(\log 4) = e^{\log 4} - \frac{1}{8}e^{\log 4^2} =$$
$$= 4 - \frac{1}{\cancel{8}} \cdot \overset{2}{\cancel{16}} = 2$$

CONCAVITA' E CONVESSITA'

$$f''(x) = e^x - \frac{e^{2x}}{2}$$

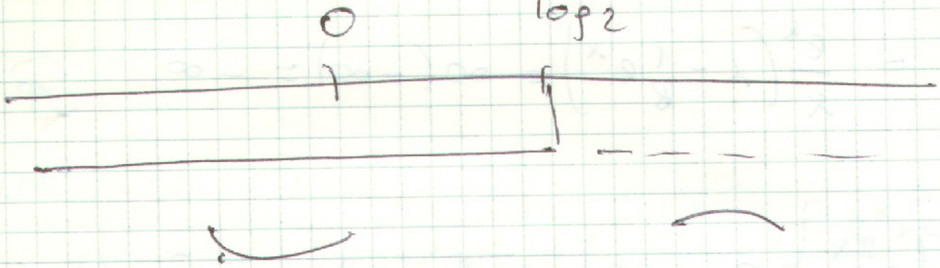
$$f''(x) = 0 \Rightarrow e^x(2 - e^{\cancel{2}x}) = 0 \quad e^{\cancel{2}x} = 2 \quad *$$

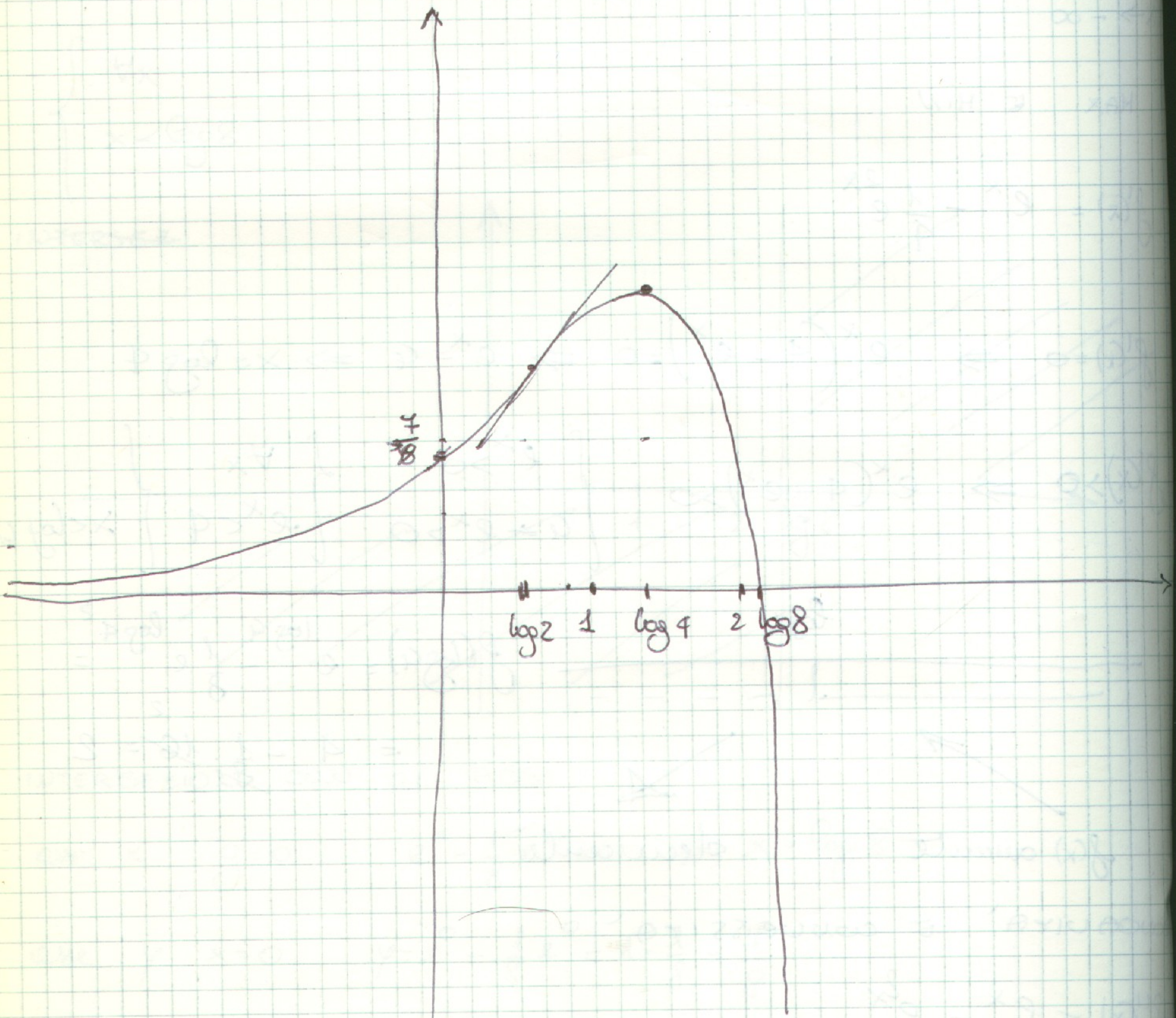
$x = \cancel{\frac{1}{2}}\log 2$

flesso

$$f(\log 2) = e^{\log 2} - \frac{1}{8}e^{\log 4} = 2 - \frac{1}{2} = \frac{3}{2}$$

$f''(x)>0 \quad \left\{ 2-e^{x}>0 \right. \quad , \quad e^{x}<2 \qquad x<\log 2$





17

$f(x) = 4 \log x + \frac{1}{8}x^2 - \frac{5}{2}x$

CE : $x > 0$ $\quad]0, +\infty[$

Positività

$4 \log x + \frac{1}{8}x^2 - \frac{5}{2}x > 0$ impossibile a risolvere

$\lim_{x \to 0^+} 4 \log x + \frac{1}{8}x^2 - \frac{5}{2}x = -\infty$

$\lim_{x \to +\infty} 4 \log x + \cancel{\frac{1}{8}}\, x\left(\frac{1}{8}x - \frac{5}{2}\right) = +\infty$

$\lim_{x \to +\infty} \frac{4 \log x}{x} + \left(\frac{1}{8}x - \frac{5}{2}\right) = 0 + \infty = +\infty$

niente asintoti

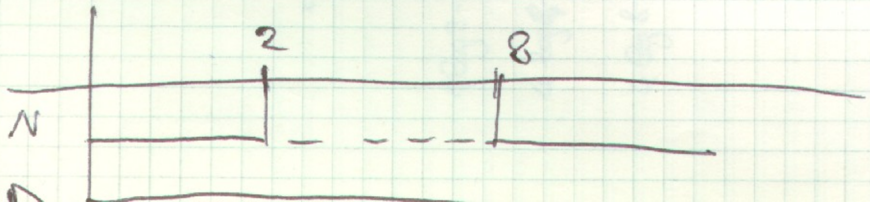
MAX E MIN

$f'(x) = \frac{4}{x} + \frac{1}{4}x - \frac{5}{2} = \frac{16 + x^2 - 10x}{4x} = 0$

$x^2 - 10x + 16 = 0 \qquad x = +5 \pm \sqrt{25 - 16} = +5 \pm 3$

$x_1 = 8 \qquad x_2 = 2$

$x^2 - 10x + 16 > 0 \qquad x < 2 \quad x > 8$



$x = 2$ max

$f(2) = 4 \log 2 + \frac{1}{2} - 5$

CE : $x > 0$ $\qquad]0, +\infty[$

Positività

$4 \log x + \frac{1}{8} x^2 - \frac{5}{2} x > 0$ impossibile a risolvere

$$\lim_{x \to 0^+} 4 \log x + \frac{1}{8} x^2 - \frac{5}{2} x = -\infty$$

$$\lim_{x \to +\infty} 4 \log x + \cancel{\frac{1}{8}} x \left(\frac{1}{8} x - \frac{5}{2} \right) = +\infty$$

$$\lim_{x \to +\infty} 4 \frac{\log x}{x} + \left(\frac{1}{8} x - \frac{5}{2} \right) = 0 + \infty = +\infty$$

niente asintoti

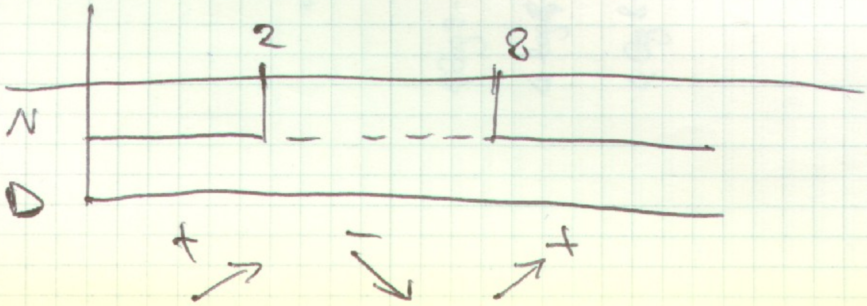
MAX E MIN

$$f'(x) = \frac{4}{x} + \frac{1}{4} x - \frac{5}{2} = \frac{16 + x^2 - 10x}{4x} = 0$$

$x^2 - 10x + 16 = 0 \qquad x = +5 \pm \sqrt{25 - 16} = +5 \pm 3$

$x_1 = 8 \qquad x_2 = 2$

$x^2 - 10x + 16 > 0 \qquad x < 2 \quad x > 8$



$x = 2$ max

$f(2) = 4 \log 2 + \frac{1}{2} - 5$

$= 4 \log 2 - \frac{9}{2}$

CE : $x > 0$ $\quad]0,+\infty[$

Positività

$4\log x + \frac{1}{8}x^2 - \frac{5}{2}x > 0$ impossibile a risolvere

$$\lim_{x\to 0^+} 4\log x + \frac{1}{8}x^2 - \frac{5}{2}x = -\infty$$

$$\lim_{x\to +\infty} 4\log x + \cancel{\frac{1}{8}}\, x\left(\frac{1}{8}x - \frac{5}{2}\right) = +\infty$$

$$\lim_{x\to +\infty} \frac{4\log x}{x} + \left(\frac{1}{8}x - \frac{5}{2}\right) = 0 + \infty = +\infty$$

niente asintoti

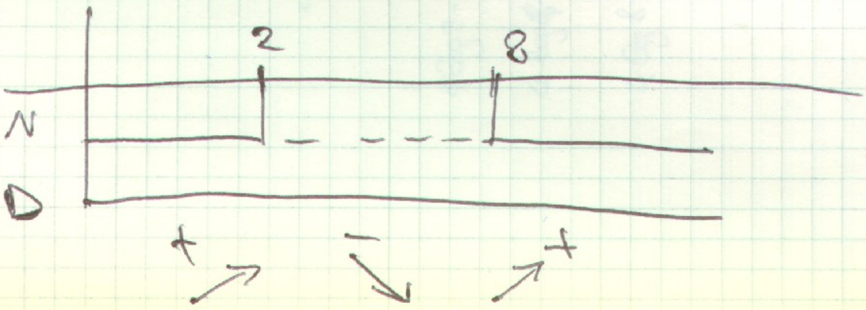
MAX E MIN

$$f'(x) = \frac{4}{x} + \frac{1}{4}x - \frac{5}{2} = \frac{16 + x^2 - 10x}{4x} = 0$$

$x^2 - 10x + 16 = 0 \qquad x = +5 \pm \sqrt{25-16} = +5 \pm 3$

$x_1 = 8 \qquad x_2 = 2$

$x^2 - 10x + 16 > 0 \qquad x < 2 \quad x > 8$



$x = 2$ max

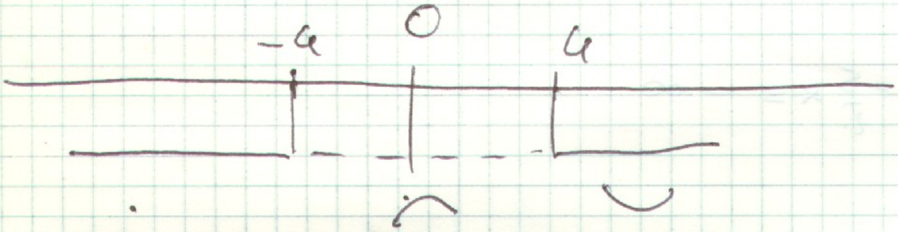
$f(2) = 4\log 2 + \frac{1}{2} - 5$

$= 4\log 2 - \frac{9}{2}$

$x = 8$ min

$$f(8) = 4 \log 8 + 8 - 20 = 4 \log 8 - 12$$

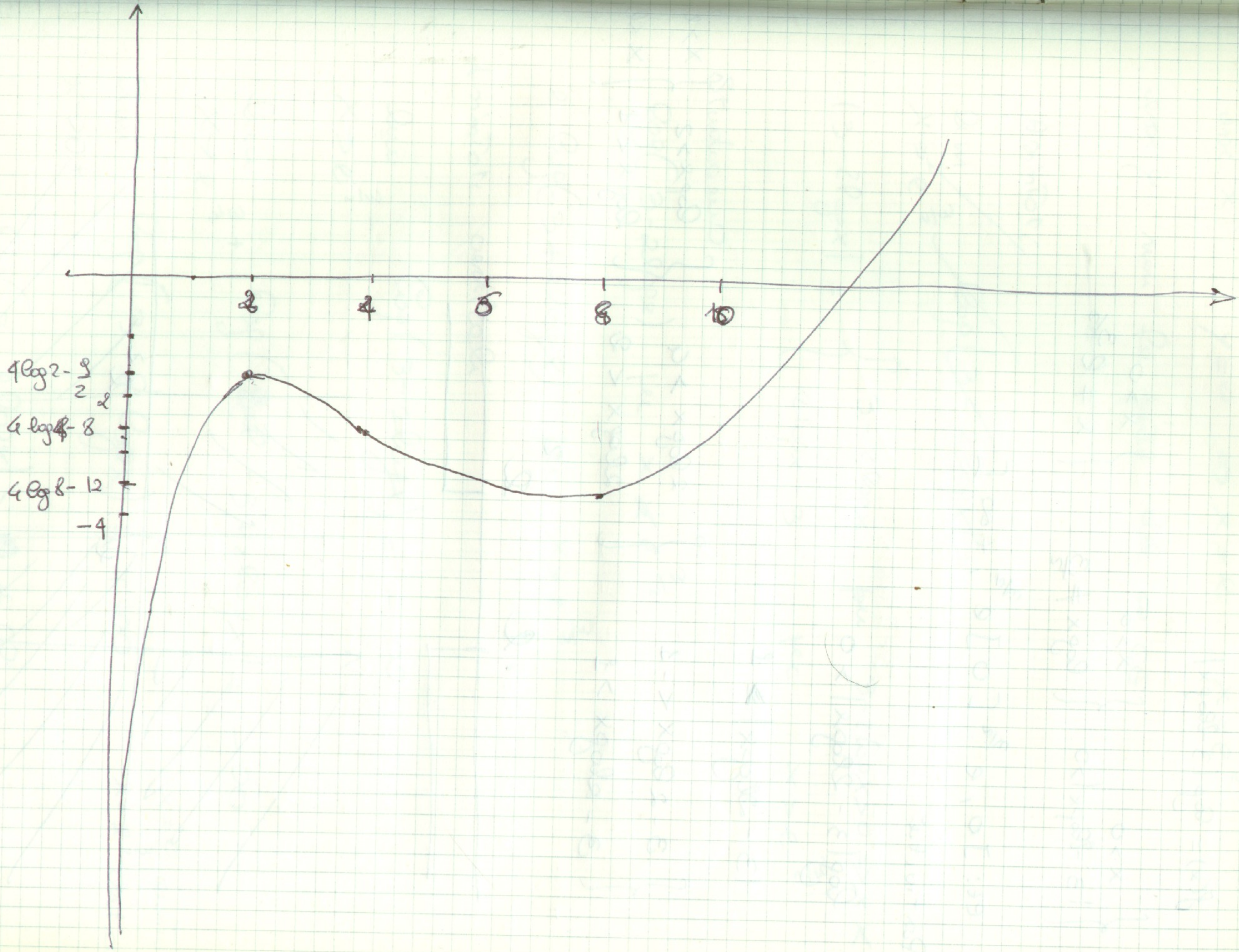
$$f''(x) = -\frac{4}{x^2} + \frac{1}{4} = \frac{-16 + x^2}{4x^2} = \frac{x^2 - 16}{4x^2} = 0 \Rightarrow x = \pm 4$$



$$f(4) = 4 \log 4 + 2 - 10 = 4 \log 4 - 8$$

Poichè dopo 8 la funzione è sempre crescente, ci sarà una sola intersezione con l'asse x

Metodo delle Tangenti


2
4
6
8
10
$4\log 2-\frac{9}{2}$
2
$4\log 4-8$
$4\log 8-12$
-4

$f(x) = \log|3-2\log x|$

$\begin{cases} x>0 \\ |3-2\log x|>0 \end{cases}$ $\begin{cases} x>0 \\ \log x \neq \frac{3}{2} \end{cases}$ $\begin{cases} x>0 \\ x \neq e^{3/2} \end{cases}$

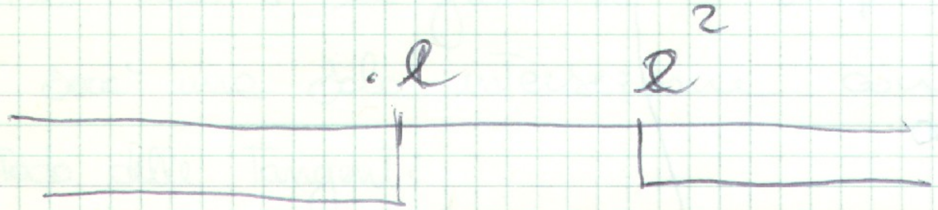
et: $]0, e^{3/2}[\cup]e^{3/2}, +\infty[$

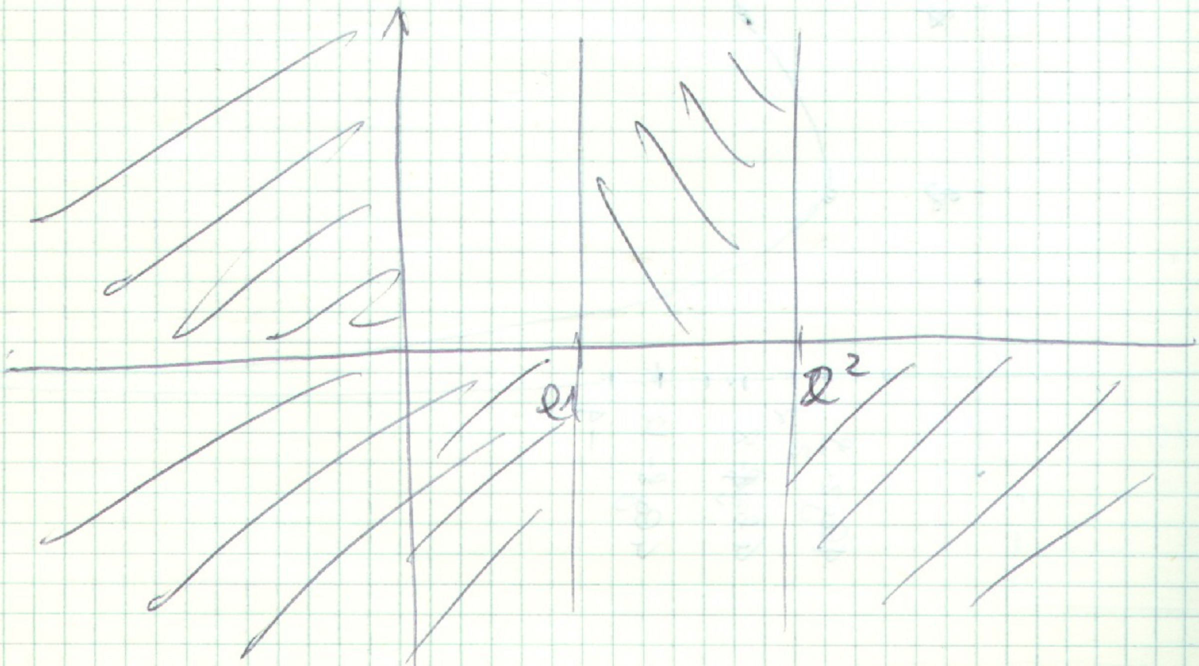
Positivité

$\log|3-2\log x|>0$

$|3-2\log x| \geq 1$

$\begin{cases} 3-2\log x<-1 \\ 3-2\log x>1 \end{cases}$ $\begin{cases} 2\log x>4 \\ 2\log x<2 \end{cases}$ $\begin{cases} \log x>2 \\ \log x<1 \end{cases}$ $\begin{cases} x> \\ x< \end{cases}$

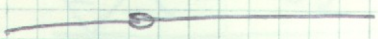




Intersezione con gli assi

asse x $\quad y = 0 \Rightarrow x = e \quad , \quad x = e^2$

asse y non fa parte del CE



Asintoti

~~$\lim_{x \to e^{\frac{3}{2}}} \log|3 - 2\log x| =$~~

$$|3 - 2\log x| = \begin{cases} 3 - 2\log x & \text{per } 3 - 2\log x > 0 \Rightarrow x < e^{\frac{3}{2}} \\ 2\log x - 3 & \text{per } x > e^{\frac{3}{2}} \end{cases}$$

Studiamo

$\log(3 - 2\log x)$ per $x < e^{\frac{3}{2}}$

e $\log(-3 + 2\log x)$ per $x > e^{\frac{3}{2}}$

Asintoti verticali

$$\lim_{x \to e^{\frac{3}{2}-}} \log(3 - 2\log x) = -\infty$$

$$\lim_{x \to e^{\frac{3}{2}+}} \log(\cancel{2e} -3 + 2\log x| = -\infty$$

$$\lim_{x \to 0+} \log(3 - 2\log x) = +\infty$$

$x = 0$ asintoti
$x = e^{\frac{3}{2}}$ verticali

$$\lim_{x \to +\infty} \log(3 - 2\log x| = \cancel{0} + \infty$$

asintoto obliquo $\quad y = mx + n$

$$\lim_{x \to +\infty} \frac{\log(3-2\log x)}{x} =$$

$$= \lim_{x \to +\infty} \frac{\log(3-2\log x)}{3-2\log x} \cdot \frac{3-\log x}{x} =$$

$$= \lim_{x \to +\infty} \frac{\log(3-2\log x)}{3-2\log x} \left(\frac{3}{x} - \frac{\log x}{x} \right) =$$

$$= 0\,(0-0) = 0$$

non c'è asintoto obliquo

MAX e MIN

$$\xcancel{f'(x) = \frac{\frac{1}{3-2\log x}\left(-\frac{2}{x}\right) \cdot x - \log(3-2\log x)}{x^2} =}$$

$$\xcancel{= - \frac{2 + (3-2\log x) \cdot \overset{+\infty}{\log(3-2\log x)}}{x^2(3-2\log x)} = 0}$$

~~$$f''(x) = -\frac{\left(-\frac{2}{x}\log(3-2x) + (3-2\right.}{x^4(3-2\log x)^2}$$~~

~~$$f''(x) = -\frac{\left[-\frac{2}{x}\log(3-2\log x) + \frac{(3-2\log x)}{3-2\log x}\left(-\frac{2}{x}\right)\right]x^2(3-2\log x)}{x^4(3-2\log x)^2}$$~~

~~$$+\frac{\left[2+(3-2\log x)\log(3-2\log x)\right]\left[2x(3-2\log x) + x^2\left(-\frac{2}{x}\right)\right]}{x^4(3-2\log x)^2}$$~~

~~$$= \frac{\frac{2}{x}\left[\log(3-2\log x)+1\right]x^2(3-2\log x) - \left[2+(3-2\log x)\log(3-2\log x)\right]\left(2x(3-2\log\right.}{x^4(3-2\log x)^2}$$~~

$$f'(x) = \frac{1}{3-2\log x}\left(-\frac{2}{x}\right) = -\frac{2}{x(3-2\log x)}$$

$$f'(x) < 0 \Rightarrow \begin{cases} x > 0 \\ 3-2\log x > 0 \end{cases} \quad \begin{cases} x > 0 \\ x < e^{\frac{3}{2}} \end{cases}$$

$0 < x < e^{\frac{3}{2}}$ $\quad f'(x) < 0$

per $x > e^{\frac{3}{2}}$

$$f'(x) = \frac{1}{2\log x - 3} \cdot \frac{2}{x}$$

$$f'(x) > 0 \qquad \begin{cases} x > 0 \\ 2\log x > 3 \end{cases} \qquad \begin{cases} x > 0 \\ x > e^{\frac{3}{2}} \end{cases}$$

0

per $x > e^{\frac{3}{2}}$ $f'(x) > 0$

non ci sono max e min

per $x < e^{\frac{3}{2}}$ $f(x)$ decrescente

per $x > e^{\frac{3}{2}}$ $f(x)$ crescente

$$f''(x) = + \frac{+2\left[3 - 2\log x + \cancel{x}\left(-\frac{2}{\cancel{x}}\right)\right]}{x^2(3-2\log x)^2}$$

$$= \frac{3(1 - 2\log x)}{x^2(3-2\log x)^2}$$

$f''(x) = 0$ $1 - 2\log x = 0$ $\log x = \frac{1}{2}$ $x = e^{\frac{1}{2}}$

$f''(x) > 0$ $1 - 2\log x > 0$ $\log x < \frac{1}{2}$ $x < e^{\frac{1}{2}}$

0 $\sqrt{e}$

$$f'(x) = \frac{1}{2\log x - 3} \cdot \frac{2}{x}$$

$$f'(x) > 0 \quad \begin{cases} x > 0 \\ 2\log x > 3 \end{cases} \quad \begin{cases} x > 0 \\ x > e^{\frac{3}{2}} \end{cases}$$

per $x > e^{\frac{3}{2}}$ $f'(x) > 0$

non ci sono max e min

per $x < e^{\frac{3}{2}}$ $f(x)$ decrescente

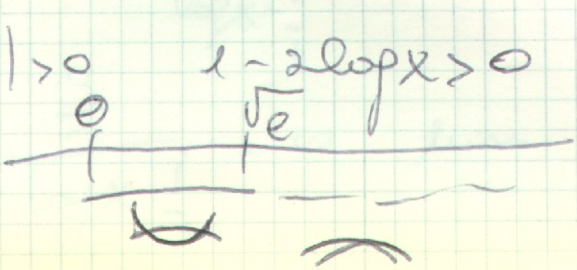
per $x > e^{\frac{3}{2}}$ $f(x)$ crescente

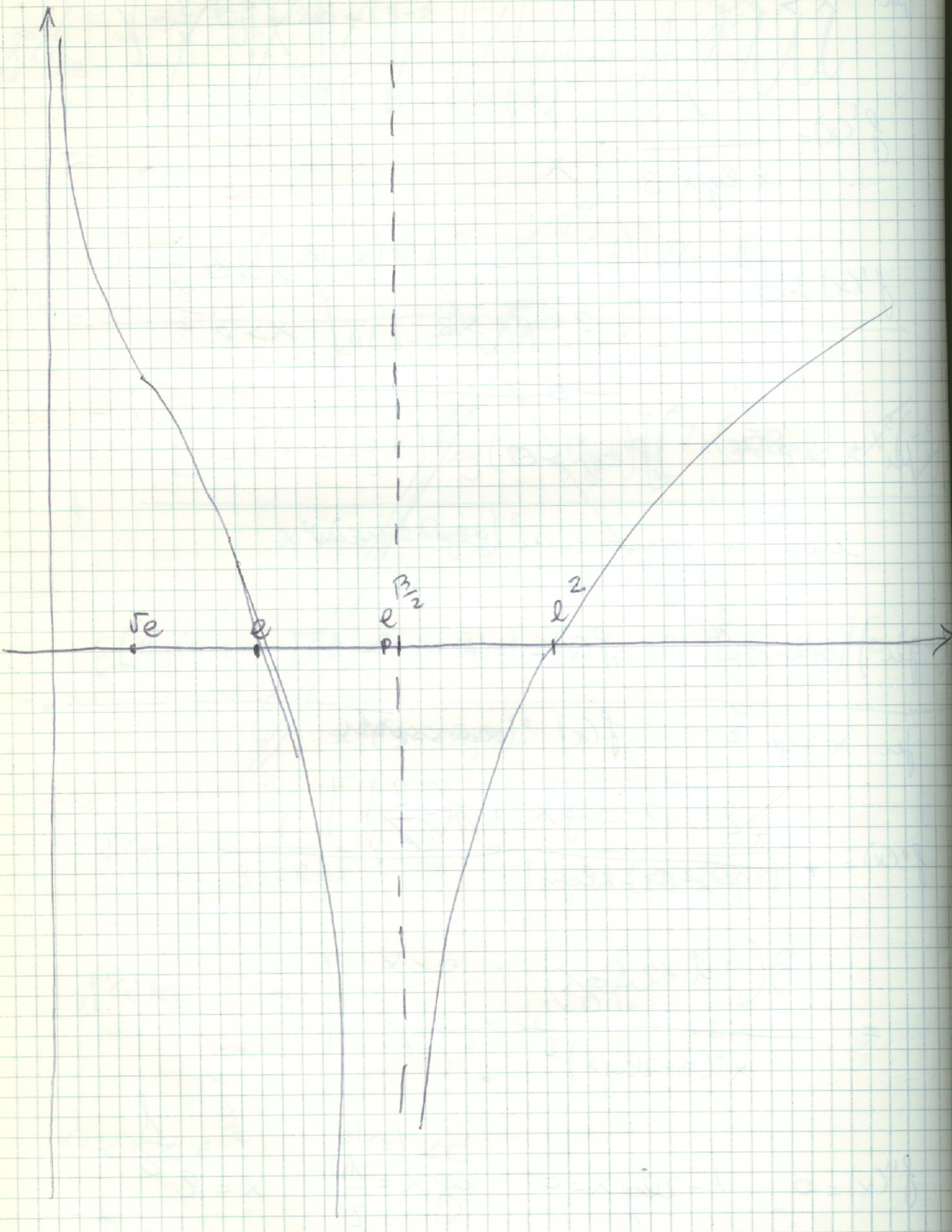
$$f''(x) = + \frac{+2\left[3 - 2\log x + \cancel{x}\left(-\frac{2}{\cancel{x}}\right)\right]}{x^2(3-2\log x)^2}$$

$$= \frac{3(1-2\log x)}{x^2(3-2\log x)^2}$$

$f''(x) = 0$ $1 - 2\log x = 0$ $\log x = \frac{1}{2}$ $x = e^{\frac{1}{2}}$

$f''(x) > 0$ $1 - 2\log x > 0$ $\log x < \frac{1}{2}$ $x < e^{\frac{1}{2}}$





$\sqrt{e}$
e
$e^{3/2}$
e^2

19

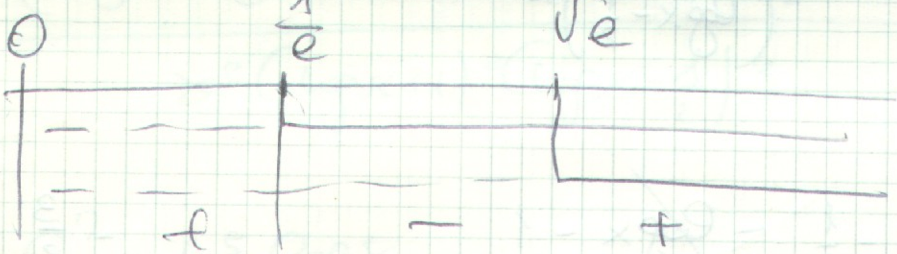
$$f(x) = \log \frac{\log x + 1}{\log x - \frac{1}{2}}$$

CE: $\begin{cases} \dfrac{\log x + 1}{\log x - \frac{1}{2}} > 0 \\ x > 0 \end{cases} \Rightarrow$ ~~$\begin{cases} \dfrac{\log x + 1 - \log x + \frac{1}{2}}{\log x - \frac{1}{2}} > 0 \\ x > 0 \end{cases}$~~

~~$\begin{cases} \dfrac{\frac{3}{2}}{\log x - \frac{1}{2}} > 0 \\ x > 0 \end{cases}$ $\begin{cases} \log x > \frac{1}{2} \\ x > 0 \end{cases}$ $\begin{cases} x > \sqrt{e} \\ x > 0 \end{cases}$~~

$\log x + 1 > 0 \qquad \log x > -1 \qquad x > e^{-1}$

$\log x - \frac{1}{2} > 0 \qquad \log x > \frac{1}{2} \qquad x > \sqrt{e}$



CE : $]0, \frac{1}{e}[\cup]\sqrt{e}, +\infty[$

$y = 0 \Rightarrow \dfrac{\log x + 1}{\log x - \frac{1}{2}} = 1 \qquad \dfrac{\log x + 1 - \log x + \frac{1}{2}}{\log x - \frac{1}{2}} = 0$

$\frac{3}{2}$ = 0 non c'è intersezione

$$\lim_{x \to 0^+} \log \frac{\log x + 1}{\log x - \frac{1}{2}} = \cancel{\log}$$

$$= \lim_{y \to -\infty} \log \frac{y+1}{y - \frac{1}{2}} = \log 1 = 0$$

$$\lim_{x \to \frac{1}{e}^-} \log \frac{\log x + 1}{\log x - \frac{1}{2}} = \log \frac{-1+1}{-1-\frac{1}{2}} = \cancel{\log} -\infty$$

$$\lim_{x \to e^{\frac{1}{2}}} \log \frac{\log x + 1}{\log x - \frac{1}{2}} = \log \frac{\frac{1}{2}+1}{\frac{1}{2} - \frac{1}{2}} = +\infty$$

$$\lim_{x \to +\infty} \log \frac{\log x + 1}{\log x - \frac{1}{2}} = \log 1 = 0$$

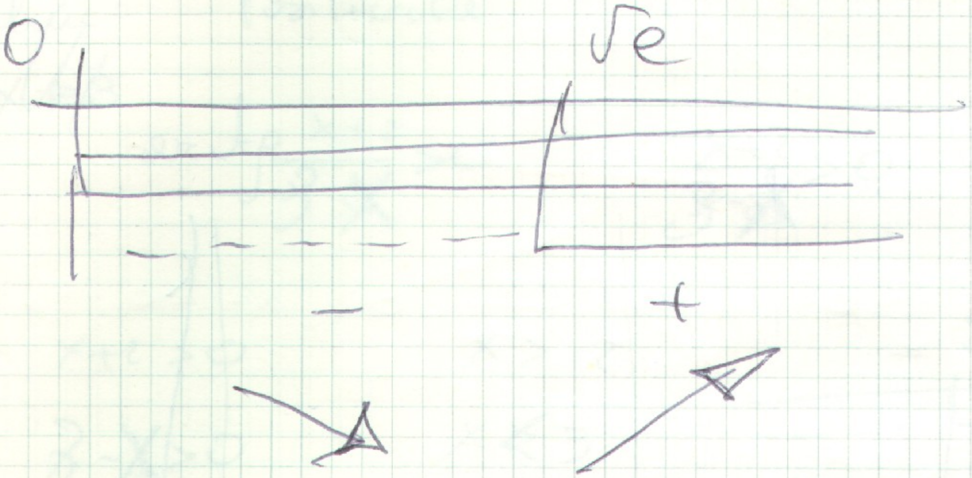
MAX E MIN

$$f'(x) = \frac{1}{\frac{\log x + 1}{\cancel{\log x - \frac{1}{2}}}} \cdot \frac{\frac{1}{x}\left(\log x - \frac{1}{2}\right) - \left(\log x + 1\right)\frac{1}{x}}{\left(\log x - \frac{1}{2}\right)^2} =$$

$$= \frac{1}{x} \frac{\cancel{\log x} - \frac{1}{2} - \cancel{\log x} - 1}{\left(\log x + 1\right)\left(\log x - \frac{1}{2}\right)} = \frac{-\frac{3}{2}}{x\left(\log x + 1\right)\left(\log x - \frac{1}{2}\right)}$$

$f'(x) \neq 0$ non ci sono max e min

$$\begin{cases} x>0 \\ \log x+1>0 \\ \log x-\frac{1}{2}>0 \end{cases} \qquad \begin{cases} x>0 \\ \log x>-1 \\ \log x>\frac{1}{2} \end{cases} \qquad \begin{cases} x>0 \\ x>0 \\ e\,x>\sqrt{e} \end{cases}$$



$$f''(x)=+\frac{3}{2}\,\frac{\left[(\log x+1)(\log x-\frac{1}{2})+x\cdot\frac{1}{x}(\log x-\frac{1}{2})+\cancel{x}(\log x+1)\frac{1}{\cancel{x}}\right]}{x^2(\log x+1)^2(\log x-\frac{1}{2})^2}$$

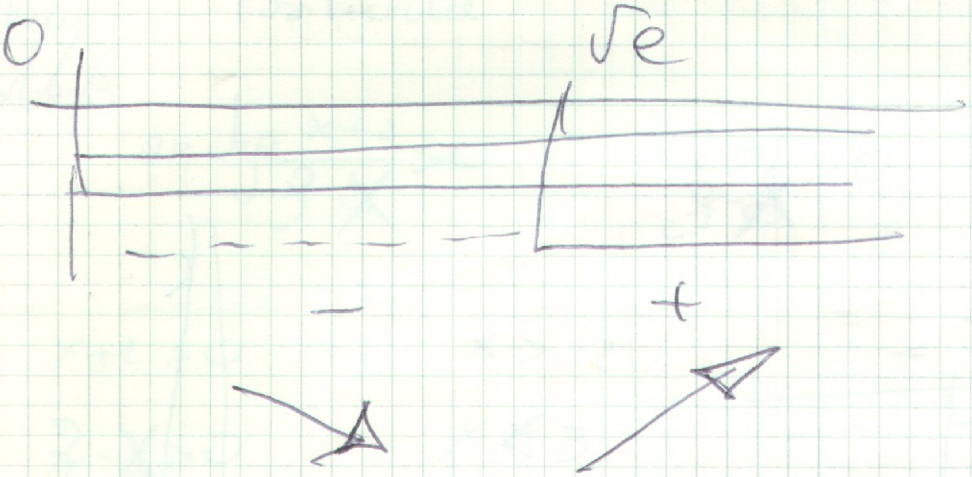
$$=\frac{3}{2}\,\frac{\left(\log^2 x-\frac{1}{2}\log x+\log x-\frac{1}{2}+\log x-\frac{1}{2}+\log x+1\right)}{x^2(\log x+1)^2(\log x-\frac{1}{2})^2}=$$

$$=\frac{3}{2}\,\frac{\log^2 x+\frac{5}{2}\log x}{x^2(\log x+1)^2(\log x-\frac{1}{2}|^2}$$

$f''(x)=0 \qquad \log^2 x+\frac{5}{2}\log x=0$

$$\log x\left(\log x+\frac{5}{2}\right)=0 \quad < \begin{matrix} x=1 \\ x=e^{-\frac{5}{2}} \end{matrix}$$

$$\begin{cases} \log x + 1 \geq 0 \\ \log x - \frac{1}{2} > 0 \end{cases} \quad \begin{cases} \log x > -1 \\ \log x > \frac{1}{2} \end{cases} \quad \begin{cases} x > 0 \\ e\,x > \sqrt{e} \end{cases}$$



$$f''(x) = + \frac{3}{2} \frac{\left[(\log x+1)(\log x - \frac{1}{2}) + x \cdot \frac{1}{x}(\log x - \frac{1}{2}) + \cancel{x}(\log x+1)\frac{1}{\cancel{x}}\right]}{x^2(\log x+1)^2(\log x - \frac{1}{2})^2}$$

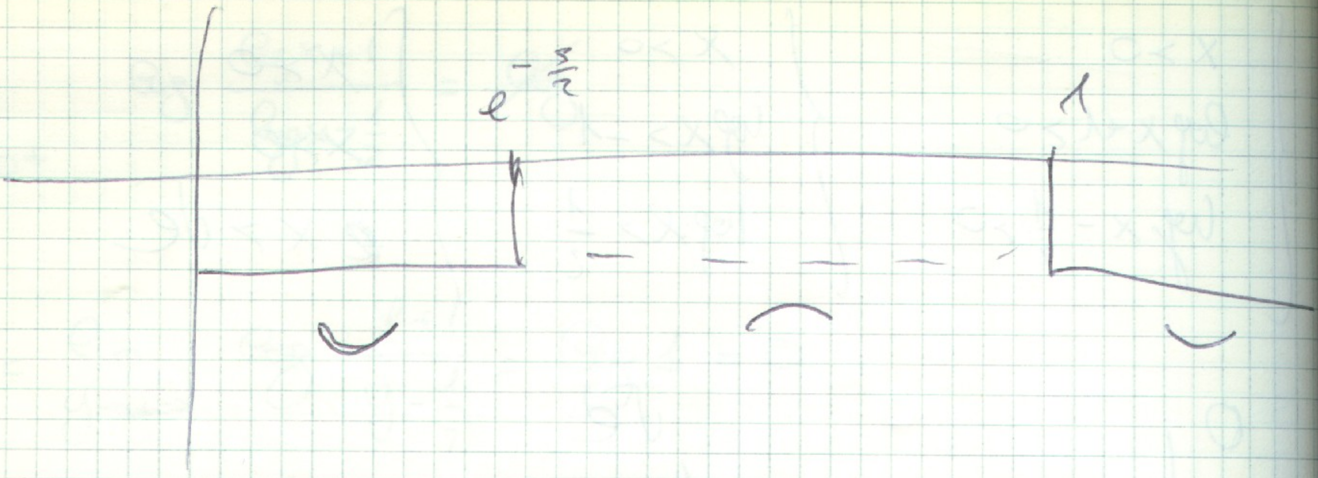
$$= \frac{3}{2} \frac{\left(\log^2 x - \frac{1}{2}\log x + \log x - \frac{1}{2} + \log x - \frac{1}{2} + \log x + 1\right)}{x^2(\log x+1)^2(\log x - \frac{1}{2})^2} =$$

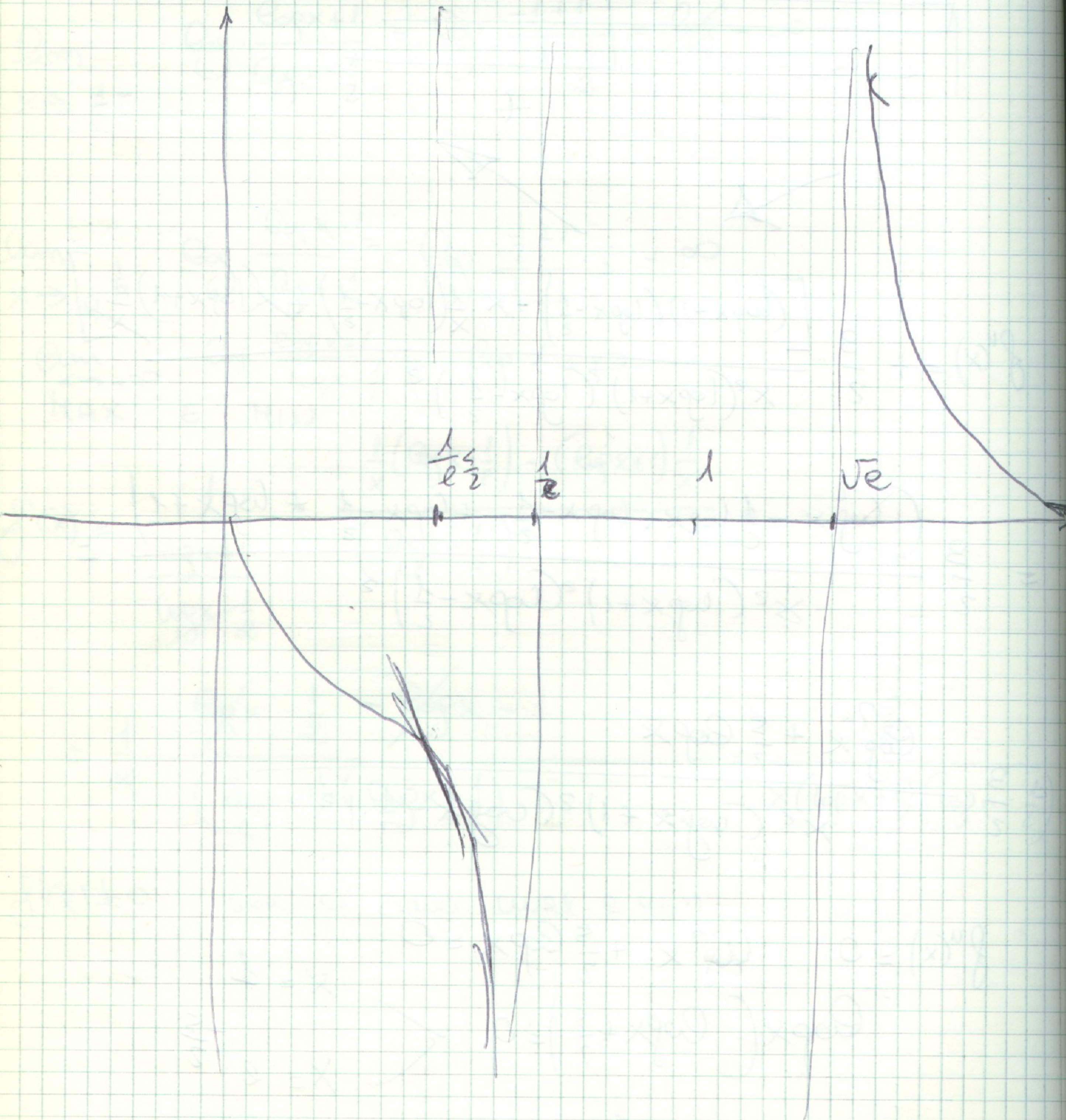
$$= \frac{3}{2} \frac{\log^2 x + \frac{5}{2}\log x}{x^2(\log x+1)^2(\log x - \frac{1}{2})^2}$$

$f''(x) = 0 \qquad \log^2 x + \frac{5}{2}\log x = 0$

$$\log x\left(\log x + \frac{5}{2}\right) = 0 \quad < \begin{matrix} x = 1 \\ x = e^{-\frac{5}{2}} \end{matrix}$$

$f''(x) \geq 0 \qquad x < e^{-5/2} \qquad x > 1$


$e^{-\frac{3}{2}}$
1


$\frac{1}{e^{\frac{5}{2}}}$
$\frac{1}{e}$
1
$\sqrt{e}$

20

$\text{arctg}\ \frac{x+2}{3-x}$

CE: $3-x \neq 0$ $x \neq 3$

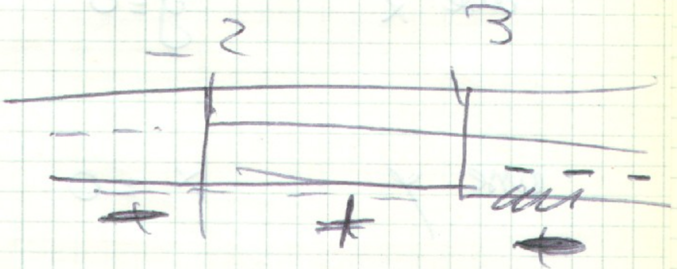
$\lim_{x \to 3}$

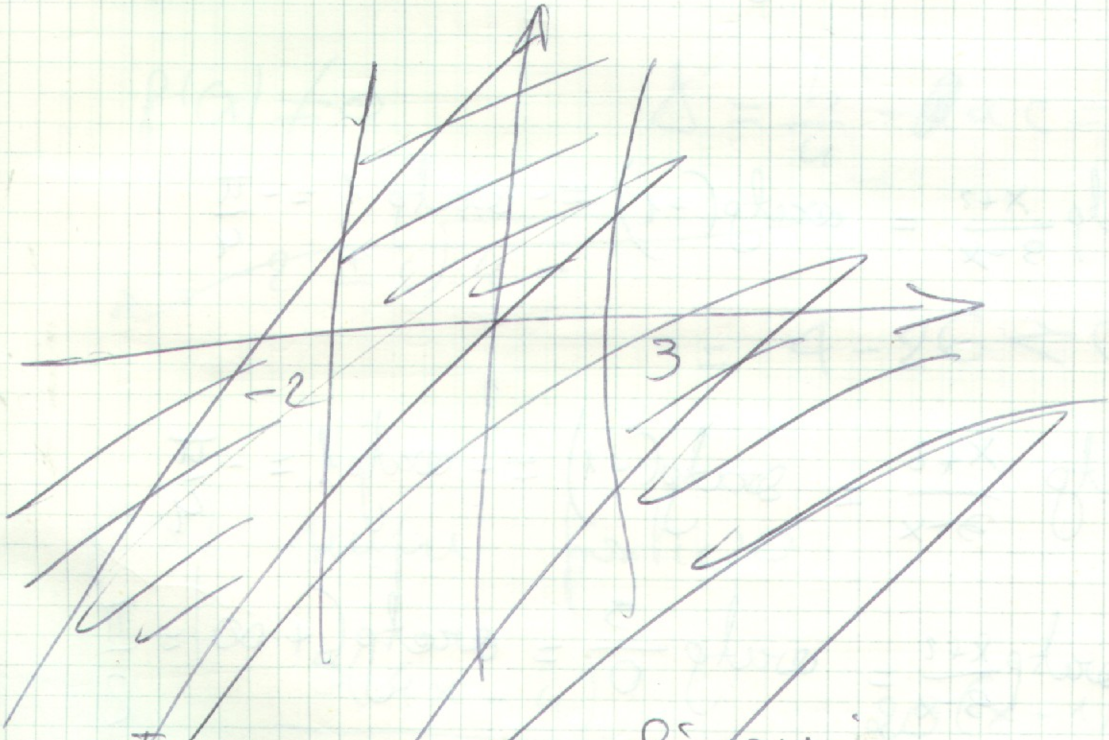
Positività

$\text{arctg}\ \frac{x+2}{3-x} > 0$ $\frac{x+2}{3-x} > 0$

$x+2 > 0$ $x > -2$

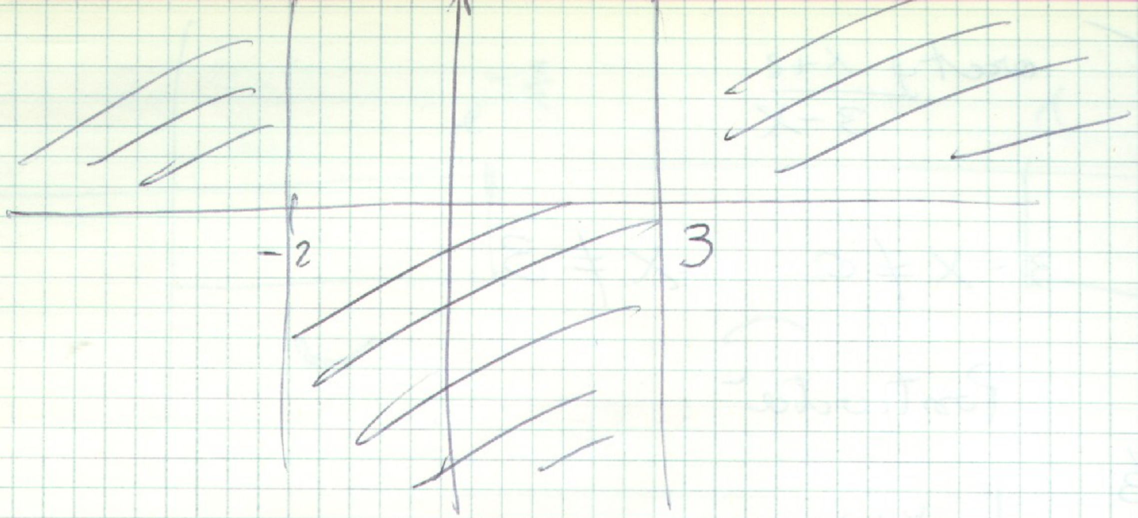
$3-x > 0$ $x < 3$





intersezione con gli assi

$x = 0$ $y = \text{arctg}\ \frac{2}{3}$



intersezioni asse x $\quad y=0 \quad \operatorname{arctg}\frac{x+2}{3-x}=0 \quad x+2=0 \quad x=-$

asse y $\quad x=0 \quad y=\operatorname{arctg}\frac{2}{3}$

asintoti

$$\lim_{x\to-\infty} \operatorname{arctg}\frac{x+2}{3-x} = \operatorname{arctg}(-1) = -\operatorname{arctg}1 = -\frac{\pi}{4}$$

$$\lim_{x\to+\infty} \operatorname{arctg}\frac{x+2}{3-x} = \operatorname{arctg}(-1) = -\operatorname{arctg}1 = -\frac{\pi}{4}$$

$$\lim_{x\to3^-} \operatorname{arctg}\frac{x+2}{3-x} = \operatorname{arctg}\frac{5}{0} = \operatorname{arctg}(+\infty) = \frac{\pi}{2}$$

$$\lim_{x\to3^+} \operatorname{arctg}\frac{x+2}{3-x} = \operatorname{arctg}(-\infty) = -\frac{\pi}{2}$$

MAX e MIN

$$f'(x) = \frac{1}{1+\left(\frac{x+2}{3-x}\right)^2} \cdot \frac{3-x-(x+2)(-1)}{(3-x)^2} =$$

$$= \frac{3-\cancel{x}+x+2}{(3-x)^2+(x+2)^2} = \frac{5}{9+x^2-6x+x^2+4+2x}$$

$$= \frac{5}{2x^2-4x+13}$$

$f'(x) \neq 0$ $\Delta = \frac{b^2}{4} - ac =$

~~$x = 2 \pm \sqrt{4 -}$~~

$= 4 - 26 < 0$

$f'(x)$ sempre positivo

$$f''(x) = -\frac{5(4x-4)}{2x^2-4x+13} = -\frac{20(x-1)}{2x^2-4x+13}$$

$f''(x) = 0$ $x-1=0$ $x=1$

$f''(x) > 0$ $x-1<0$ $x<1$

$$f'(x) = \frac{1}{1+\left(\frac{x+2}{3-x}\right)^2} \cdot \frac{}{(3-x)^2} =$$

$$= \frac{3 - \cancel{x} + x + 2}{(3-x)^2 + (x+2)^2} = \frac{5}{9 + x^2 - 6x + x^2 + 4 + 2x}$$

$$= \frac{5}{2x^2 - 4x + 13}$$

$f'(x) \neq 0$ $\qquad$ $\Delta = \frac{b^2}{4} - ac =$

~~$x = 2 \pm \sqrt{4 -}$~~

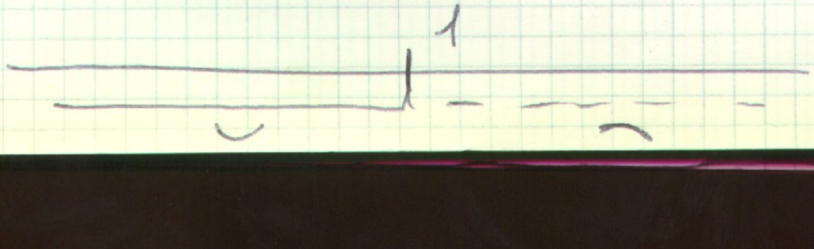
$= 4 - 26 < 0$

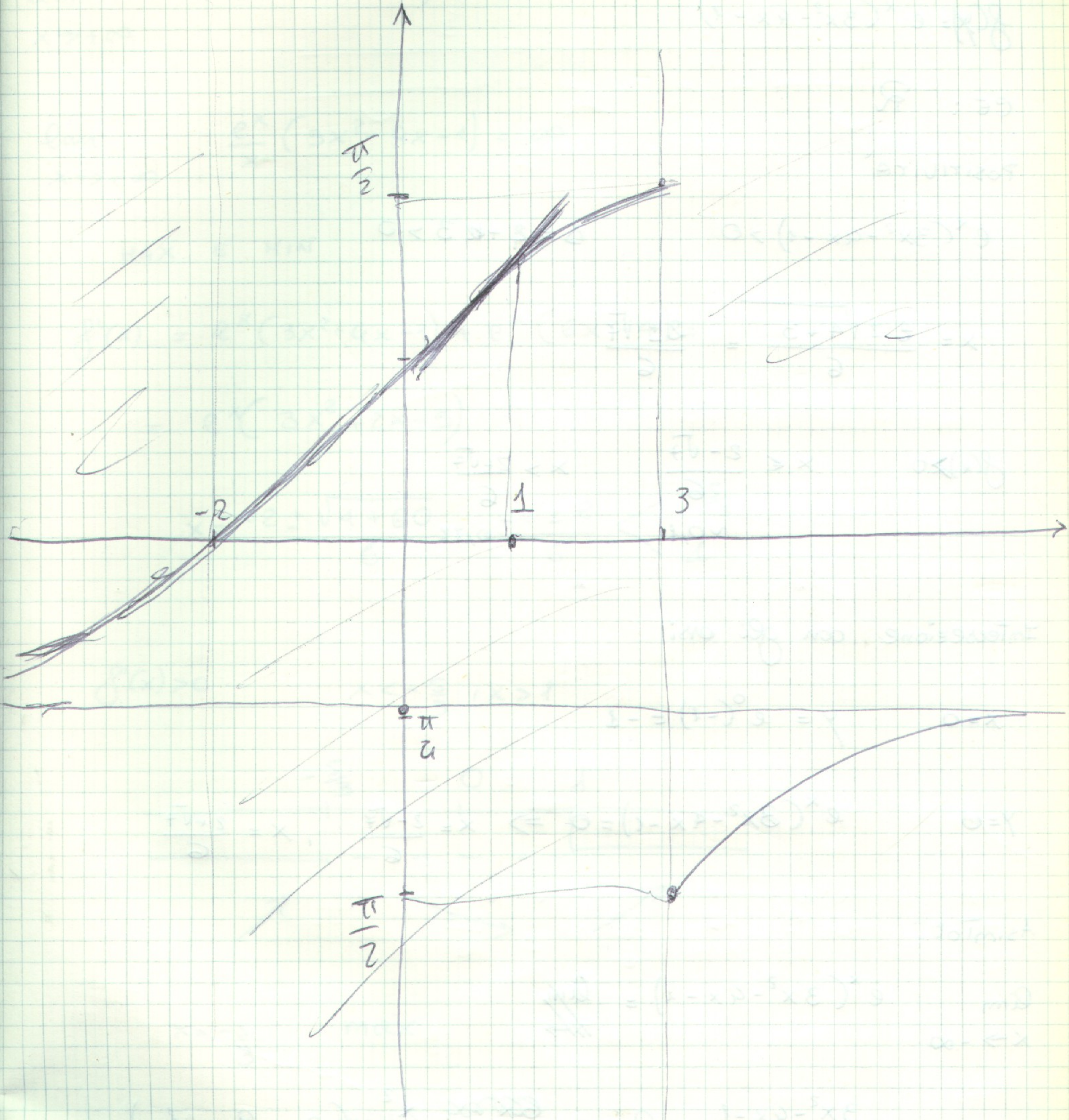
$f'(x)$ sempre positivo

$$f''(x) = -\frac{5(4x-4)}{2x^2-4x+13} = -\frac{20(x-1)}{2x^2-4x+13}$$

$f''(x) = 0 \qquad x - 1 = 0 \qquad x = 1$

$f''(x) > 0 \qquad x - 1 < 0 \qquad x < 1$




π/2
-2
1
3
π/4
π/2

21

$f(x) = e^x(3x^2 - 4x - 1)$

CE: $\mathbb{R}$

POSITIVITÀ

$e^x(3x^2-4x-1) > 0$ $\qquad$ $\Delta = 4 + 3 > 0$

$$x = \frac{2 \pm \sqrt{4+3}}{6} = \frac{2 \pm \sqrt{7}}{6}$$

$f(x) > 0$ $\qquad$ $x < \frac{2-\sqrt{7}}{6}$ $\qquad$ $x > \frac{2+\sqrt{7}}{6}$

$\sim 0{,}1$ $\qquad$ $\sim 0{,}77$

Intersezione con gli assi

$x = 0$ $\qquad$ $y = e^0(-1) = -1$

$y = 0$ $\qquad$ $e^x(3x^2-4x-1) = 0 \Rightarrow x = \frac{2-\sqrt{7}}{6}$, $x = \frac{2+\sqrt{7}}{6}$

Asintoti

$$\lim_{x \to -\infty} e^x(3x^2-4x-1) =$$

$$= \lim_{x \to -\infty} \frac{3x^2-4x-1}{e^{-x}} = \lim_{x \to -\infty} \frac{x^2}{e^{-x}}\left(3 - \frac{4}{x} - \frac{1}{x^2}\right) =$$

$$= \lim_{y \to +\infty} \frac{\left(3 + \frac{4}{y} - \frac{1}{y^2}\right)}{\frac{e^y}{y^2}} = \frac{3+0+0}{+\infty} = 0$$

$$\lim_{x \to +\infty} e^x(3x^2-4x-1) = \infty$$

$$\lim_{x \to +\infty} \frac{e^x}{x}(3x^2-4x-1) = \infty$$

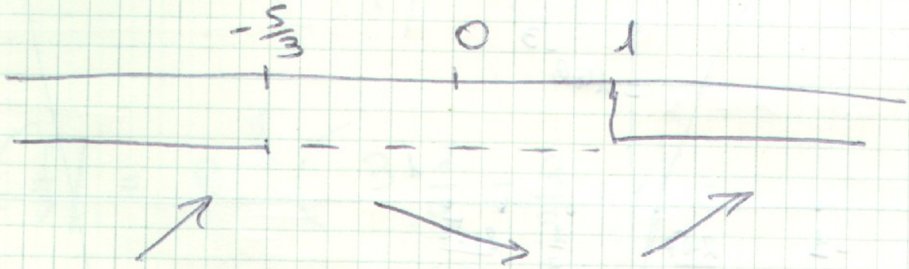
MAX E MIN

$$f'(x) = e^x(3x^2-4x-1) + e^x(6x-4) =$$

$$= e^x(3x^2+2x-5)$$

$$x = \frac{-2 \pm \sqrt{4+60}}{6} = \frac{-2 \pm 8}{6} \left\langle \begin{matrix} -\frac{5}{3} \\ 1 \end{matrix} \right.$$

$f'(x)>0$ $\quad x < -\frac{5}{3}$, $x > 1$



$x = -\frac{5}{3}$ max

$x = 1$ min

$$f''(x) = e^x(3x^2+2x-5) + e^x(6x+2) = e^x(3x^2+8x-3)$$

$$x = \frac{-4 \pm \sqrt{16+9}}{3} = \frac{-4 \pm 5}{3} \left\langle \begin{matrix} -3 \\ 1 \end{matrix} \right.$$

$x \to +\infty$

$$\lim_{x \to +\infty} \frac{e^x}{x}(3x^2-4x-1) = \infty$$

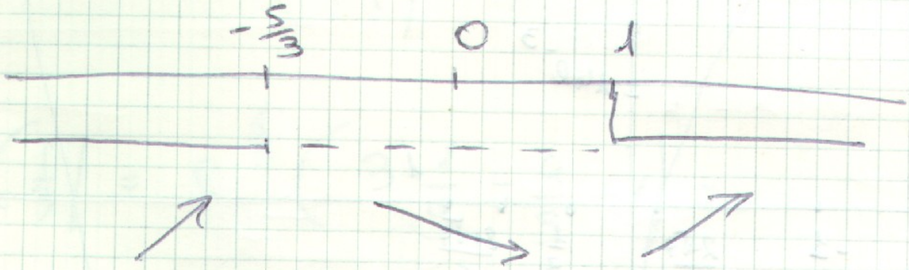
MAX E MIN

$$f'(x) = e^x(3x^2-4x-1) + e^x(6x-4) =$$

$$= e^x(3x^2+2x-5)$$

$$x = \frac{-2 \pm \sqrt{4+60}}{6} = \frac{-2 \pm 8}{6} < \begin{matrix} -\frac{5}{3} \\ 1 \end{matrix}$$

$f'(x) > 0$ $\quad x < -\frac{5}{3}, x > 1$



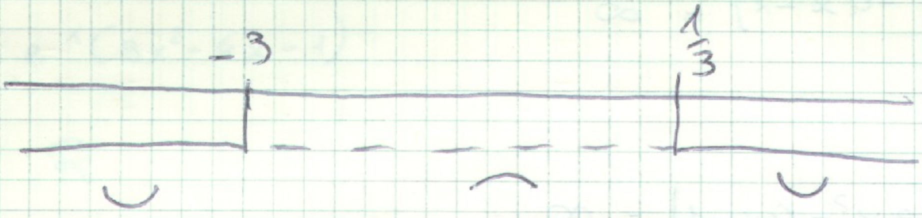
$x = -\frac{5}{3}$ max

$x = 1$ min

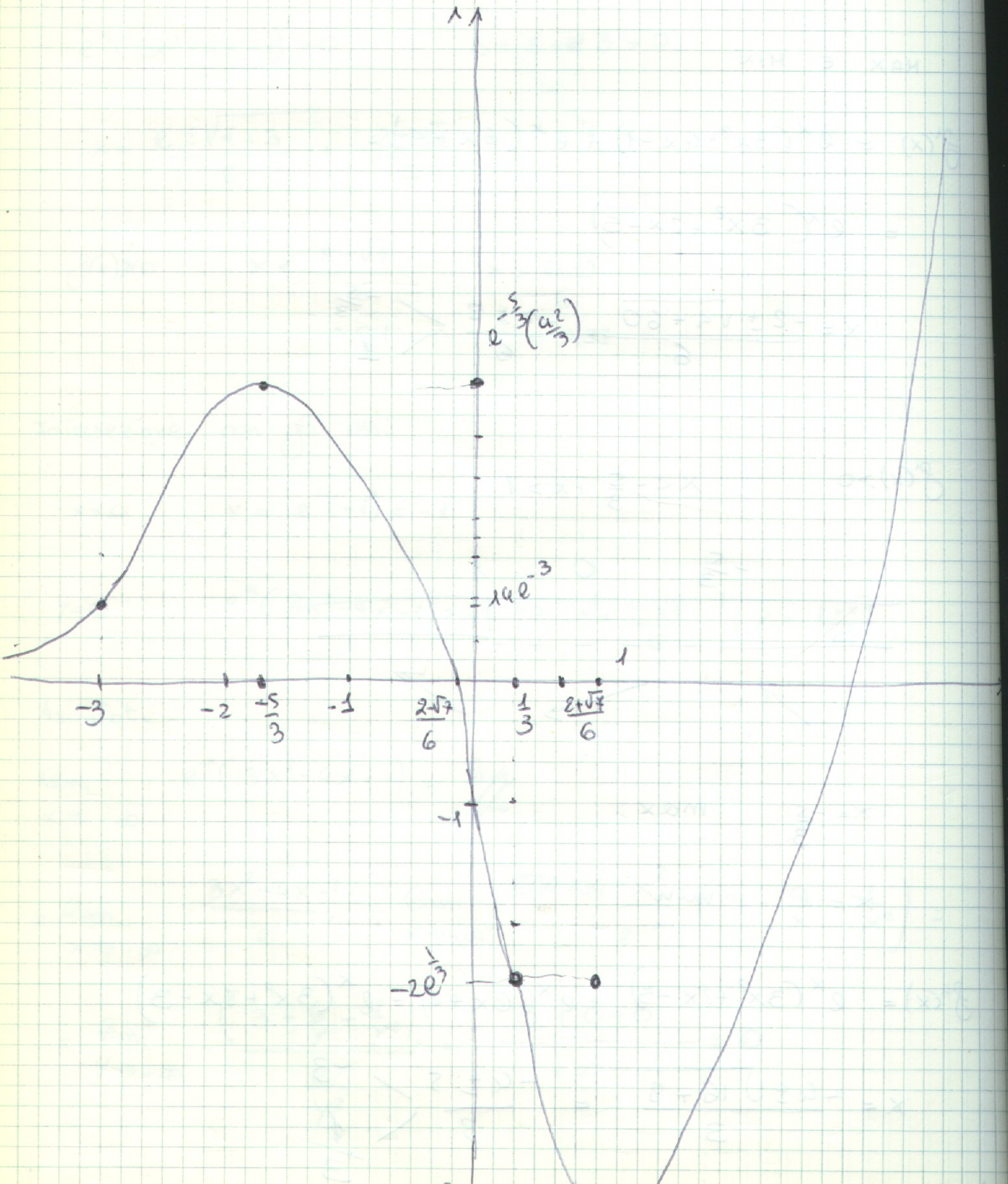
$$f''(x) = e^x(3x^2+2x-5) + e^x(6x+2) = e^x(3x^2+8x-3)$$

$$x = \frac{-4 \pm \sqrt{16+9}}{3} = \frac{-4 \pm 5}{3} < \begin{matrix} -3 \\ \frac{1}{3} \end{matrix}$$

$f''(x) > 0$ $\quad x < -3$ $\quad x > \frac{1}{3}$



-3
$\frac{1}{3}$



1
$e^{-\frac{5}{3}}\left(\frac{42}{3}\right)$
$14e^{-3}$
1
-3
-2
$-\frac{5}{3}$
-1
$\frac{2-\sqrt{7}}{6}$
$\frac{1}{3}$
$\frac{2+\sqrt{7}}{6}$
-1
$-2e^{\frac{1}{3}}$

$$f\left(-\frac{5}{3}\right) = e^{-\frac{5}{3}}\left(3\cdot\frac{25}{\cancel{9}_{3}} + 4\cdot\frac{5}{3} - 1\right) =$$

$$= e^{-\frac{5}{3}}\left(\frac{25}{3} + \frac{20}{3} - 1\right) =$$

$$= e^{-\frac{5}{3}}\left(\frac{42}{3}\right) =$$

$$f(-3) = e^{-3}(3\cdot 9 - 4\cdot 3 - 1) =$$

$$= e^{-3}(27 - 12 - 1) =$$

$$= e^{-3}(14) = 0{,}69$$

$$f\left(\frac{1}{3}\right) = e^{\frac{1}{3}}\left(3\cdot\frac{1}{9} - \frac{4}{3} - 1\right) =$$

$$= e^{\frac{1}{3}}\left(\frac{1}{3} - \frac{4}{3} - 1\right) = e^{\frac{1}{3}}(-2) =$$

$$f(1) = e(3 - 4 - 1) = -2e$$

22

$$f(x)=\frac{x^2}{16}\left(4\log^2\frac{x}{4}-10\log\frac{x}{4}+5\right)$$

CE: $\frac{x}{4}>0$ $\quad x\in\,]0,+\infty[$

Positività

$$4\log^2\frac{x}{4}-10\log\frac{x}{4}+5>0$$

$$4t^2-10t+5>0$$

$$t=\frac{5\pm\sqrt{25-20}}{4}=\frac{5\pm\sqrt{5}}{4}\begin{cases}\frac{5-\sqrt{5}}{4}\\ \frac{5+\sqrt{5}}{4}\end{cases}$$

$$\log x=\frac{5-\sqrt{5}}{4} \qquad x=e^{\frac{5-\sqrt{5}}{4}}$$

$$\log x=\frac{5+\sqrt{5}}{4} \qquad x=e^{\frac{5+\sqrt{5}}{4}}$$

$$f(x)>0 \qquad x<e^{\frac{5-\sqrt{5}}{4}} \qquad x>e^{\frac{5+\sqrt{5}}{4}}$$

Intersezioni con l'asse x per $x=e^{\frac{5-\sqrt{5}}{4}}$ e $x=e^{\frac{5+\sqrt{5}}{4}}$

Asint

$$\lim_{x\to 0}\frac{x^2}{16}\left(4\log^2\frac{x}{4}-10\log\frac{x}{4}+5\right)=0$$

$$\lim_{x\to+\infty}\frac{x^2}{16}\left(4\log^2\frac{x}{4}-10\log\frac{x}{4}+5\right)=$$

$$=\lim\frac{x^2}{16}4\log^2\frac{x}{4}\left(4-\frac{10}{\log\frac{x}{4}}+\frac{5}{\log^2 x}\right)=\infty\cdot\infty\left(4-0\right)=$$

$f(x) = \frac{x}{16}\left(4\log^2\frac{x}{4} - 10\log\frac{x}{4} + 5\right)$

CE: $\frac{x}{4} > 0$ $\quad x \in]0, +\infty[$

Positività

$4\log^2\frac{x}{4} - 10\log\frac{x}{4} + 5 > 0$

$4t^2 - 10t + 5 > 0$

$t = \frac{5 \pm \sqrt{25-20}}{4} = \frac{5 \pm \sqrt{5}}{4} \begin{cases} \frac{5-\sqrt{5}}{4} \\ \frac{5+\sqrt{5}}{4} \end{cases}$

$\log x = \frac{5-\sqrt{5}}{4} \qquad x = e^{\frac{5-\sqrt{5}}{4}}$

$\log x = \frac{5+\sqrt{5}}{4} \qquad x = e^{\frac{5+\sqrt{5}}{4}}$

$f(x) > 0 \qquad x < e^{\frac{5-\sqrt{5}}{4}} \qquad x > e^{\frac{5+\sqrt{5}}{4}}$

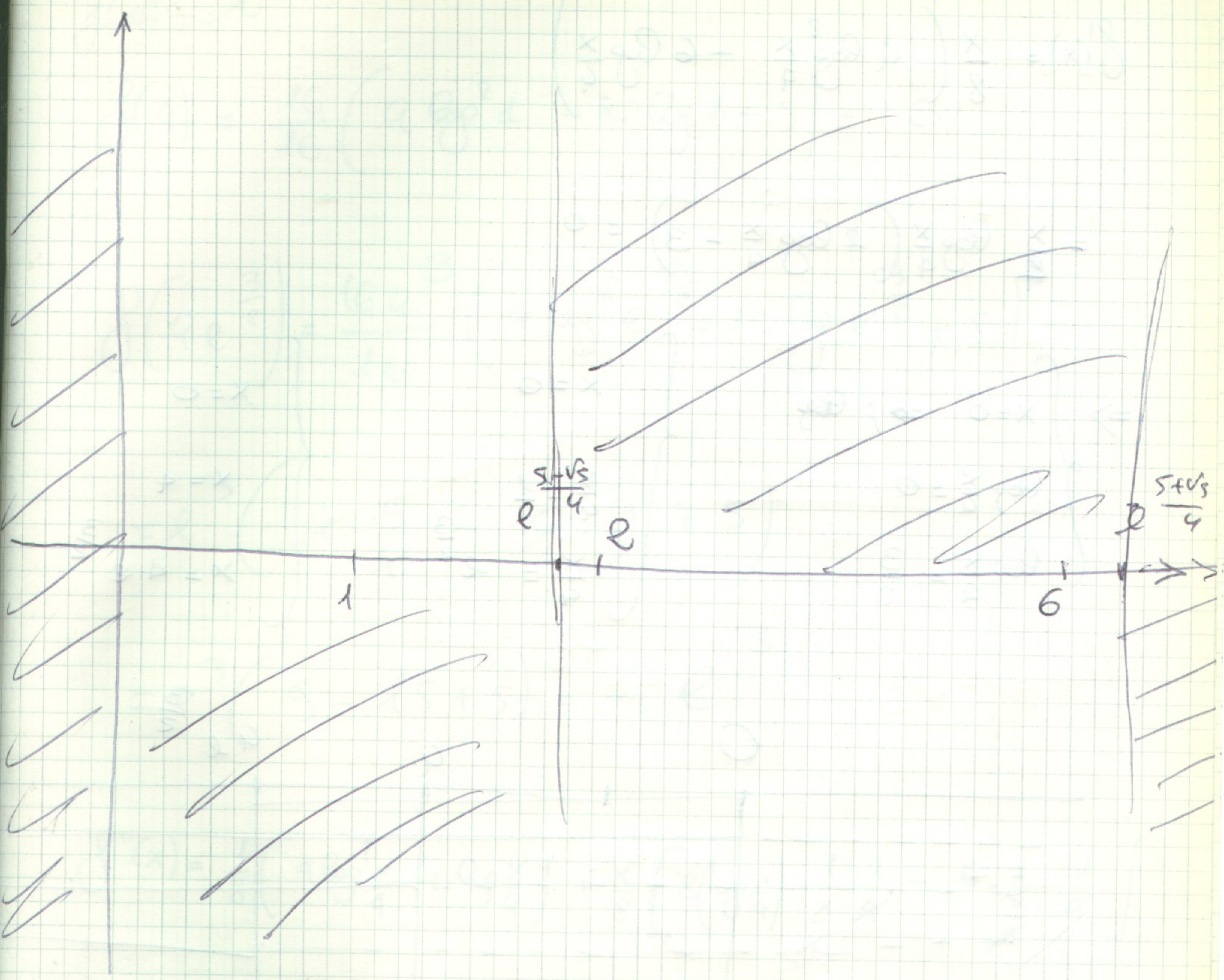
Intersezioni con l'asse x per $x = e^{\frac{5-\sqrt{5}}{4}}$ e $x = e^{\frac{5+\sqrt{5}}{4}}$

~~Asint~~

$\lim_{x \to 0} \frac{x^2}{16}\left(4\log^2\frac{x}{4} - 10\log\frac{x}{4} + 5\right) = 0$

$\lim_{x \to +\infty} \frac{x^2}{16}\left(4\log^2\frac{x}{4} - 10\log\frac{x}{4} + 5\right) =$

$= \lim_{x \to \infty} \frac{x^2}{16} 4\log^2\frac{x}{4}\left(4 - \frac{10}{\log\frac{x}{4}} + \frac{5}{\log^2\frac{x}{4}}\right) = \infty \cdot \infty (4-0)$



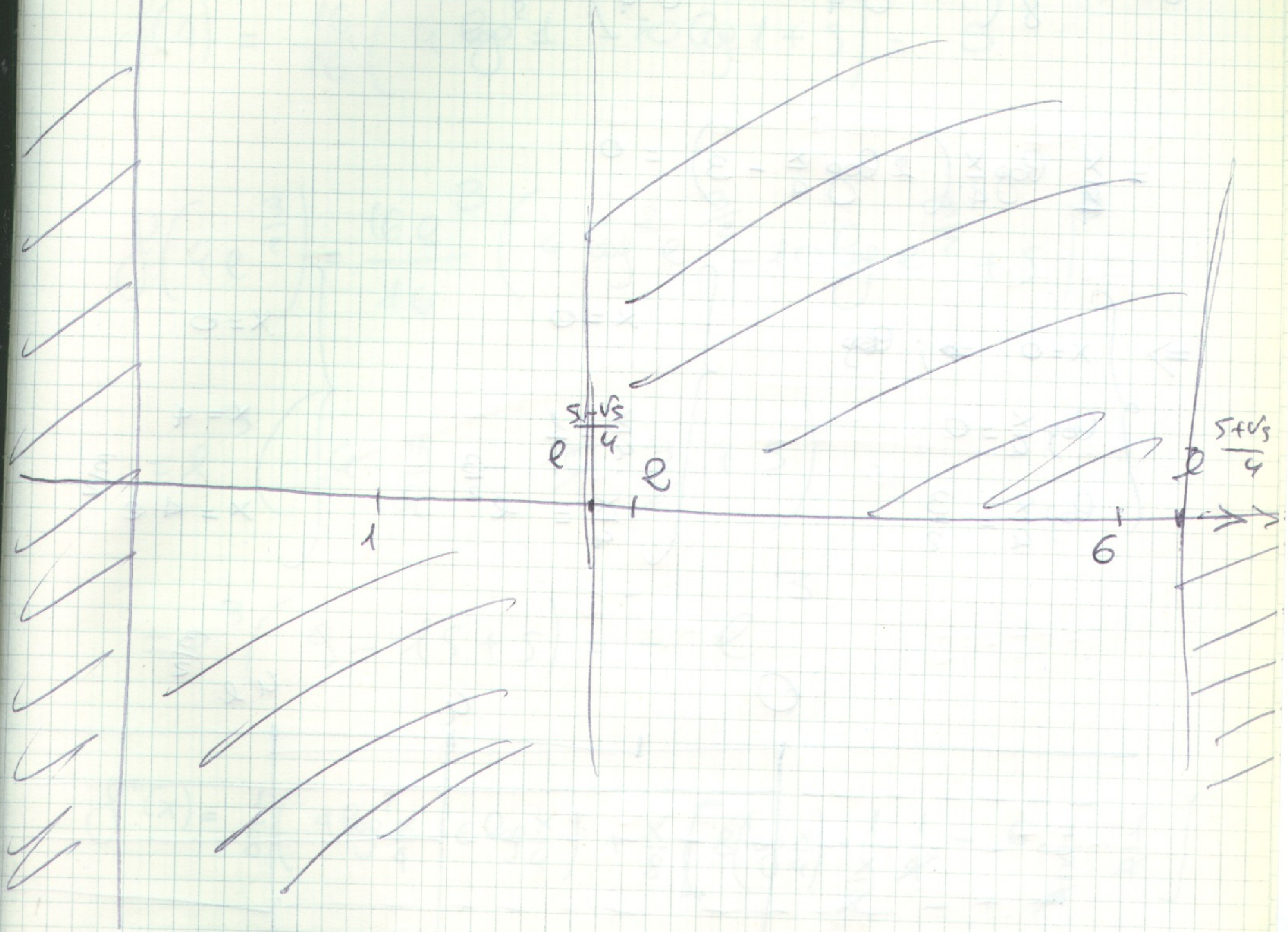
$$\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to+\infty}\frac{x}{16}\left(4\log^2\frac{x}{4}-10\log\frac{x}{4}+5\right)=+\infty$$

non ci sono asintoti orizzontali

MAX E MIN

$$f'(x)=\frac{x}{8}\left(4\log^2\frac{x}{4}-10\log\frac{x}{4}+5\right)+\frac{x^2}{16}\left(8\left(\log\frac{x}{4}\right)\cdot\frac{1}{\frac{x}{\cancel{4}}}\cdot\frac{1}{\cancel{4}}-10\cdot\frac{1}{\frac{x}{\cancel{4}}}\cdot\frac{1}{\cancel{4}}\right)=$$

$$=\frac{x}{8}\left(4\log^2\frac{x}{4}-10\log\frac{x}{4}+5\right)+\frac{x}{16}\left(8\log\frac{x}{4}-10\right)=$$



$$\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{x}{16}\left(4\log^2\frac{x}{4} - 10\log\frac{x}{4} + 5\right) = +\infty$$

non ci sono asintoti orizzontali

MAX E MIN

$$f'(x) = \frac{x}{8}\left(4\log^2\frac{x}{4} - 10\log\frac{x}{4} + 5\right) + \frac{x^2}{16}\left(8\left(\log\frac{x}{4}\right)\cdot\frac{1}{\frac{x}{\cancel{4}}}\cdot\frac{1}{4} - 10\cdot\frac{1}{\frac{x}{\cancel{4}}}\cdot\frac{1}{\cancel{4}}\right) =$$

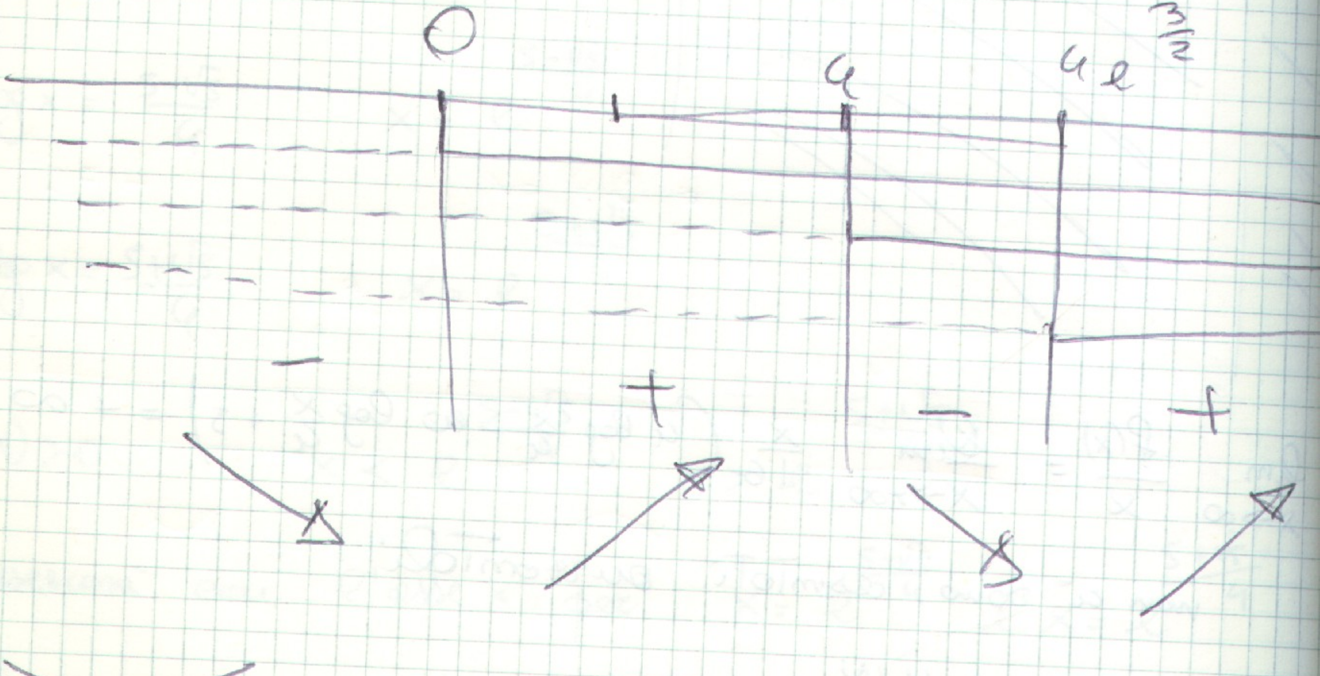
$$= \frac{x}{8}\left(4\log^2\frac{x}{4} - 10\log\frac{x}{4} + 5\right) + \frac{x}{16}\left(8\log\frac{x}{4} - 10\right) =$$

$$= \frac{x}{8}\left(4\log^2\frac{x}{4} - 10\log\frac{x}{4} + \cancel{5} + 4\log\frac{x}{4} - \cancel{5}\right)$$

$$f'(x) = \frac{x}{8}\left(4\log^2\frac{x}{4} - 6\log\frac{x}{4}\right)$$

$$= \frac{x}{4}\log\frac{x}{4}\left(2\log\frac{x}{4} - 3\right) = 0$$

$$\Rightarrow \begin{cases} x = 0 \quad \cancel{\Rightarrow ;\ \log} \\ \log\frac{x}{4} = 0 \\ \log\frac{x}{4} = \frac{3}{2} \end{cases} \qquad \begin{cases} x = 0 \\ \frac{x}{4} = 1 \\ \frac{x}{4} = e^{\frac{3}{2}} \end{cases} \qquad \begin{cases} x = 0 \\ x = 4 \\ x = 4e^{\frac{3}{2}} \end{cases}$$



~~x = 0 min~~

$x = 4$ max

$x = 4e^{\frac{3}{2}}$ min

$f(0) = 0$

$$f(4) = \frac{16}{16}\left(4\log^2 1 - 10\log 1 + 5\right) = 5$$

$$f\left(4e^{\frac{3}{2}}\right) = \frac{16e^3}{16}\left(4\log^2 e^{\frac{3}{2}} - 10\log e^{\frac{3}{2}} + 5\right) =$$

$$= e^3\left(\cancel{4}\cdot\frac{9}{\cancel{4}} - \cancel{10}\cdot\frac{3}{\cancel{2}} + 5\right) =$$

$$= e^3\left(9 - 15 + 5\right) = -e^3$$

$$f''(x) = \frac{1}{8}\left(4\log^2\frac{x}{4} - 6\log\frac{x}{4}\right) + \frac{x}{8}\left[8\left(\log\frac{x}{4}\right)\cdot\frac{1}{\frac{x}{\cancel{4}}}\cdot\frac{1}{\cancel{4}} - 6\cdot\frac{1}{\frac{x}{\cancel{4}}}\cdot\frac{1}{\cancel{4}}\right]$$

$$= \frac{1}{8}\left(4\log^2\frac{x}{4} - 6\log\frac{x}{4}\right) + \frac{1}{8}\left(8\log\frac{x}{4} - 6\right) =$$

$$= \frac{1}{8}\left(4\log^2\frac{x}{4} + 2\log\frac{x}{4} - 6\right)$$

$$4\log^2\frac{x}{4} - 5\log\frac{x}{4} - 6 = 0$$

$$4t^2 - 5t - 6 = 0$$

$$t = 5 \pm \sqrt{25 + 96} = 5 \pm 11$$

$\frac{7}{4}$

$$f(4) = \frac{16}{16}\left(4 \log^2 1 - 10 \log 1 + 5\right) = 5$$

$$f\left(4e^{\frac{3}{2}}\right) = \frac{16e^3}{16}\left(4 \log^2 e^{\frac{3}{2}} - 10 \log e^{\frac{3}{2}} + 5\right) =$$

$$= e^3\left(\cancel{4} \cdot \frac{9}{\cancel{4}} - \cancel{10} \cdot \frac{3}{\cancel{2}} + 5\right) =$$

$$= e^3\left(9 - 15 + 5\right) = -e^3$$

$$f''(x) = \frac{1}{8}\left(4 \log^2 \frac{x}{4} - 6 \log \frac{x}{4}\right) + \frac{x}{8}\left[8\left(\log \frac{x}{4}\right) \cdot \frac{1}{\frac{x}{\cancel{4}}} \cdot \frac{1}{\cancel{4}} - 6 \cdot \frac{1}{\frac{x}{\cancel{4}}} \cdot \frac{1}{\cancel{4}}\right]$$

$$= \frac{1}{8}\left(4 \log^2 \frac{x}{4} - 6 \log \frac{x}{4}\right) + \frac{1}{8}\left(8 \log \frac{x}{4} - 6\right) =$$

$$= \frac{1}{8}\left(4 \log^2 \frac{x}{4} + 2 \log \frac{x}{4} - 6\right)$$

$$4 \log^2 \frac{x}{4} - 5 \log \frac{x}{4} - 6 = 0$$

$$4t^2 - 5t - 6 = 0$$

$$t = \frac{5 \pm \sqrt{25 + 96}}{8} = \frac{5 \pm 11}{8} \begin{array}{l} \nearrow \frac{7}{4} \\ \searrow \frac{9}{2} \end{array}$$

~~$t = \frac{7}{4} \rightarrow x = e^{\frac{7}{4}}$~~

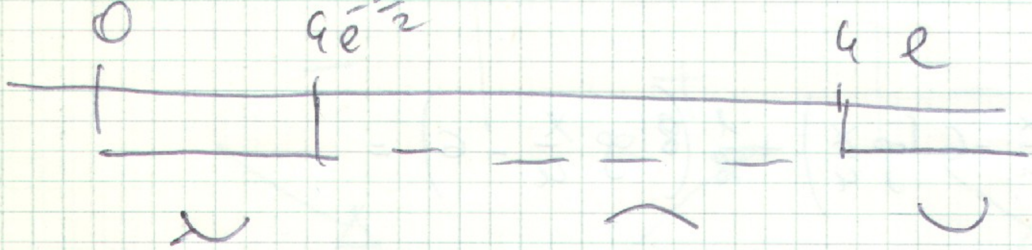
$$4 \log^2 \frac{x}{4} + 2 \log \frac{x}{4} - 6 = 0$$

$$4t^2 + 2t - 6 = 0$$

$$t = \frac{-1 \pm \sqrt{1+24}}{4} = \frac{-1 \pm 5}{4}$$

$t_1 = -\frac{6}{4} = -\frac{3}{2}$ $\qquad x = 4e^{-\frac{3}{2}}$

$t_2 = 1$ $\qquad x = 4e$



$$f(4e^{-\frac{3}{2}}) =$$

$$= \frac{16}{16} e^{-3}\left(4 \log^2 e^{-\frac{3}{2}} - 10 \log e^{-\frac{3}{2}} + 5\right) =$$

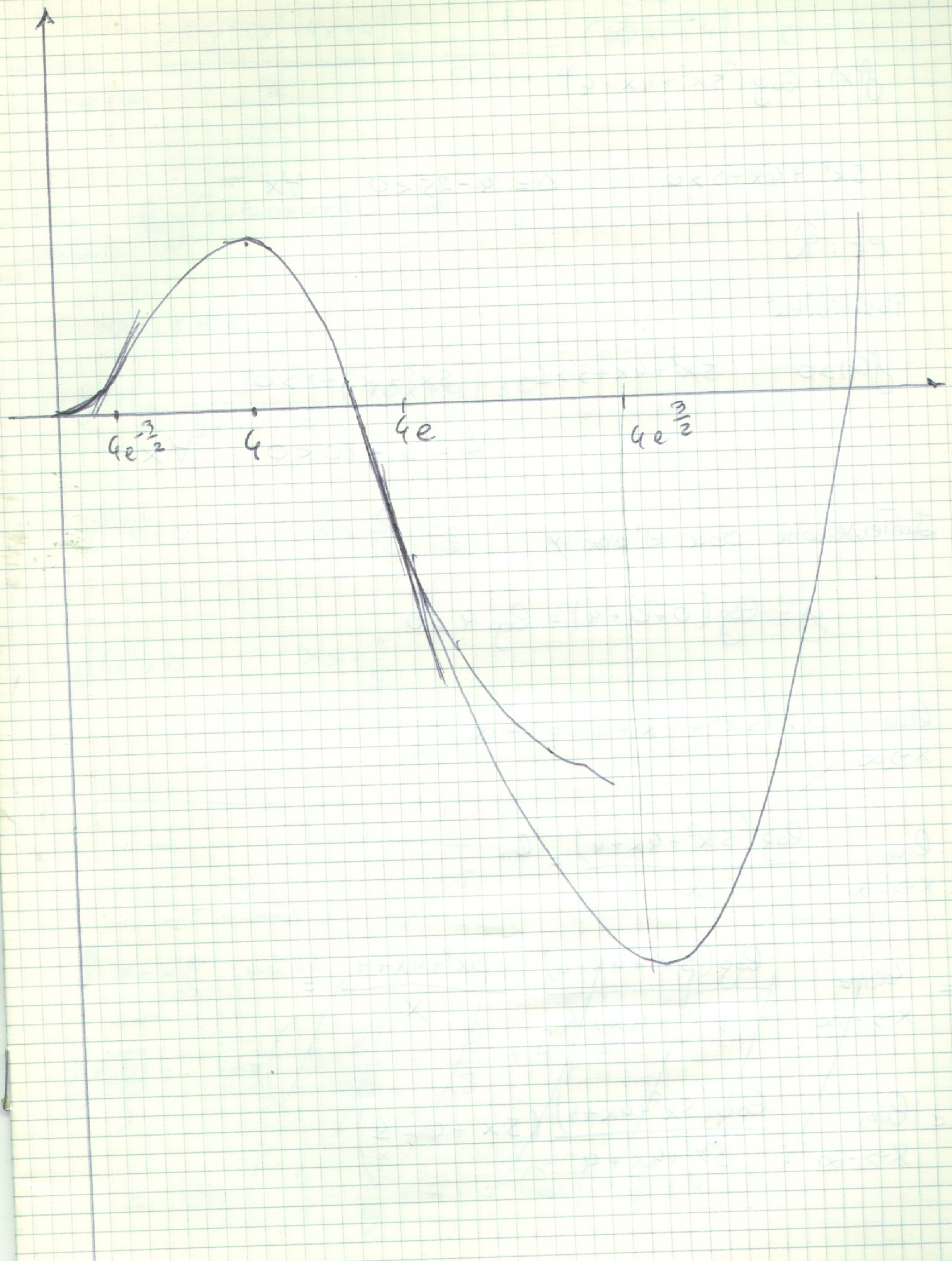
$$= e^{-3}\left(4 \cdot \frac{9}{4} + 10 \cdot \frac{3}{2} + 5\right) =$$

$$= e^{-3}\left(9 + 15 + 5\right) = 29 e^{-3}$$

$$f(4e) = \frac{16e^2}{16}\left(4 \log^2 \cancel{4} e - 10 \log e + 5\right) =$$

$$= e^2\left(4 - 10 + 5\right) =$$

$$= e^2(-1) = -e^2$$


$4e^{-\frac{3}{2}}$
4
$4e$
$4e^{\frac{3}{2}}$

23

$f(x) = \log(5x^2+4x+4)$

$5x^2+4x+5>0$ $\Delta = 4-25<0$ $\forall x$

CE: $\mathbb{R}$

Positività

$f(x)>0$ $5x^2+4x+3>1$ $5x^2+4x+2>0$

$\Delta = 4-10<0$ $\forall x$

Intersezione con l'asse y

$$y = \log(0+0+4) = \log 4 > 0$$

$$\lim_{x\to -\infty} \log(5x^2+4x+4) = +\infty$$

$$\lim_{x\to -\infty} \frac{\log(5x^2+4x+4)}{x} = \lim$$

~~$= \lim_{x\to -\infty} \frac{\log(5x^2+4x+4)}{5x^2+4x+4} \cdot \frac{5x^2+4x+4}{x} =$~~

~~$= \lim_{x\to -\infty} \frac{\log(5x^2+4x+4)}{5x^2+4x+4} \left(5x+4+\frac{4}{x}\right)$~~

$$= \lim_{x \to -\infty} \frac{D\log(5x^2+4x+4)}{1} = \frac{1}{5x^2+4x+4}(10x+4) = 0$$

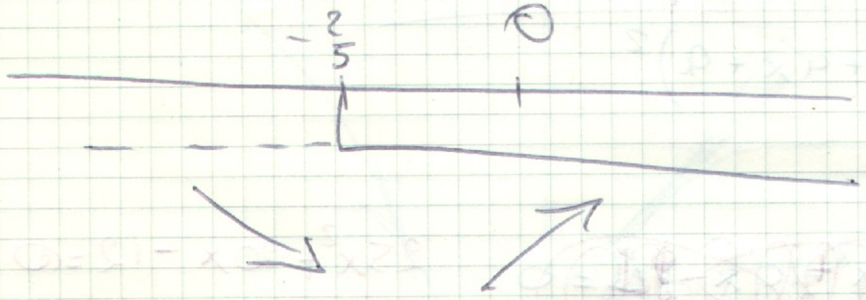
$$\lim_{x \to +\infty} \log(5x^2+4x+4) = +\infty$$

MAX E MIN

$$f'(x) = \frac{1}{5x^2+4x+4}(10x+4) = 0$$

$f'(x)=0$ $\quad x = -\frac{4}{10} = -\frac{2}{5}$

$f'(x)>0$ $\quad x > -\frac{2}{5}$



$x = -\frac{2}{5}$ min

$$f\left(-\frac{2}{5}\right) = \log\left(5\cdot\frac{4}{25} - \frac{8}{5} + 4\right) =$$

$$= \log\left(\frac{4}{5} - \frac{8}{5} + 4\right) = \log\left(-\frac{4}{5} + 4\right) =$$

$x \to -\infty \quad 1 \quad 5x^2+4x+4$

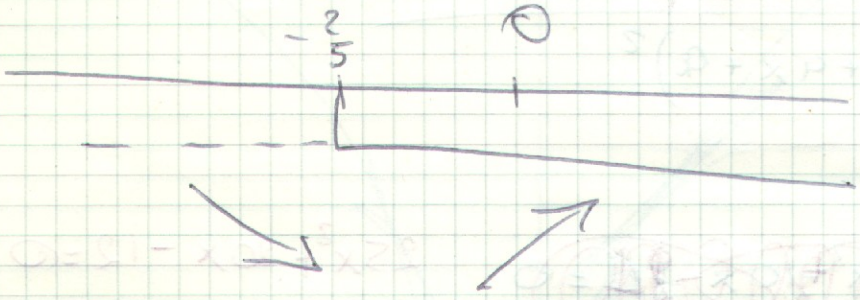
$$\lim_{x \to +\infty} \log(5x^2+4x+4) = +\infty$$

MAX E MIN

$$f'(x) = \frac{1}{5x^2+4x+4}(10x+4) = 0$$

$f'(x)=0 \qquad x = -\frac{4}{10} = -\frac{2}{5}$

$f'(x)>0 \qquad x > -\frac{2}{5}$



$f \; x=-\frac{2}{5}$ min

$$f\left(-\frac{2}{5}\right) = \log\left(5 \cdot \frac{4}{25} - \frac{8}{5} + 4\right) =$$

$$= \log\left(\frac{4}{5} - \frac{8}{5} + 4\right) = \log\left(-\frac{4}{5} + 4\right) =$$

$$= \log \frac{16}{5}$$

$$f''(x) = \left(\frac{10x+4}{5x^2+4x+4} \right)' =$$

$$= \frac{10(5x^2+4x+4) - (10x+4)(10x+4)}{(5x^2+4x+4)^2} =$$

$$= \frac{\cancel{50}\,10(5x^2+4x+4) - (100x^2+16+80x)}{(5x^2+4x+4)^2} =$$

$$= \frac{50x^2+40x+40-100x^2-16-80x}{(5x^2+4x+4)^2} =$$

$$= \frac{-50x^2-40x+24}{(5x^2+4x+4)^2}$$

$f''(x) = 0$ ~~$25x^2+10x-16=0$~~ $25x^2+20x-12=0$

~~$x = -10 \pm \sqrt{100+}$~~ $x = \frac{-10 \pm \sqrt{100+300}}{25}$

$$x_1 = \frac{-10-20}{25} = -\frac{30}{25} = -\frac{6}{5}$$

$x_2 = \; -10+20 \quad 10 \quad 2$

$$\left(\frac{}{5x^2+4x+4} \right) =$$

$$= \frac{10(5x^2+4x+4)-(10x+4)(10x+4)}{(5x^2+4x+4)^2} =$$

$$= \frac{\cancel{50}\,10(5x^2+4x+4)-(100x^2+16+80x)}{(5x^2+4x+4)^2} =$$

$$= \frac{50x^2+40x+40-100x^2-16-80x}{(5x^2+4x+4)^2} =$$

$$= \frac{-50x^2+40x+24}{(5x^2+4x+4)^2}$$

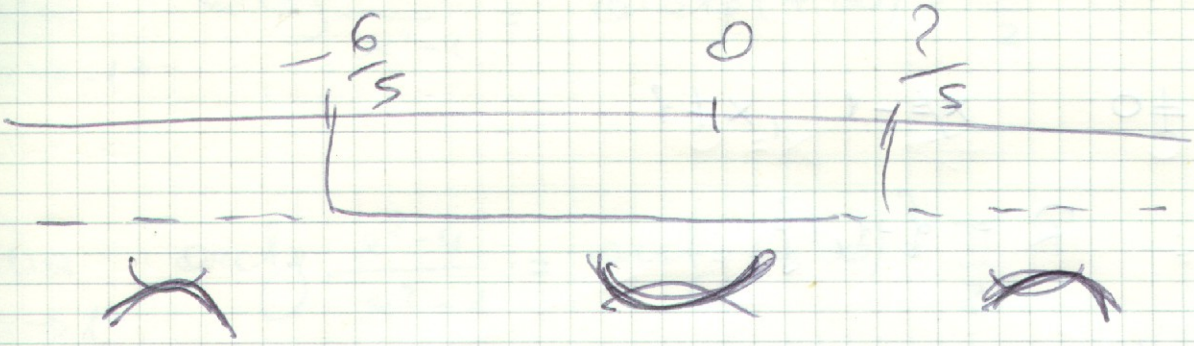
$f''(x) = 0$ ~~$25x^2+10x-16=0$~~ $25x^2+20x-12=0$

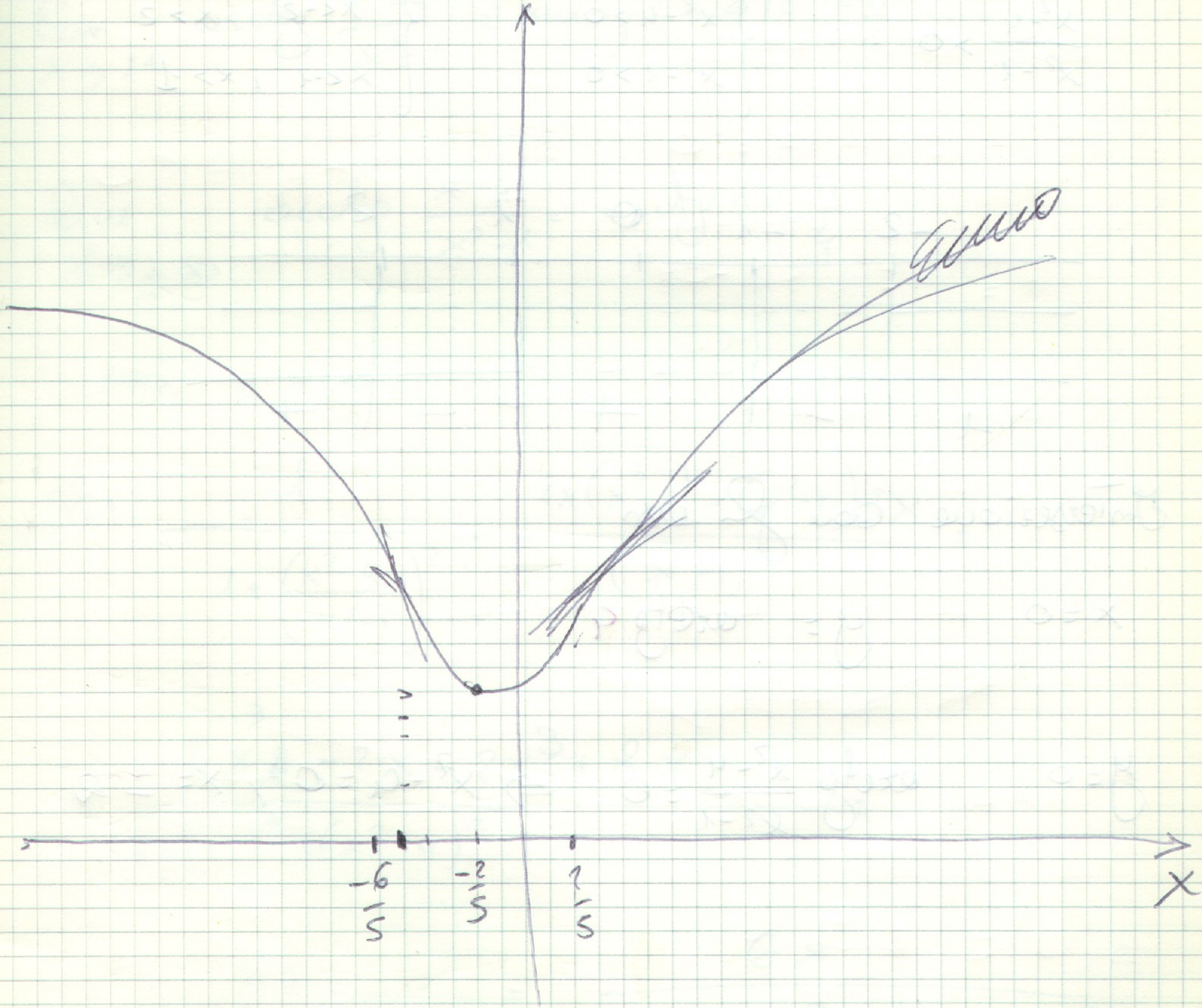
$x =$ ~~$-10 \pm \sqrt{100+}$~~ $x = \frac{-10 \pm \sqrt{100+300}}{25}$

$$x_1 = \frac{-10-20}{25} = -\frac{30}{25} = -\frac{6}{5}$$

$$x_2 = \frac{-10+20}{25} = \frac{10}{25} = \frac{2}{5}$$

$f''(x) > 0 \qquad -\frac{6}{5} < x < \frac{2}{5}$





23

$f(x) = \log(5x^2+4x+4)$

$5x^2+4x+5>0 \qquad \Delta = 4-25<0 \qquad \forall x$

$C.E.: \mathbb{R}$

Positività

$f(x)>0 \qquad 5x^2+4x+3>1 \qquad 5x^2+4x+2>0$

$\Delta = 4-10<0 \qquad \forall x$

Intersezione con l'asse y

$$y = \log(0+0+4) = \log 4 > 0$$

$$\lim_{x\to\pm\infty} \log(5x^2+4x+4) = +\infty$$

$$\lim_{x\to-\infty} \frac{\log(5x^2+4x+4)}{x} = \lim$$

~~$= \lim_{x\to-\infty} \frac{\log(5x^2+4x+4)}{5x^2+4x+4} \cdot \frac{5x^2+4x+4}{x} =$~~

~~$= \lim_{x\to-\infty} \frac{\log(5x^2+4x+4)}{5x^2+4x+4} \left(5x+4+\frac{4}{x}\right)$~~

$$= \lim_{x \to -\infty} \frac{D \log(5x^2+4x+4)}{1} = \frac{1}{5x^2+4x+4}(10x+4) = 0$$

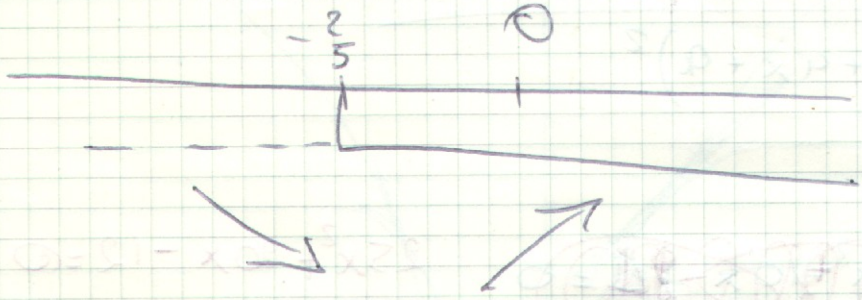
$$\lim_{x \to +\infty} \log(5x^2+4x+4) = +\infty$$

MAX E MIN

$$f'(x) = \frac{1}{5x^2+4x+4}(10x+4) = 0$$

$f'(x) = 0$ $\qquad x = -\frac{4}{10} = -\frac{2}{5}$

$f'(x) > 0$ $\qquad x > -\frac{2}{5}$



$x = -\frac{2}{5}$ min

$$f\left(\frac{-2}{5}\right) = \log\left(5 \cdot \frac{4}{25} - \frac{8}{5} + 4\right) =$$

$$= \log\left(\frac{4}{5} - \frac{8}{5} + 4\right) = \log\left(-\frac{4}{5} + 4\right) =$$

$x \to -\infty$ 1 $5x^2+4x+4$

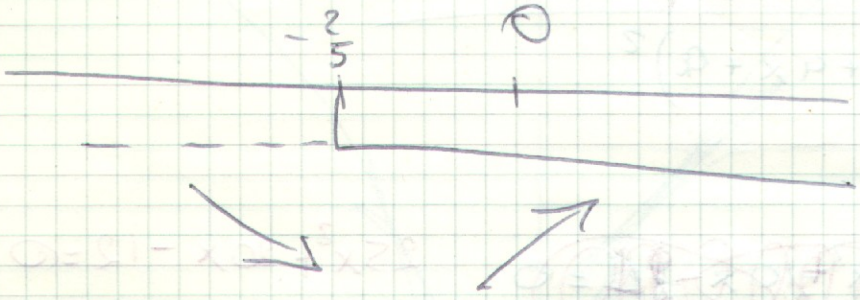
$$\lim_{x \to +\infty} \log(5x^2+4x+4) = +\infty$$

MAX E MIN

$$f'(x) = \frac{1}{5x^2+4x+4}(10x+4) = 0$$

$f'(x)=0$ $x = -\frac{4}{10} = -\frac{2}{5}$

$f'(x)>0$ $x > -\frac{2}{5}$



f $x = -\frac{2}{5}$ min

$$f\left(-\frac{2}{5}\right) = \log\left(5 \cdot \frac{4}{25} - \frac{8}{5} + 4\right) =$$

$$= \log\left(\frac{4}{5} - \frac{8}{5} + 4\right) = \log\left(-\frac{4}{5} + 4\right) =$$

$$= \log \frac{16}{5}$$

$$f''(x) = \left(\frac{10x+4}{5x^2+4x+4} \right)' =$$

$$= \frac{10(5x^2+4x+4) - (10x+4)(10x+4)}{(5x^2+4x+4)^2} =$$

$$= \frac{\cancel{50}\,10(5x^2+4x+4) - (100x^2+16+80x)}{(5x^2+4x+4)^2} =$$

$$= \frac{50x^2+40x+40-100x^2-16-80x}{(5x^2+4x+4)^2} =$$

$$= \frac{-50x^2-40x+24}{(5x^2+4x+4)^2}$$

$f''(x) = 0$ ~~$25x^2+10x-16=0$~~ $25x^2+20x-12=0$

~~$x = -10 \pm \sqrt{100+}$~~ $x = \frac{-10 \pm \sqrt{100+300}}{25}$

$$x_1 = \frac{-10-20}{25} = -\frac{30}{25} = -\frac{6}{5}$$

$x_2 = -10+20 \qquad 10 \qquad 2$

$$\left(\frac{\quad}{5x^2+4x+4} \right) =$$

$$= \frac{10(5x^2+4x+4) - (10x+4)(10x+4)}{(5x^2+4x+4)^2} =$$

$$= \frac{\cancel{50}\,10(5x^2+4x+4) - (100x^2+16+80x)}{(5x^2+4x+4)^2} =$$

$$= \frac{50x^2+40x+40-100x^2-16-80x}{(5x^2+4x+4)^2} =$$

$$= \frac{-50x^2-40x+24}{(5x^2+4x+4)^2}$$

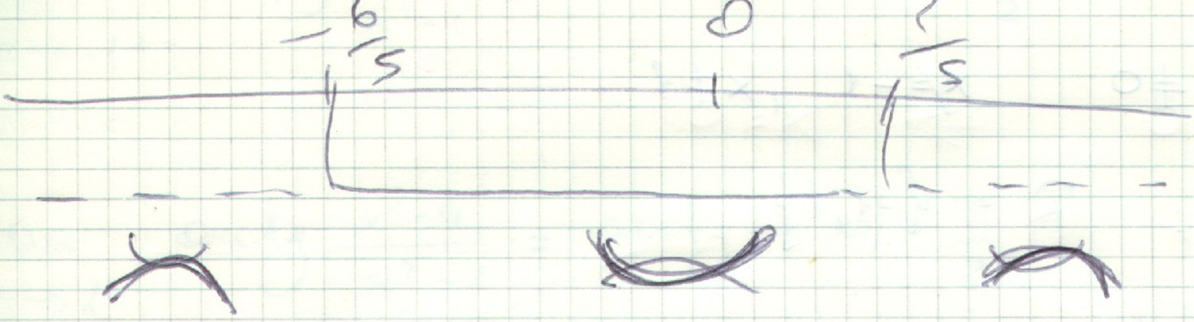
$f''(x) = 0$ ~~$25x^2+10x-16=0$~~ $25x^2+20x-12=0$

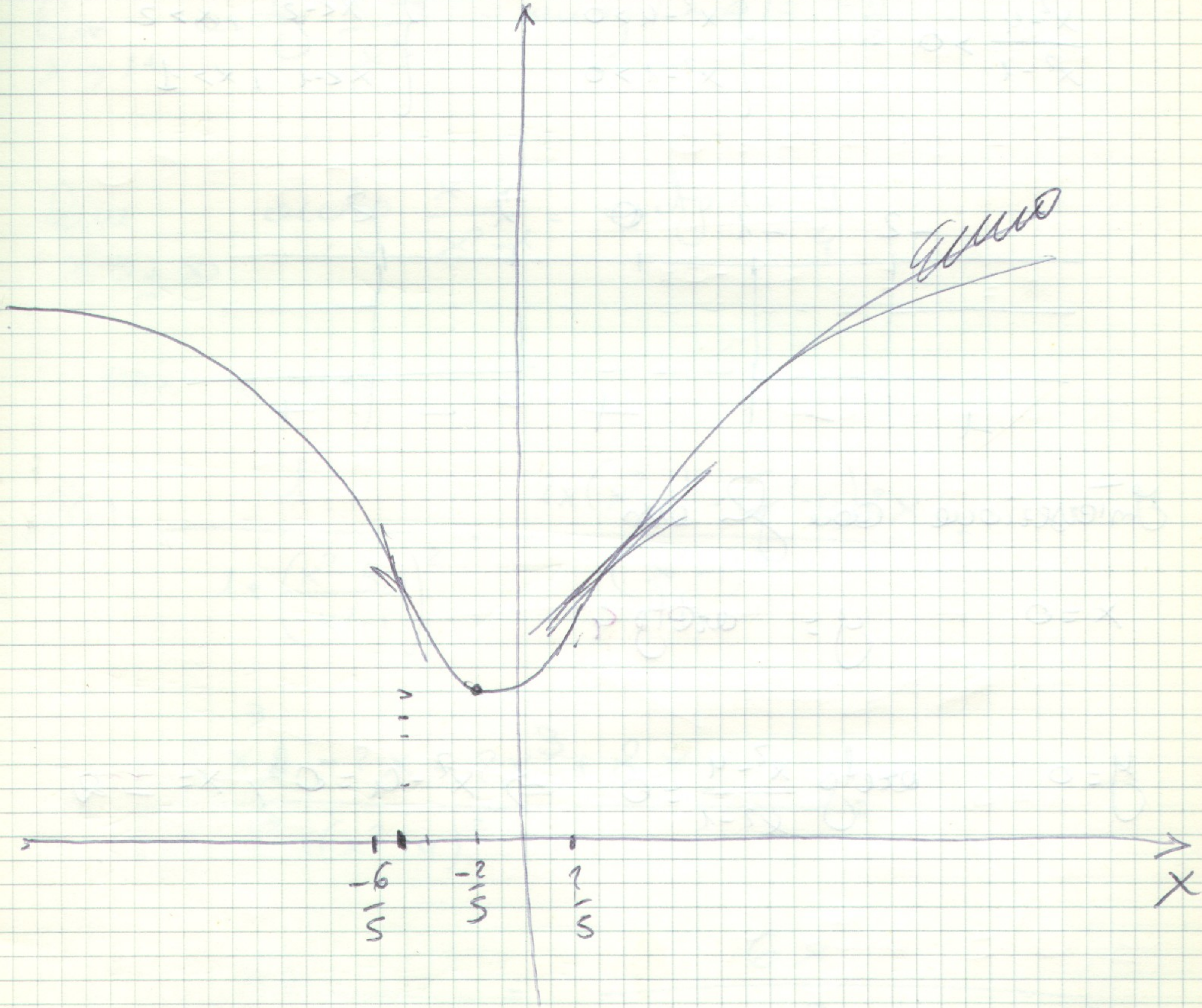
$x =$ ~~$-10 \pm \sqrt{100+}$~~ $x = \frac{-10 \pm \sqrt{100+300}}{25}$

$$x_1 = \frac{-10-20}{25} = -\frac{30}{25} = -\frac{6}{5}$$

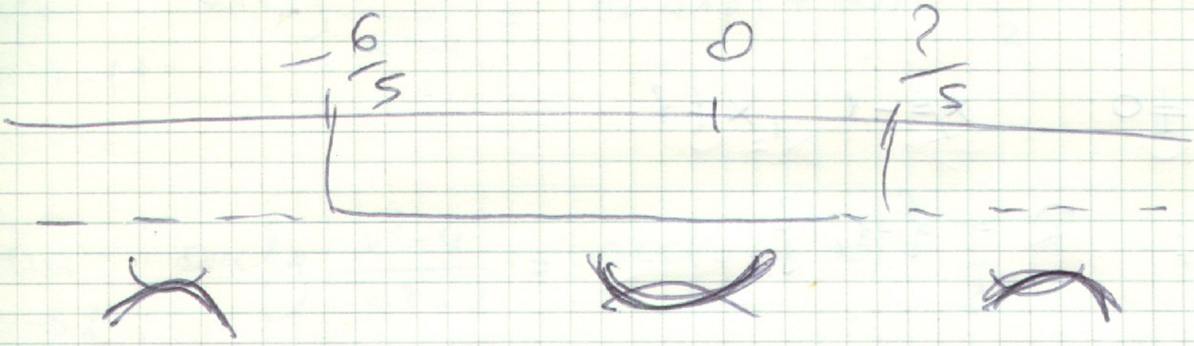
$$x_2 = \frac{-10+20}{25} = \frac{10}{25} = \frac{2}{5}$$

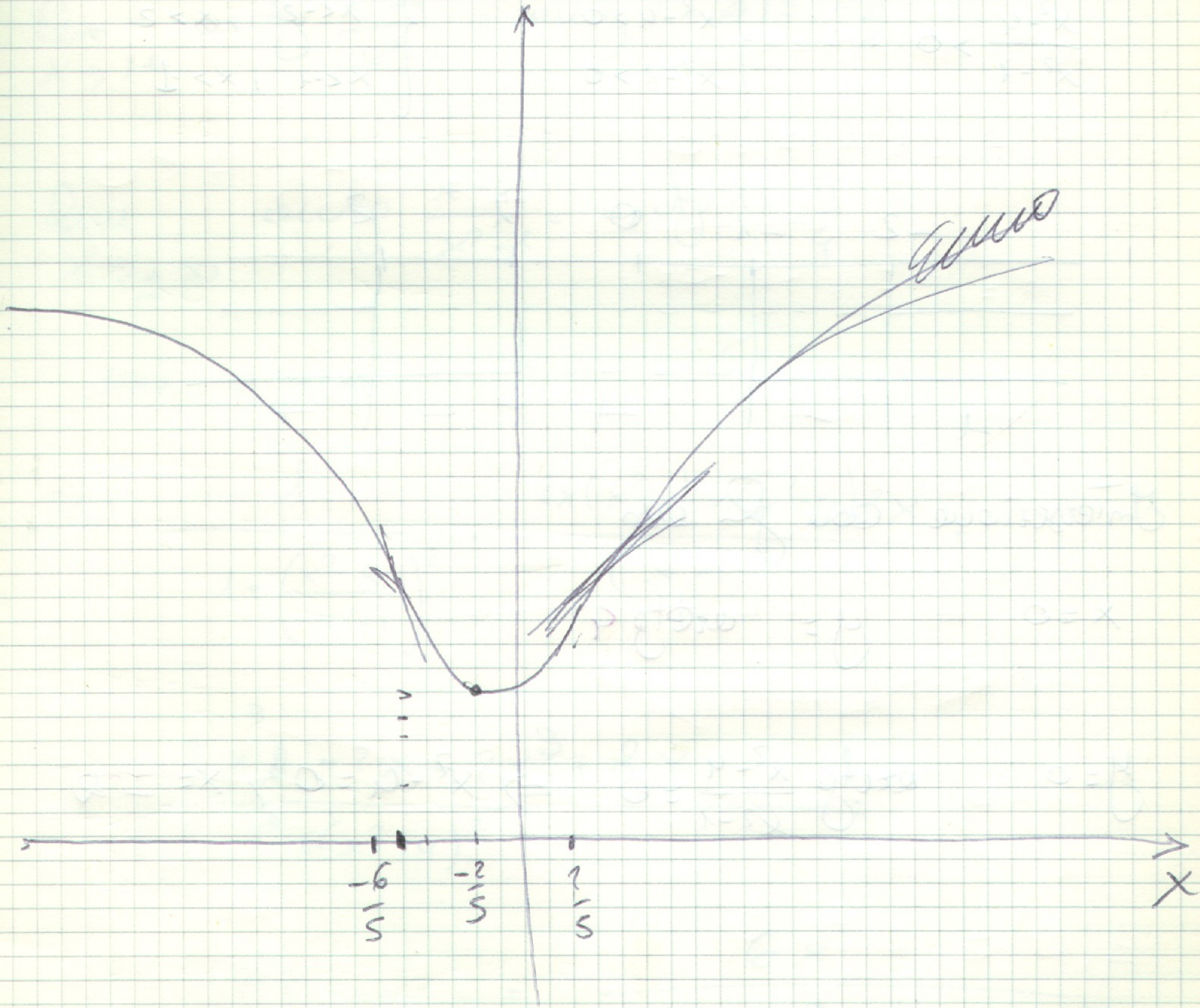
$f''(x) > 0$ $\quad -\frac{6}{5} < x < \frac{2}{5}$





$f''(x) > 0 \quad \Leftrightarrow \quad -\frac{6}{5} < x < \frac{2}{5}$





24

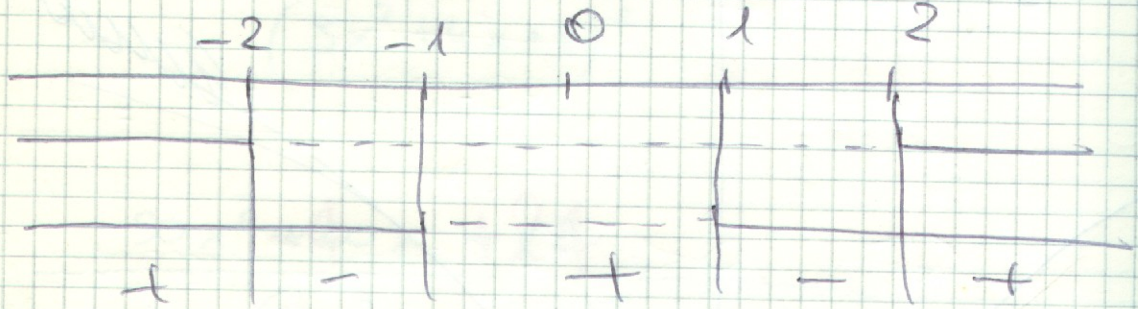
$f(x) = \operatorname{arctg} \frac{x^2-4}{x^2-1}$

$x^2-1 \neq 0$ $\quad x \neq -1 \ , x \neq 1$

Ce: $\mathbb{R} - \{-1,1\}$

Positività

$\frac{x^2-4}{x^2-1} > 0 \qquad \begin{cases} x^2-4>0 \\ x^2-1>0 \end{cases} \qquad \begin{cases} x<-2 \ , x>2 \\ x<-1 \ , x>1 \end{cases}$



Intersezione con gli assi

$x=0 \qquad y = \operatorname{arctg} 4$

$y=0 \qquad \operatorname{arctg} \frac{x^2-4}{x^2-1} = 0 \Rightarrow x^2-4=0 \ , x = \pm 2$

$$\lim_{x\to -1^-} \operatorname{arctg} \frac{x^2-4}{x^2-1} = \cancel{\operatorname{arctg}}\ \operatorname{arctg}(-\infty) = -\frac{\pi}{2}$$

$$\lim_{x\to -1^+} \operatorname{arctg} \frac{x^2-4}{x^2-1} = \operatorname{arctg}(+\infty) = \frac{\pi}{2}$$

$$\lim_{x\to 1^-} \operatorname{arctg} \frac{x^2-4}{x^2-1} = \operatorname{arctg}(+\infty) = \frac{\pi}{2}$$

$$\lim_{x\to 1^+} \operatorname{arctg} \frac{x^2-4}{x^2-1} = \operatorname{arctg}(-\infty) = -\frac{\pi}{2}$$

$$\lim_{x\to \pm\infty} \operatorname{arctg} \frac{x^2-4}{x^2-1} = \operatorname{arctg} 1 = \frac{\pi}{4}$$

MAX e MIN

$$f'(x) = \frac{1}{1+\left(\frac{x^2-4}{x^2-1}\right)^2} \cdot \frac{2x(x^2-1)-(x^2-4)2x}{(x^2-1)^2} =$$

$$= \frac{2x^{\overset{3}{\cancel{1}}} - 2x - 2x^3 + 8x}{(x^2-1)^2+(x^2-4)^2} =$$

$$= \frac{6x}{(x^2-1)^2+(x^2-4)^2} = \frac{6x}{x^4+1-2x^2+x^4+16-8x^2}$$

$x \to 1^-$

$$\lim_{x \to -1^+} \operatorname{arctg} \frac{x^2-4}{x^2-1} = \operatorname{arctg}(+\infty) = \frac{\pi}{2}$$

$$\lim_{x \to 1^-} \operatorname{arctg} \frac{x^2-4}{x^2-1} = \operatorname{arctg}(+\infty) = \frac{\pi}{2}$$

$$\lim_{x \to 1^+} \operatorname{arctg} \frac{x^2-4}{x^2-1} = \operatorname{arctg}(-\infty) = -\frac{\pi}{2}$$

$$\lim_{x \to \pm\infty} \operatorname{arctg} \frac{x^2-4}{x^2-1} = \operatorname{arctg} 1 = \frac{\pi}{4}$$

MAX e MIN

$$f'(x) = \frac{1}{1+\left(\frac{x^2-4}{x^2-1}\right)^2} \cdot \frac{2x(x^2-1)-(x^2-4)2x}{(x^2-1)^2} =$$

$$= \frac{2x^{3} - 2x - 2x^3 + 8x}{(x^2-1)^2+(x^2-4)^2} =$$

$$= \frac{6x}{(x^2-1)^2+(x^2-4)^2} = \frac{6x}{x^4+1-2x^2+x^4+16-8x^2}$$

$$f'(x) = \frac{6x}{2x^4 - 10x^2 + 17}$$

$f'(x) = 0 \quad \Rightarrow \quad x = 0$

f

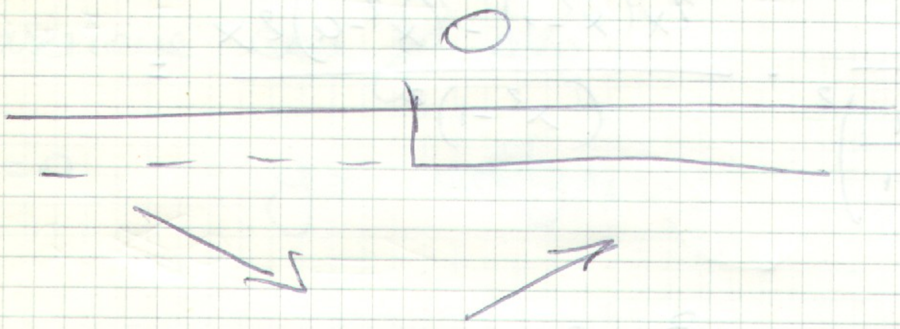
~~2x~~ $f'(x) > 0$

$2x^4 - 10x^2 + 17 = 0$

$2t^2 - 10t + 17 = 0$ $\qquad \Delta = 25 - 2 \cdot 17 < 0$

~~$t = \frac{5 \pm \sqrt{25 - 34}}{4}$~~

il denominatore è > 0 $\forall x$



$x = 0$ min

$f(0) = \text{arctg } 4$

$$f''(x) = \frac{6(2x^4-10x^2+17)-6x(8x^3-20x)}{(2x^4-10x^2+17)^2} =$$

$$= \frac{12x^4-60x^2+102-48x^4+120x^2}{(2x^4-10x^2+17)^2} =$$

$$= \frac{-36x^4+60x^2+102}{(2x^4-10x^2+17)^2}$$

$f''(x)=0 \qquad 18x^4-30x^2-51=0$

$$18t^2-30t-51=0$$

$$t = \frac{15 \pm \sqrt{225+918}}{18} =$$

$$= \frac{15 \pm 3\sqrt{127}}{18} = \frac{5 \pm \sqrt{127}}{6}$$

$f''(x) \qquad x = \pm\sqrt{5 \pm \sqrt{127}}$

$$f''(x) = \frac{6(2x^4-10x+17)-6x(8x-20x)}{(2x^4-10x^2+17)^2} =$$

$$= \frac{12x^4-60x^2+102-48x^4+120x^2}{(2x^4-10x^2+17)^2} =$$

$$= \frac{-36x^4+60x^2+102}{(2x^4-10x^2+17)^2}$$

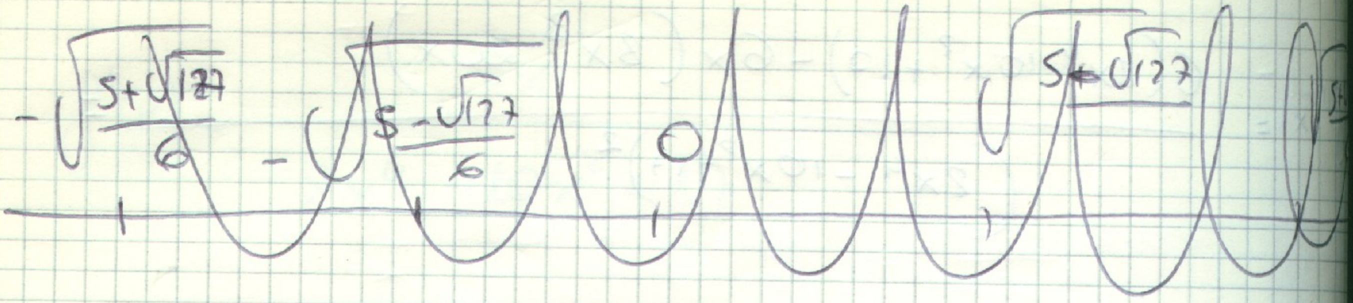
$$f''(x)=0 \qquad 18x^4-30x^2-51=0$$

$$18t^2-30t-51=0$$

$$t = \frac{15 \pm \sqrt{225+918}}{18} =$$

$$= \frac{15 \pm 3\sqrt{127}}{18} = \frac{5 \pm \sqrt{127}}{6}$$

$$\cancel{f''(x)} \qquad \cancel{x = \pm\sqrt{\frac{5 \pm \sqrt{127}}{6}}}$$



$x = \pm\sqrt{t_1}$ $\qquad$ $x = \pm\sqrt{t_2}$

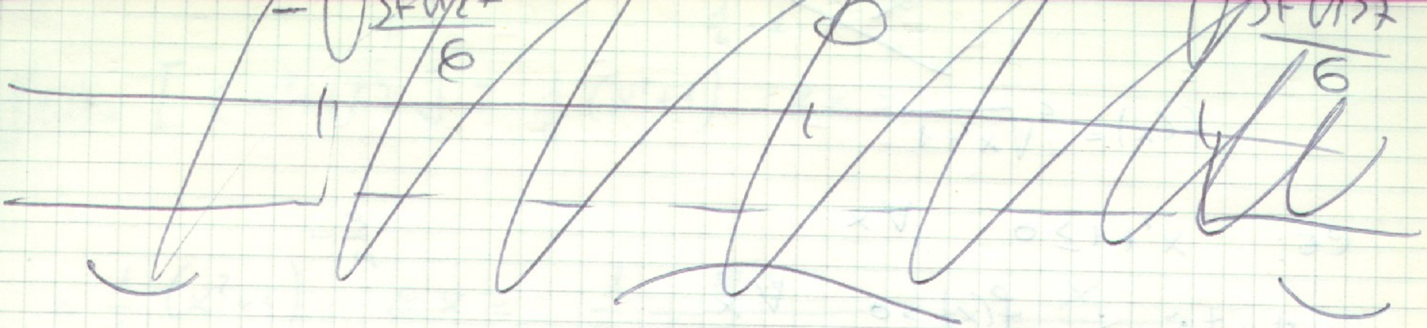
$t_1 = \frac{5+\sqrt{127}}{6} > 0$

$t_2 = \frac{5-\sqrt{127}}{6} < 0 \Rightarrow$ non da luogo a radici x

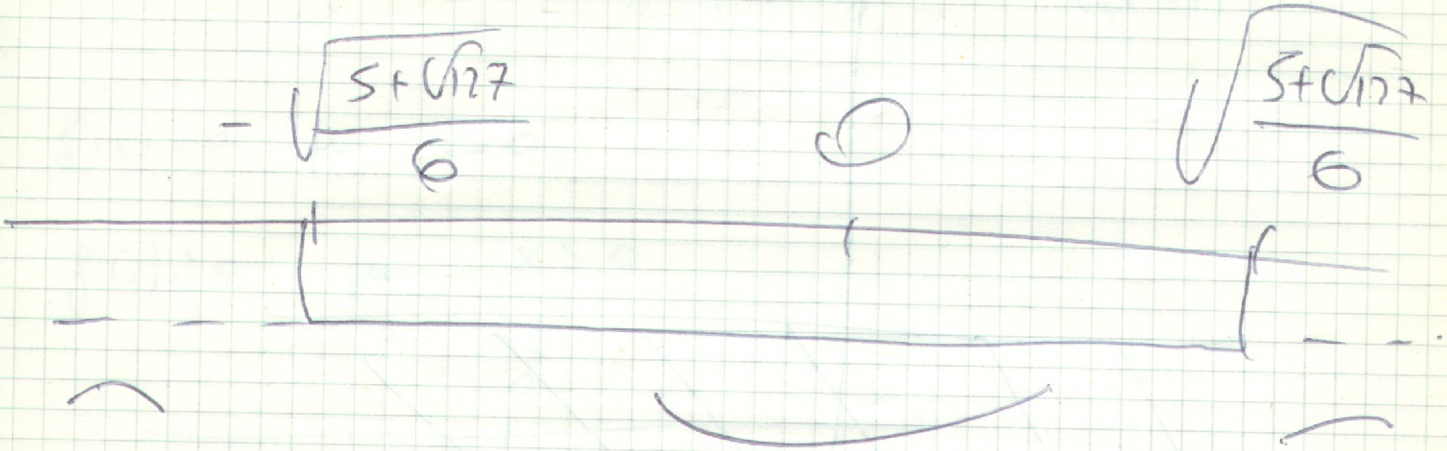
$x_1 = -\sqrt{\frac{5+\sqrt{127}}{6}}$ $\qquad$ $x_2 = +\sqrt{\frac{5+\sqrt{127}}{6}}$

$f''(x) > 0 \Longrightarrow$

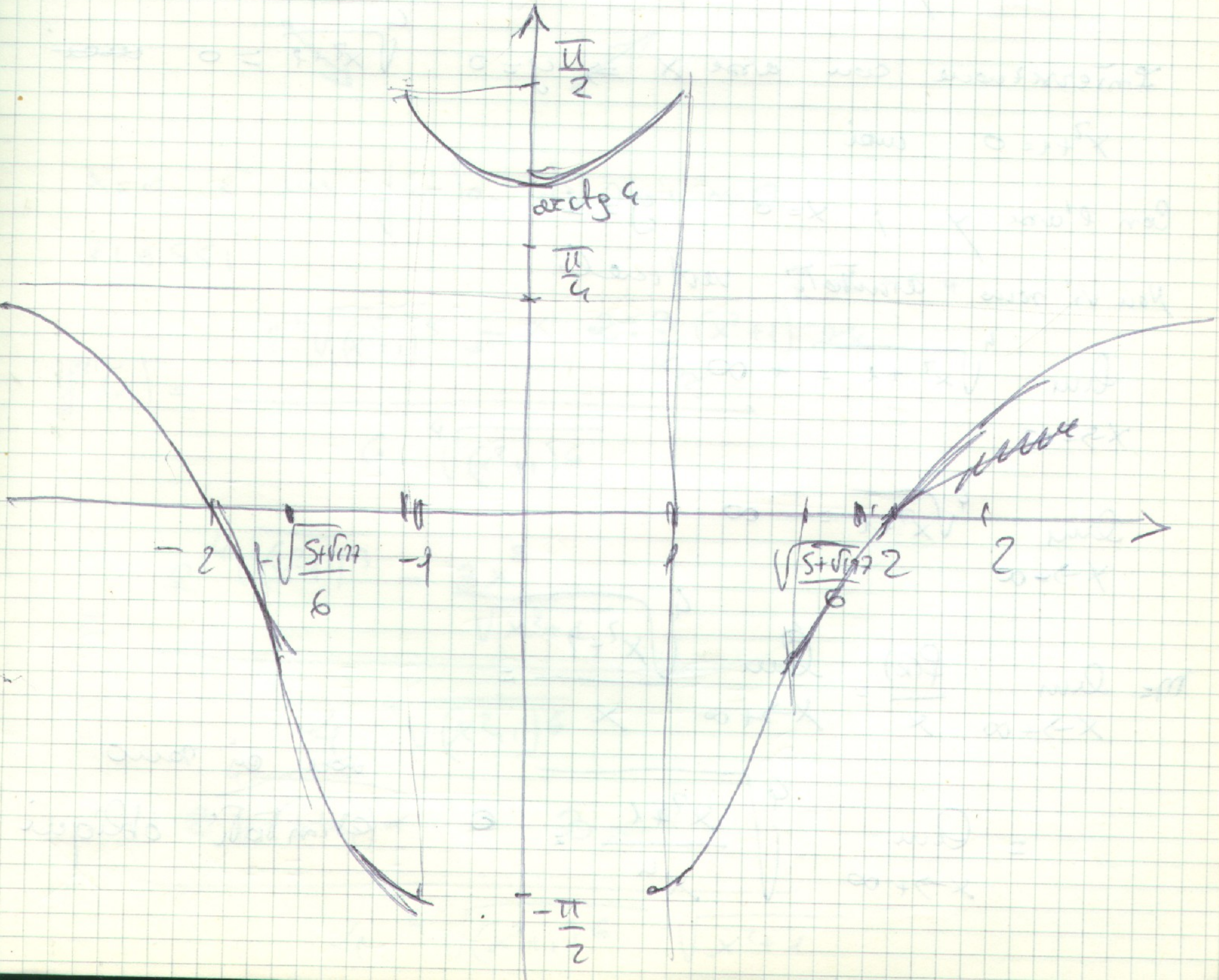
$x < -\sqrt{\frac{5+\sqrt{127}}{6}}$ $\vee$ $x > \sqrt{\frac{5+\sqrt{127}}{6}}$



$-\sqrt{\frac{5+\sqrt{127}}{6}}$
0
$\sqrt{\frac{5+\sqrt{127}}{6}}$



$-\sqrt{\frac{5+\sqrt{127}}{6}}$
0
$\sqrt{\frac{5+\sqrt{127}}{6}}$



$\frac{\pi}{2}$
arctg 4
$\frac{\pi}{4}$
-2
$-\sqrt{\frac{5+\sqrt{127}}{6}}$
-1
1
$\sqrt{\frac{5+\sqrt{127}}{6}}$
2
2
$-\frac{\pi}{2}$

25

$f(x) = \sqrt[4]{x^2+1}$

C.E.: $x^2+1 \geq 0 \quad \forall x$

Positività: $f(x) > 0 \quad \forall x$

Intersezione con asse x $\Rightarrow y=0 \quad \sqrt[4]{x^2+1} = 0$ ~~mai~~

$x^2+1=0$ mai

Con l'asse y ; $x=0 \quad y=1$

Non vi sono asintoti verticali

$$\lim_{x \to +\infty} \sqrt[4]{x^2+1} = +\infty$$

$$\lim_{x \to -\infty} \sqrt[4]{x^2+1} = -\infty$$

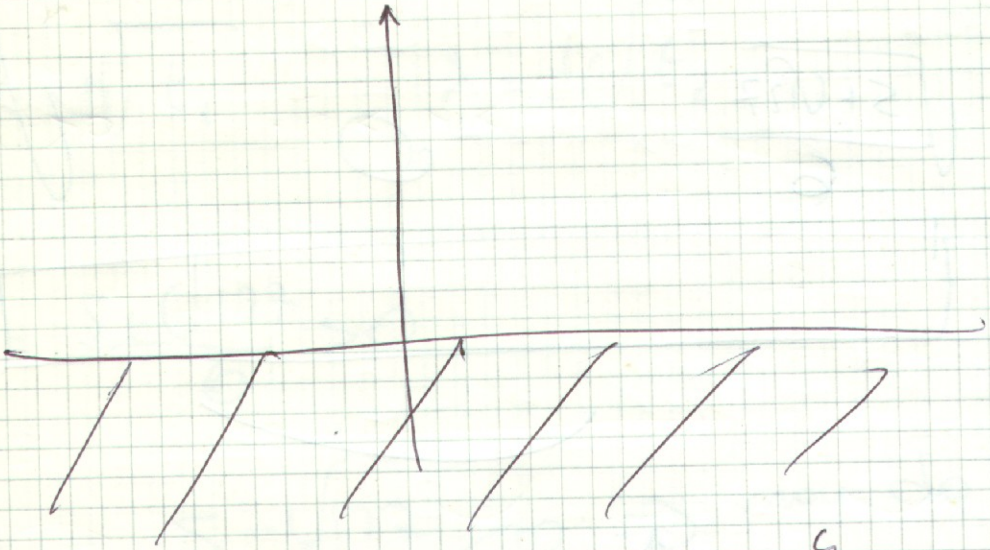
$$m = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{\sqrt[4]{x^2+1}}{x} =$$

$$= \lim_{x \to +\infty} \sqrt[4]{\frac{x^2+1}{x^4}} = 0$$ non vi sono asintoti obliqu

$f(x) = \sqrt[4]{x^2+1}$

C.E.: $x^2+1 \geq 0 \quad \forall x$

Positività: $f(x) > 0 \quad \forall x$



Intersezione con asse $x \Rightarrow y = 0 \quad \sqrt[4]{x^2+1} = 0$ ~~mai~~

$x^2+1=0$ mai

Con l'asse y; $x=0 \quad y=1$

Non vi sono asintoti verticali

$$\lim_{x\to+\infty} \sqrt[4]{x^2+1} = +\infty$$

$$\lim_{x\to-\infty} \sqrt[4]{x^2+1} = -\infty$$

$$m = \lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\sqrt[4]{x^2+1}}{x} =$$

$$= \lim_{x\to+\infty} \sqrt[4]{\frac{x^2+1}{x^4}} = 0$$ non ci sono asintoti obliqu

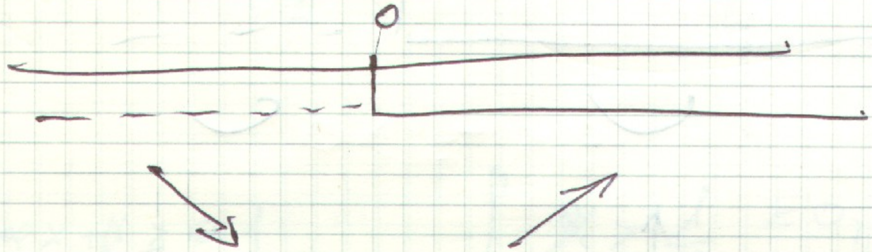
MAX e MIN

$$f'(x) = \left[(x^2+1)^{\frac{1}{4}} \right] = \frac{1}{4}(x^2+1)^{\frac{1}{4}-1} \cdot 2x =$$

$$= \frac{1}{4}(x^2+1)^{-\frac{3}{4}} 2x = \frac{1}{\cancel{4}_2} \frac{\cancel{2}x}{\sqrt[4]{(x^2+1)^3}} = \frac{x}{2\sqrt[4]{(x^2+1)^3}}$$

$f'(x) = 0$ per $x = 0$

$f'(x) > 0$ per $x > 0$



$x = 0$ è un punto di minimo

FLESSI

$$f''(x) = \frac{2\sqrt[4]{(x^2+1)^3} - x \cdot \cancel{2} \cdot \frac{3}{\cancel{4}_2}(x^2+1)^{\frac{3}{4}-1 = -\frac{1}{4}} \cancel{2x}}{4\sqrt[4]{(x^2+1)^6}} =$$

$$= \frac{2\sqrt[4]{(x^2+1)^3} - \frac{3x^2}{\sqrt{x^2+1}}}{4\sqrt[4]{(x^2+1)^6}} =$$

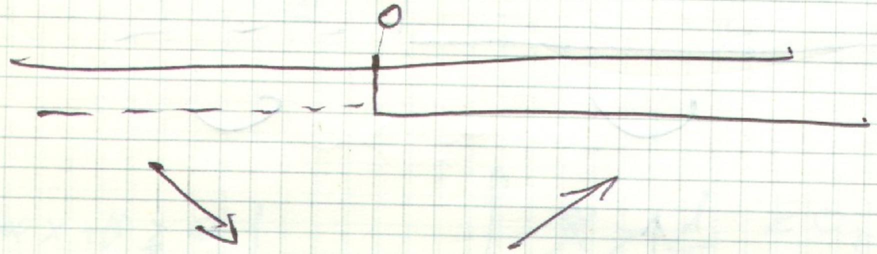
$$2\sqrt[4]{(x^2+1)^4} - 3x^2$$

$$f'(x) = \left[(x^2+1)^{\frac{}{4}} \right] = \frac{1}{4}(x^2+1) \cdot 2x =$$

$$= \frac{1}{4}(x^2+1)^{-\frac{3}{4}} 2x = \frac{1}{\cancel{4}_2} \frac{\cancel{2}x}{\sqrt[4]{(x^2+1)^3}} = \frac{x}{2\sqrt[4]{(x^2+1)^3}}$$

$f'(x) = 0$ per $x = 0$

$f'(x) > 0$ per $x > 0$



$x = 0$ è un punto di minimo

FLESSI

$$f''(x) = \frac{2\sqrt[4]{(x^2+1)^3} - x \cdot \cancel{2} \cdot \frac{3}{\cancel{4}_2}(x^2+1)^{\frac{3}{4}-1=-\frac{1}{4}} \cancel{2x}}{4\sqrt[4]{(x^2+1)^6}} =$$

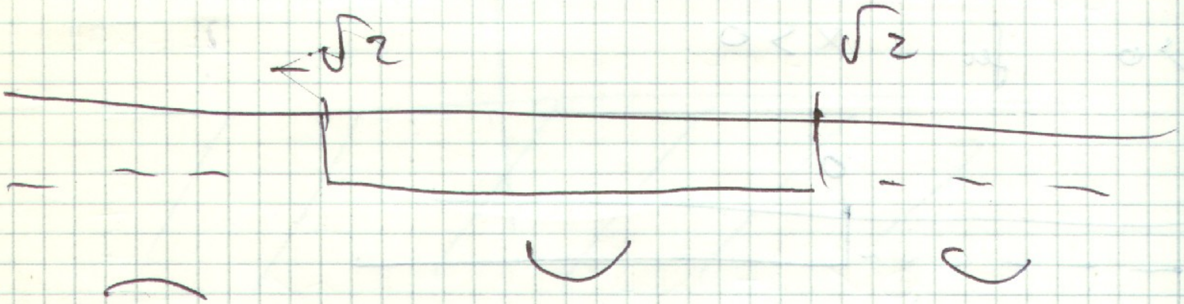
$$= \frac{2\sqrt[4]{(x^2+1)^3} - \frac{3x^2}{\sqrt{x^2+1}}}{4\sqrt[4]{(x^2+1)^6}} =$$

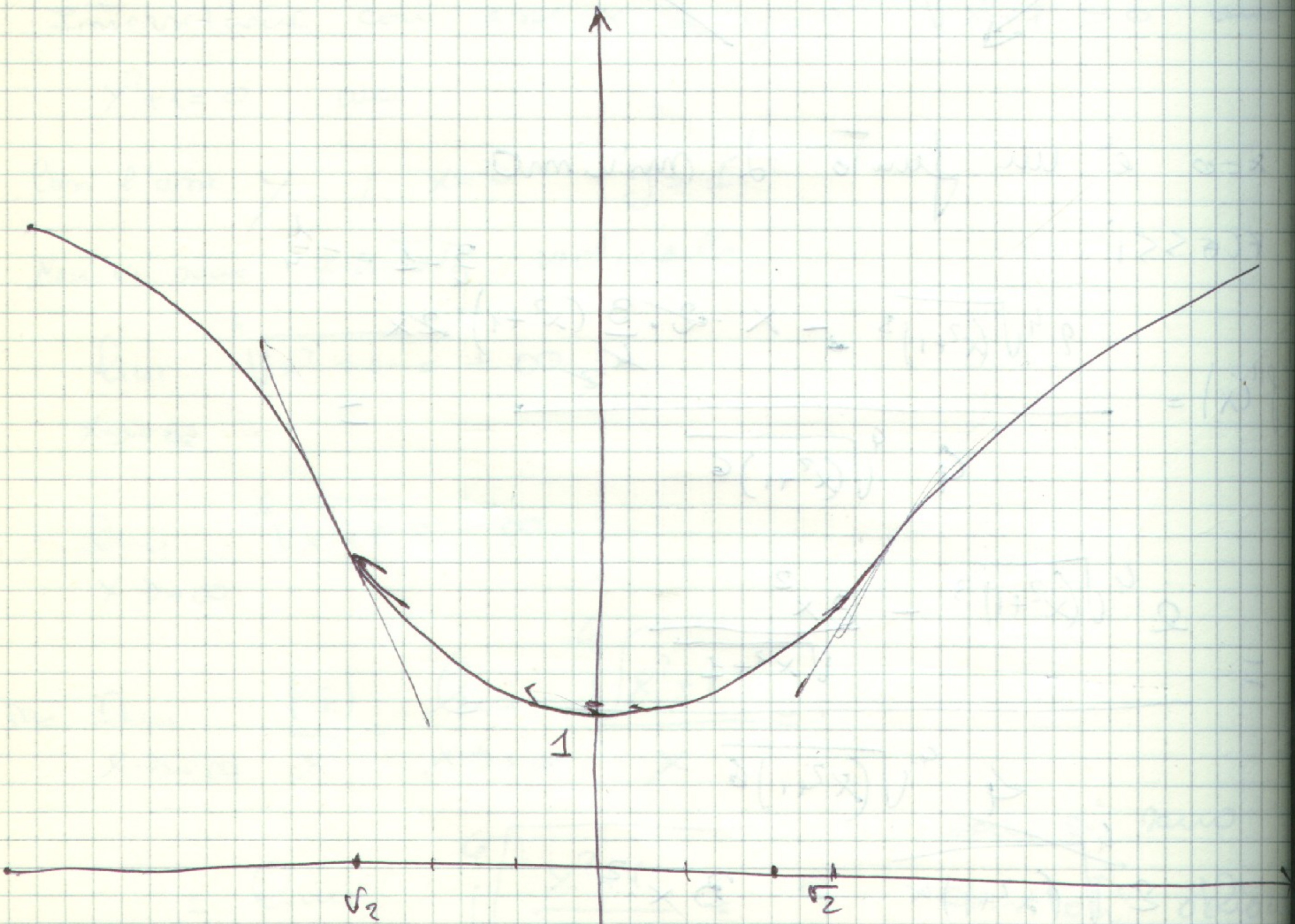
$$= \frac{2\cancel{\sqrt[4]{(x^2+1)^4}} - 3x^2}{4\sqrt[4]{(x^2+1)^6}\sqrt{x^2+1}}$$

$$f''(x) = \frac{2x^2+2-3x^2}{4\sqrt[4]{(x^2+1)^7}} = \frac{-x^2+2}{4\sqrt[4]{(x^2+1)^7}}$$

$f''(x) = 0 \qquad x^2-2=0 \qquad x = \pm\sqrt{2}$

$f''(x) > 0 \qquad -\sqrt{2} < x < \sqrt{2}$



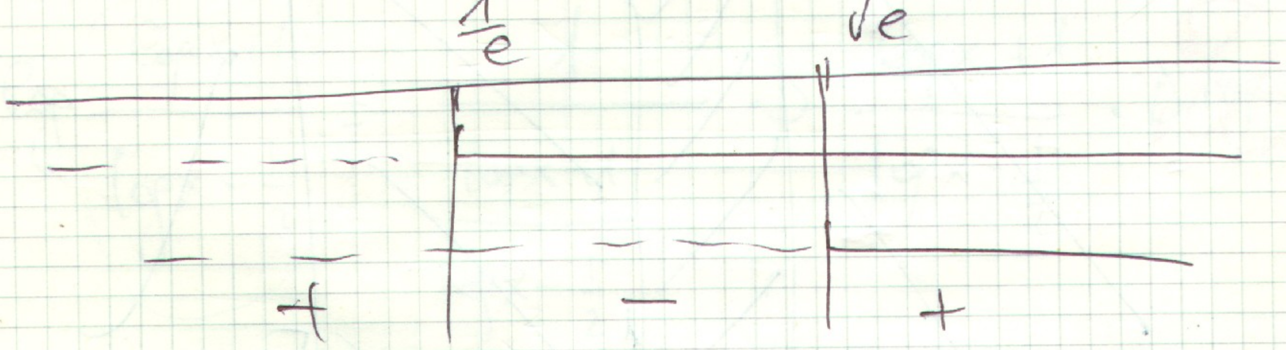


28

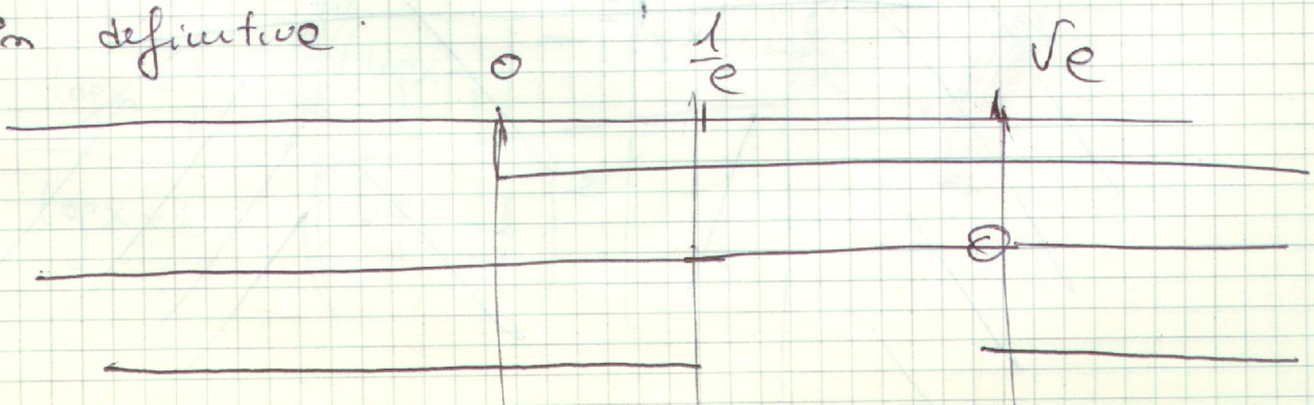
$$f(x) = \log \frac{\log x + 1}{\log x - \frac{1}{2}}$$

C.E.: $\begin{cases} \dfrac{\log x + 1}{\log x - \frac{1}{2}} > 0 \\ x > 0 \\ \log x \neq \frac{1}{2} \end{cases}$ $\quad\begin{cases} \dfrac{\log x + 1}{\log x - \frac{1}{2}} > 0 \\ x > 0 \\ x \neq e^{\frac{1}{2}} = \sqrt{e} \end{cases}$

$\begin{cases} \log x + 1 \geq 1 \\ \log x > +\frac{1}{2} \end{cases}$ $\quad\begin{cases} x > \frac{1}{e} = 0,367 \\ x > \sqrt{e} = 1,648 \end{cases}$



In definitiva



$C E:]0, \frac{1}{e}[\cup]\sqrt{e} , +\infty[$

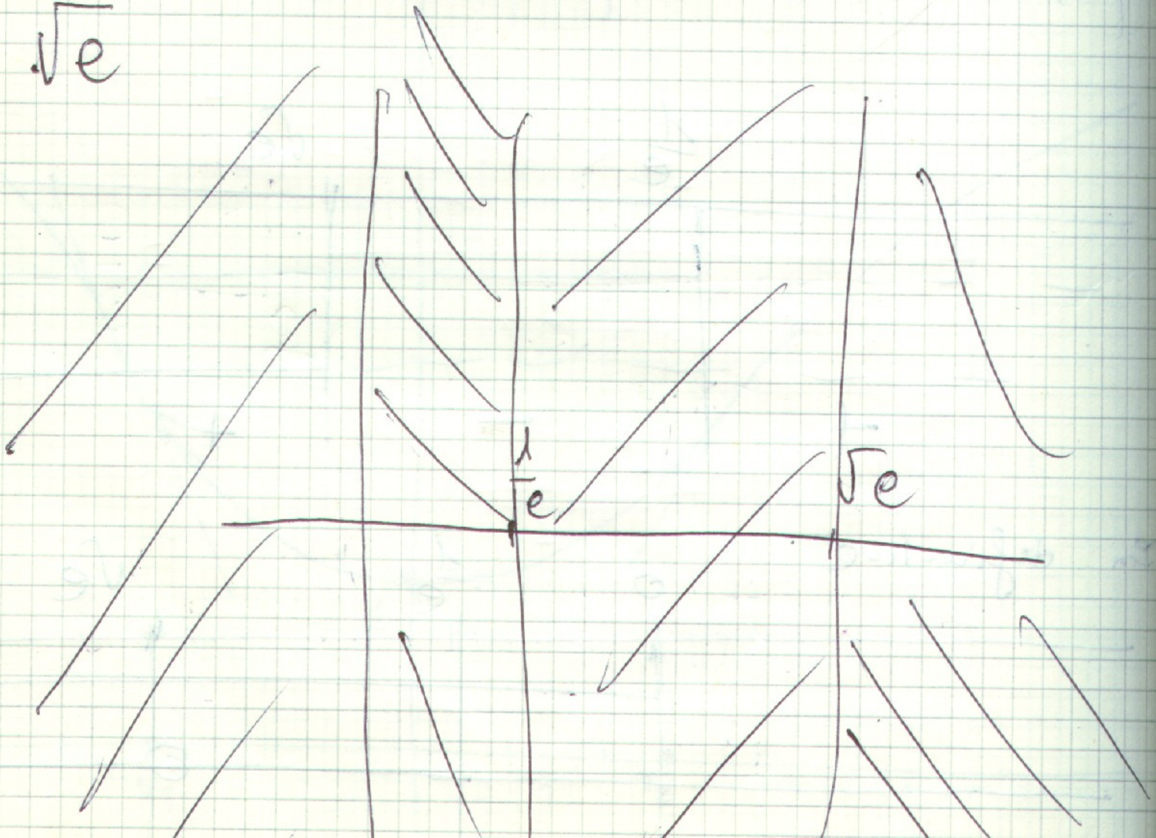
Positività

$\frac{\log x + 1}{\log x - \frac{1}{2}} > 1$; ~~$\log x + 1 > \log x - \frac{1}{2}$~~

$\frac{\log x + 1}{\log x - \frac{1}{2}} - 1 > 0$; $\frac{\log x + 1 - \log x + \frac{1}{2}}{\log x - \frac{1}{2}} > 0$;

$\frac{\frac{3}{2}}{\log x - \frac{1}{2}} > 0$; $\log x - \frac{1}{2} > 0$

$x > \sqrt{e}$



$$\lim_{x\to 0^+} \log \frac{\log x + 1}{\log x - \frac{1}{2}} = \log 1 = 0$$

$$\lim_{x\to \frac{1}{e}^-} \log \frac{\log x + 1}{\log x - \frac{1}{2}} = \log \frac{-1+1}{-1+\frac{1}{2}} = \log 0 = -\infty$$

asintoto verticale

$$\lim_{x\to \sqrt{e}^+} \log \frac{\log x + 1}{\log x - \frac{1}{2}} = \log \frac{\frac{1}{2}+1}{\frac{1}{2}-\frac{1}{2}} = \cancel{\log} \ +\infty$$

asintoto verticale

$$\lim_{x\to +\infty} \log \frac{\log x + 1}{\log x - \frac{1}{2}} = 0$$

asintoto orizzontale

MAX E MIN

$$D \log \frac{\log x + 1}{\log x - \frac{1}{2}} = \frac{1}{\dfrac{\log x + 1}{\log x - \frac{1}{2}}} \cdot D \frac{\log x + 1}{\log x - \frac{1}{2}} \cdot \cancel{D \log x}$$

$$= \frac{\cancel{\log x - \frac{1}{2}}}{\log x + 1} \cdot \frac{\cancel{\frac{1}{x}}\left(\log x - \frac{1}{2}\right) - \left(\log x + 1\right)\cancel{\frac{1}{x}}}{\left(\log x - \frac{1}{2}\right)^{\cancel{2}}} \cdot \cancel{\frac{1}{x}} =$$

$$= \frac{\frac{1}{x}\left(\cancel{\log x} - \frac{1}{2} - \cancel{\log x} - 1\right)}{\left(\log x + 1\right)\left(\log x - \frac{1}{2}\right)} =$$

$$= \frac{-\frac{3}{2}}{\underset{>0}{x^{2}}\,\underset{>0}{\left(\log x + 1\right)}\,\underset{>0}{\left(\log x - \frac{1}{2}\right)}}$$

$f'(x) < 0 \quad \forall x \in$ cē

la f e' sempre decrescente

$$f''(x) = +\frac{3}{2}\,\frac{D\left[x\left(\log x + 1\right)\left(\log x - \frac{1}{2}\right)\right]}{x^{\overset{2}{\cancel{4}}}\left(\log x + 1\right)^2\left(\log x - \frac{1}{2}\right)^2} =$$

$$= \frac{3}{2}\,\cancel{\frac{\frac{1}{x}\left(\log x - \frac{1}{2}\right) + \left(\log x + 1\right)\frac{1}{x}}{x^4\left(\log x + 1\right)^2\left(\log x - \frac{1}{2}\right)^2}}$$

~~$$= \frac{3}{2}\left(\frac{2\log x + \frac{1}{2}}{x^5(\log x + 1)^2(\log x - \frac{1}{2})^2}\right)$$~~

~~$f''(x) = 0 \qquad 2\log x + \frac{1}{2} = 0$~~

~~$\log x = -\frac{1}{4}$~~

~~$x = e^{-\frac{1}{4}} \qquad e^{-1}$~~

$$= \frac{3}{2}\,\frac{\left[2x(\log x+1)(\log x-\frac{1}{2}) + x^{2}\left(\frac{1}{x}(\log x-\frac{1}{2}) + \frac{1}{x}(\log x+1)\right)\right]}{x^{2}(\log x+1)^2(\log x-\frac{1}{2})^2} =$$

$$= \frac{3}{2}\,\frac{\left[x\left(\log^2 x - \frac{1}{2}\log x + \log x - \frac{1}{2}\right) + x\left(\log x - \frac{1}{2} + \log x + 1\right)\right]}{x^{2}(\log x+1)^2(\log x-\frac{1}{2})^2}$$

$$= \frac{3}{2}\,\frac{\log^2 x + \frac{1}{2}\log x - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} + 2\log x}{x^{3}(\log x+1)^2(\log x-\frac{1}{2})^2}$$

$$= \frac{3}{2}\,\frac{2\log^2 x + \frac{5}{2}\log x - \overset{0}{\cancel{\frac{5}{2}}}}{x^{2}(\log x + 1)^2(\log x - \frac{1}{2})^2}$$

~~$2y^2 + \frac{1}{2}y - \frac{5}{2} = 0$~~ ~~$2y^2 + y - 4 = 0$~~

~~$4y^2 + 2y - 5 = 0$~~

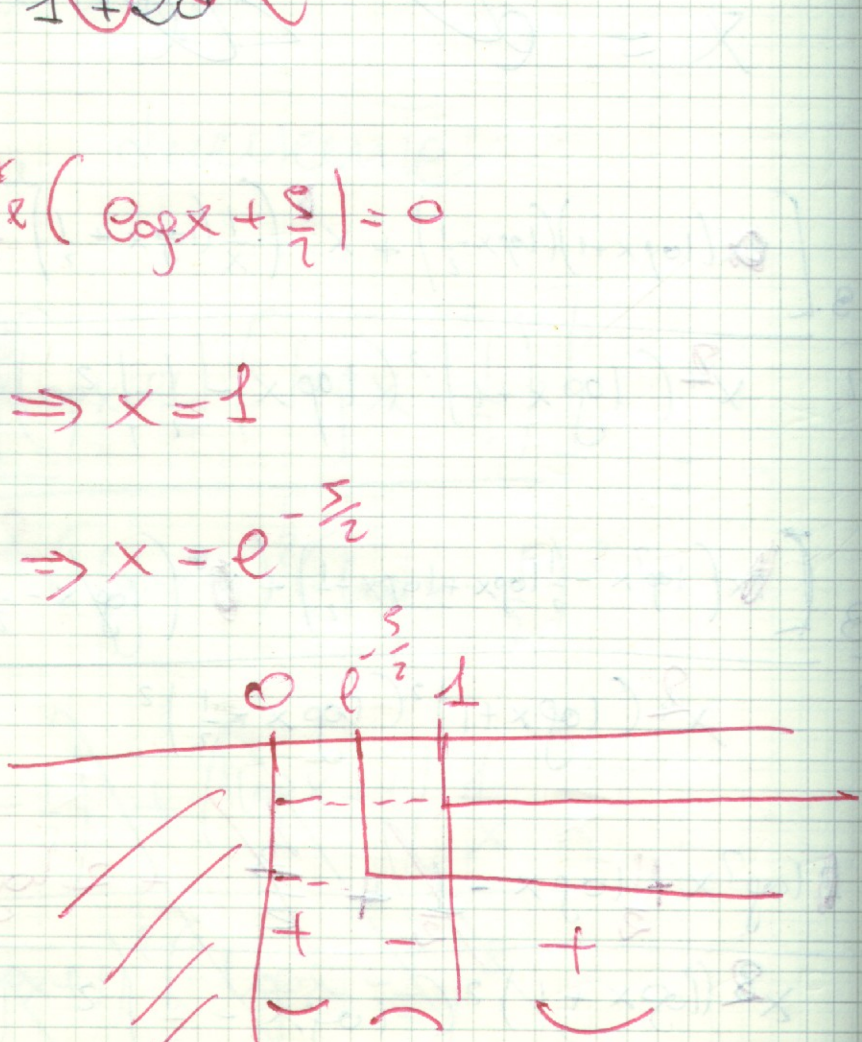
~~$y = -1 \pm \sqrt{1+20}$~~

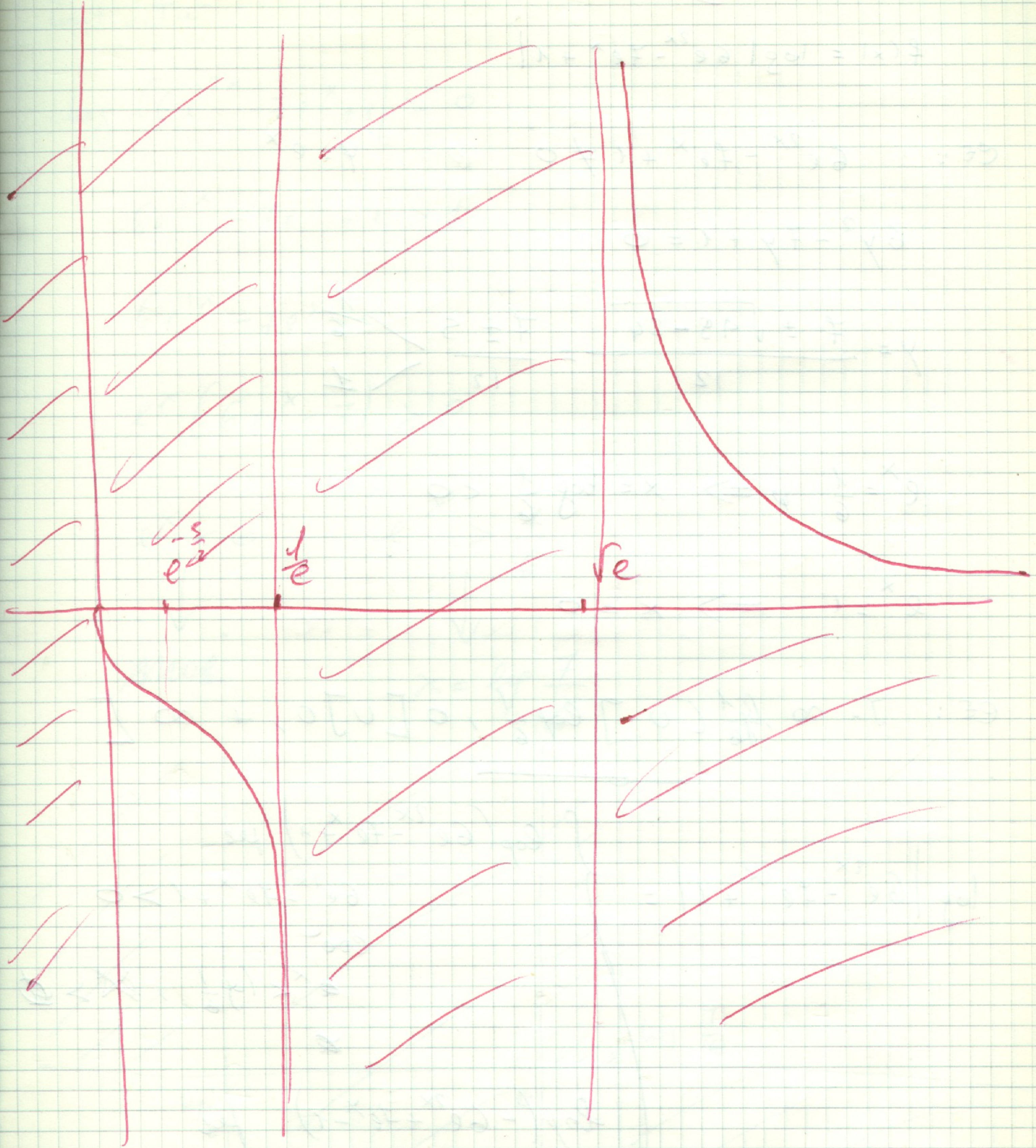
$f''(x) = 0 \Rightarrow \log x\left(\log x + \frac{5}{2}\right) = 0$

$\log x = 0 \Rightarrow x = 1$

$\log x = -\frac{5}{2} \Rightarrow x = e^{-\frac{5}{2}}$

$f''(x) > 0$




$e^{-\frac{5}{2}}$
$\frac{1}{e}$
$\sqrt{e}$

N° 27

$$f(x) = \log|6e^{2x} - 7e^x + 1|$$

CE: $6e^{2x} - 7e^x + 1 \neq 0$ $\qquad y = e^x$

$$6y^2 - 7y + 1 = 0$$

$$y = \frac{7 \pm \sqrt{49 - 24}}{12} = \frac{7 \pm 5}{12} \begin{cases} \frac{1}{6} \\ 1 \end{cases}$$

$e^x = \frac{1}{6} \Longrightarrow x = \log\frac{1}{6} < 0$

$e^x = 1 \Longrightarrow x = 0$

CE: $]-\infty, \log\frac{1}{6}[\cup]\log\frac{1}{6}, 0[\cup]0, +\infty[$

$$\log|6e^{2x} - 7e^x + 1| = \begin{cases} \log(6e^{2x} - 7e^x + 1) \text{ per } \\ \quad 6e^{2x} - 7e^x + 1 > 0 \\ \quad \text{cioè} \\ \quad x < \log\frac{1}{6} \,;\, x > 0 \\ \log(-6e^{2x} + 7e^x - 1) \text{ per } \\ \quad 6e^{2x} - 7e^x + 1 < 0 \\ \quad \text{cioè} \\ \quad \log\frac{1}{6} < x < 0 \end{cases}$$

Studiamo la 1ª funzione

Positività

$$\log\left(6e^{2x}-7e^{x}+1\right)>0$$

$$6e^{2x}-7e^{x}+1>1$$

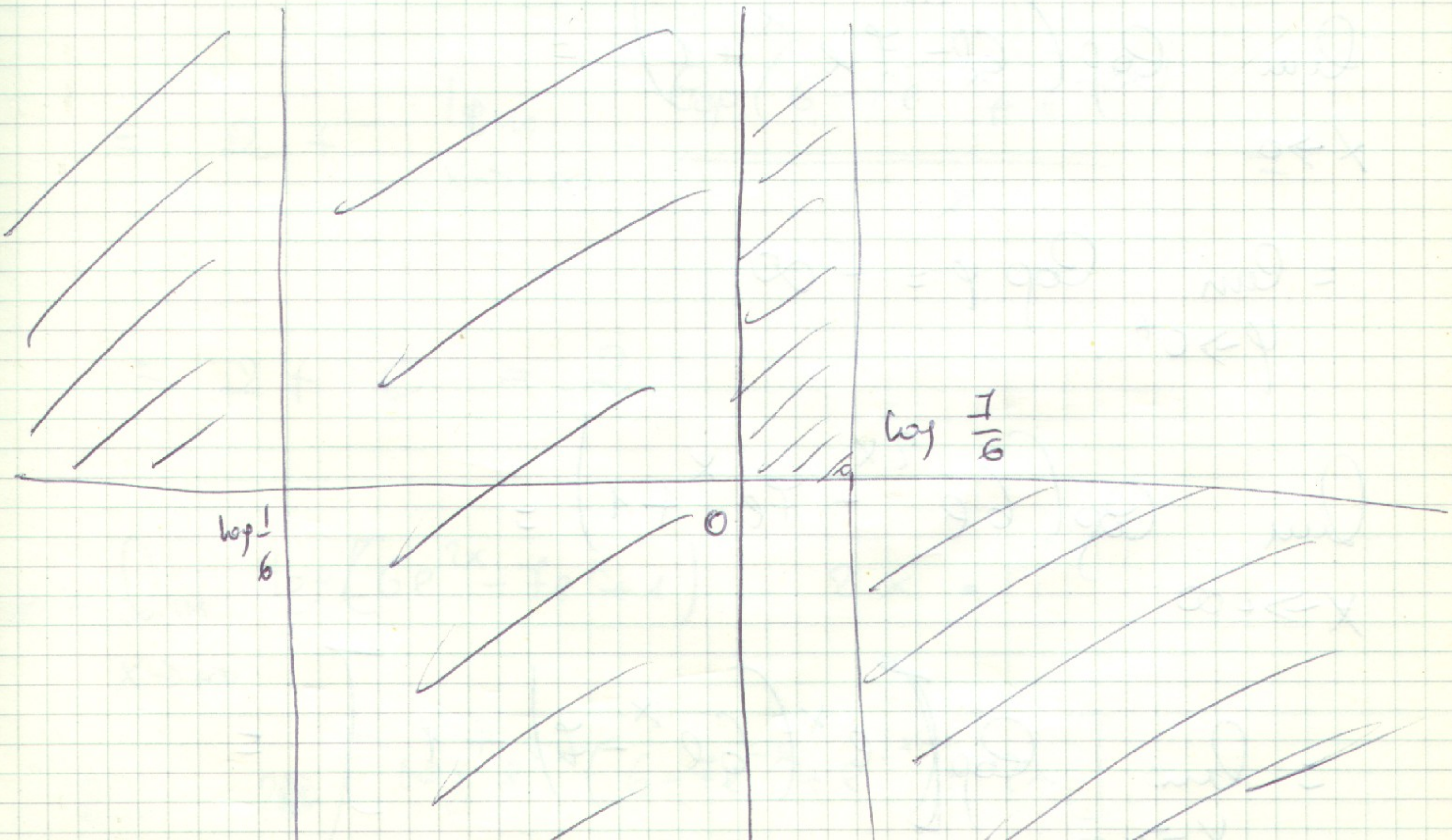
$$6e^{2x}-7e^{x}>0$$

$$e^{x}\left(6e^{x}-7\right)>0$$

$e^{x}<0$ $\quad\quad$ $e^{x}>\frac{7}{6}$

$\downarrow$ $\quad\quad$ $\downarrow$

~~MAI~~ $\quad\quad$ $x=\log\frac{7}{6}$



$$\lim_{x\to-\infty} \log(6e^{2x}-7e^{x}+1) =$$

$$= \lim_{x\to-\infty} \log[e^{x}(6e^{x}-7)+\frac{1}{\text{[illegible]}}) =$$

$$= \lim_{x} \log[0(0-7)+1) = \log 1 = 0$$

$$\lim_{x\to\log\frac{1}{6}^-} \log(6e^{2x}-7e^{x}+1) =$$

$$= \cancel{\log(6}\ \lim_{y\to0^+} \log y = -\infty$$

$$\lim_{x\to0} \log(6e^{2x}-7e^{x}+1) =$$

$$= \lim_{y\to0^+} \log y = -\infty$$

$$\lim_{x\to+\infty} \log(6e^{2x}-7e^{x}+1) =$$

$$= \lim_{x\to+\infty} \log[e^{x}(6e^{x}-7)+1] =$$

$x \to -\infty$

$$= \lim_{x \to -\infty} \log\left[e^{x}(6e^{x} - 7) + \frac{1}{\cancel{\infty}}\right) =$$

$$= \lim_{x} \log\left[0(0-7) + 1\right) = \log 1 = 0$$

$$\lim_{x \to \log\frac{1}{6}^{-}} \log(6e^{2x} - 7e^{x} + 1) =$$

$$= \log(\cancel{6} \quad \lim_{y \to 0^{+}} \log y = -\infty$$

$$\lim_{x \to 0} \log(6e^{2x} - 7e^{x} + 1) =$$

$$= \lim_{y \to 0^{+}} \log y = -\infty$$

$$\lim_{x \to +\infty} \log(6e^{2x} - 7e^{x} + 1) =$$

$$= \lim_{x \to +\infty} \log\left[e^{x}(6e^{x} - 7) + 1\right] =$$

$$= \cancel{\log\left(+\infty(+\infty)+1\right) = +\infty}$$

$$= \lim_{y \to +\infty} \log(y+1) = +\infty$$

$$m = \lim_{x \to +\infty} \frac{\log\left[6e^{2x} + 7e^{x} + 1\right]}{x} =$$

$$= \lim_{x \to +\infty} \frac{\log e^{2x}\left[6\cancel{e^{-x}} - 7e^{-\cancel{2}x} + e^{-2x}\right]}{x} =$$

$$= \lim_{x \to +\infty} \frac{2x}{x} + \frac{\log\left(6 - 7e^{-x} + e^{-2x}\right)}{x} =$$

$$= 2 + \lim_{x \to +\infty} \frac{\log\left(6 - 7e^{-x} + e^{-2x}\right)}{x} =$$

$$= 2 + 0 = 2$$

$$m = \lim_{x \to +\infty} \log\left[6e^{2x} - 7e^{x} + 1\right) - 2x =$$

$$= \lim_{x \to +\infty} \cancel{2\otimes}\left[\log e^{2x}\left(6 - 7e^{-x} + e^{-2x}\right) - 2x\right] =$$

$$= \lim_{x \to +\infty} \left[2x + \log(6 - 7e^{-x} + e^{-2x}) - 2x \right] =$$

$$= \lim_{x \to +\infty} \log(6 - 7e^{-x} + e^{-2x}) =$$

$$= \log 6$$

Asintoto obliquo di equazione

$$y = 2x + \log 6$$

MAX E MIN

$$Df(x) = D \log(6e^{2x} - 7e^{x} + 1) =$$

$$= \frac{1}{6e^{2x} - 7e^{x} + 1}(12e^{2x} - 7e^{x}) =$$

$$= \frac{e^{x}(12e^{x} - 7)}{6e^{2x} - 7e^{x} + 1}$$

$f'(x)=0 \Rightarrow \quad e^x=0$ mai

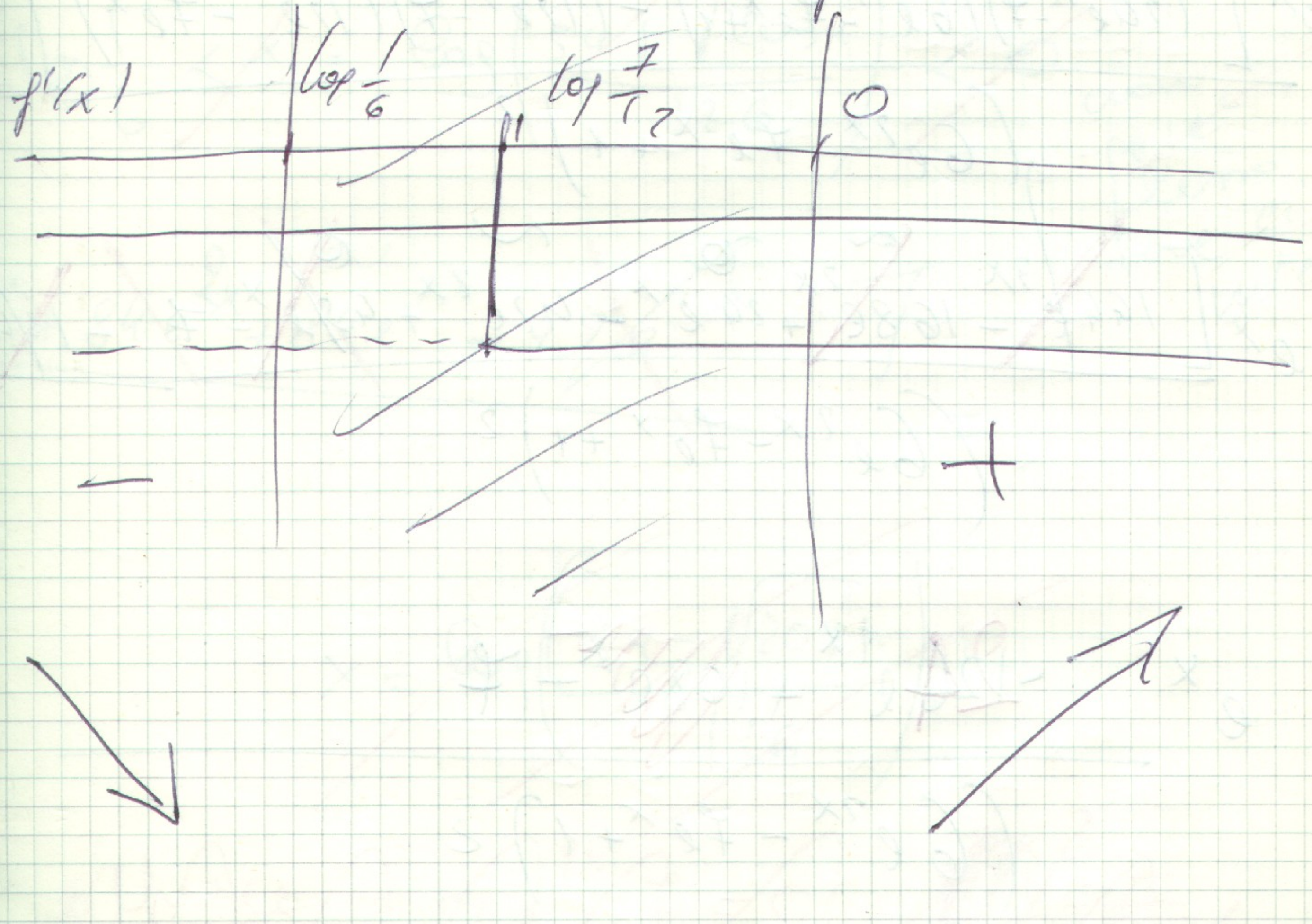
$$12e^x-7=0$$

$$\Downarrow$$

$$e^x=\frac{7}{12}$$

$$\Downarrow$$

$x=\log\frac{7}{12}$ che non appartiene al campo in cui stiamo studiando la funzione



$$f''(x) = D\,\frac{12e^{2x}-7e^{x}}{6e^{2x}-7e^{x}+1} =$$

$$= \frac{(24e^{2x}-7e^{x})(6e^{2x}-7e^{x}+1)-(12e^{2x}-7e^{x})(12e^{2x}-7e^{x}}{(6e^{2x}-7e^{x}+1)^2}$$

$$= \frac{e^{x}\left[(24e^{x}-7)(6e^{2x}-7e^{x}+1)-(12e^{x}-7)(12e^{2x}-7e^{x})\right]}{(6e^{2x}-7e^{x}+1)^2}$$

$$= \frac{e^{x}\left[144\cancel{e}^{3x}\overset{\sim}{-168\cancel{e}^{2x}}\overset{\circ}{+24e^{x}}\overset{\sim}{-42e^{2x}}\overset{\circ}{+4\cancel{9e}^{x}}-7\ -1\right.}{(6e^{2x}-7e^{x}+1)^2}$$

$$= e^{x}\,\frac{-124e^{2x}+\cancel{108e^{x}}-7}{(6e^{2x}-7e^{x}+1)^2}$$

$f''(x)=0 \Rightarrow e^{x}=0$ mai

$g'(x) = D \dfrac{\quad}{6e^{2x}-7e^{x}+1} =$

$$= \frac{(24e^{2x}-7e^{x})(6e^{2x}-7e^{x}+1)-(12e^{2x}-7e^{x})(12e^{2x}-7}{(6e^{2x}-7e^{x}+1)^{2}}$$

$$= \frac{e^{x}\left[(24e^{x}-7)(6e^{2x}-7e^{x}+1)-(12e^{x}-7)(12e^{2x}-7e^{x})\right]}{(6e^{2x}-7e^{x}+1)^{2}}$$

$$= \frac{e^{x}\left[\cancel{144e^{3x}}-\cancel{168e^{2x}}+24e^{x}-42e^{2x}+4\cancel{9e^{x}}-7 \quad -1\right.}{(6e^{2x}-7e^{x}+1)^{2}}$$

$$= \frac{e^{x} \quad -124e^{2x}+108e^{x}-7}{(6e^{2x}-7e^{x}+1)^{2}}$$

$g''(x)=0 \Rightarrow e^{x}=0$ ααί

$f''(x)=0$ $e^{x}=0$ mai

$126e^{2x}-108e^{x}+7=0$

↓

$$e^{x}=\frac{54\pm\sqrt{2916-882}}{126}=$$

$$=\frac{54\pm\sqrt{2034}}{126}=$$

$$\log\frac{1}{6}<x=\log\left(\frac{54\pm\sqrt{2034}}{126}\right)<0$$

non
~~e'e~~
~~lemo~~
dove ~~ci~~
interessa

$$\left[\frac{{}^{3x}+84\cancel{e^{2x}}+8\cancel{4}e^{2x}-49\cancel{e^{x}}}{}\right]=$$

$$x=\log\left(\frac{54-\sqrt{2034}}{126}\right)$$

~~$84e^{2x}-7=0$~~ ~~$x^2=\frac{1}{84}$~~

~~non c'è~~

~~$y = \pm\sqrt{\frac{7}{24}}$~~ ~~$e^x = \pm\sqrt{\frac{7}{24}}$~~

- oppure

$24e^{2x} + 7 = 0$ mai

$f''(x) < 0 \quad \forall x$

la $f(x)$ è sempre concava

Studiamo ove

$\log(-6e^{2x} + 7e^x - 1)$

per $\log\frac{1}{6} < x < 0$

Positività

$-6e^{2x} + 7e^x - 1 > 1$

$-6e^{2x} + 7e^x - 2 > 0$

$6e^{2x} - 7e^x + 2 < 0$

$6y^2 - 7y + 2 = 0$

$$y = \frac{7 \pm \sqrt{49-48}}{12} = \frac{7 \pm 1}{12} \begin{cases} \frac{1}{2} \\ \frac{2}{3} \end{cases}$$

$e^x = \frac{1}{2}$ $\qquad$ $x = \log \frac{1}{2}$

$e^{x} = \frac{2}{3}$ $\qquad$ $X = \log \frac{2}{3}$

$f(x) > 0$ per

$$\log \frac{1}{2} < x < \log \frac{2}{3}$$

Intersezione con l'asse x

$x = \log \frac{1}{2}$, $x = \log \frac{2}{3}$

$$\lim_{x \to \log \frac{1}{6}^-} \log(-6e^{2x} + 7e^x + 1) =$$

$$= -\infty$$

$$\lim_{x \to 0} \log(-6e^{2x} + 7e^x + 1) = -\infty$$

MAX e MIN

$$D \log\left(-6e^{2x}+7e^{x}-1\right) =$$

$$= \frac{1}{-6e^{2x}+7e^{x}-1}\left(-12e^{2x}+7e^{x}\right)$$

$$= \frac{12e^{2x}-7e^{x}}{6e^{2x}+7e^{x}+1} = 0 \Rightarrow e^{x}=0 \quad \text{mai'}$$

oppure

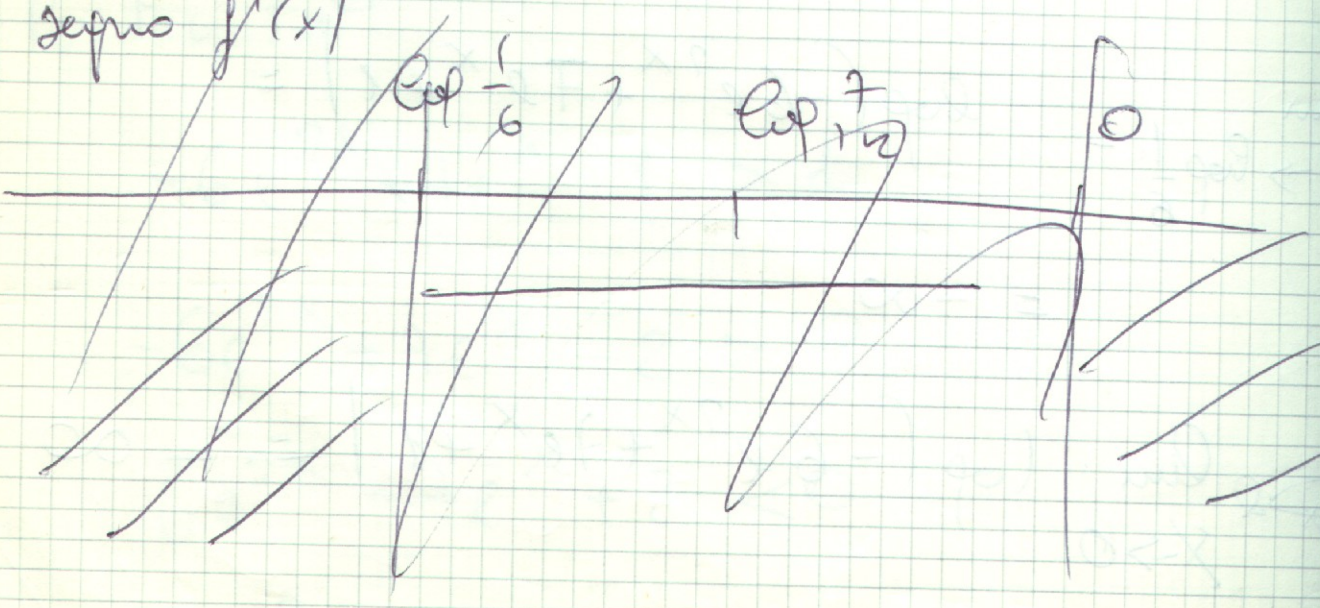
$$12e^{x}-7=0$$

$$\Downarrow$$

$$e^{x}=\frac{7}{12}$$

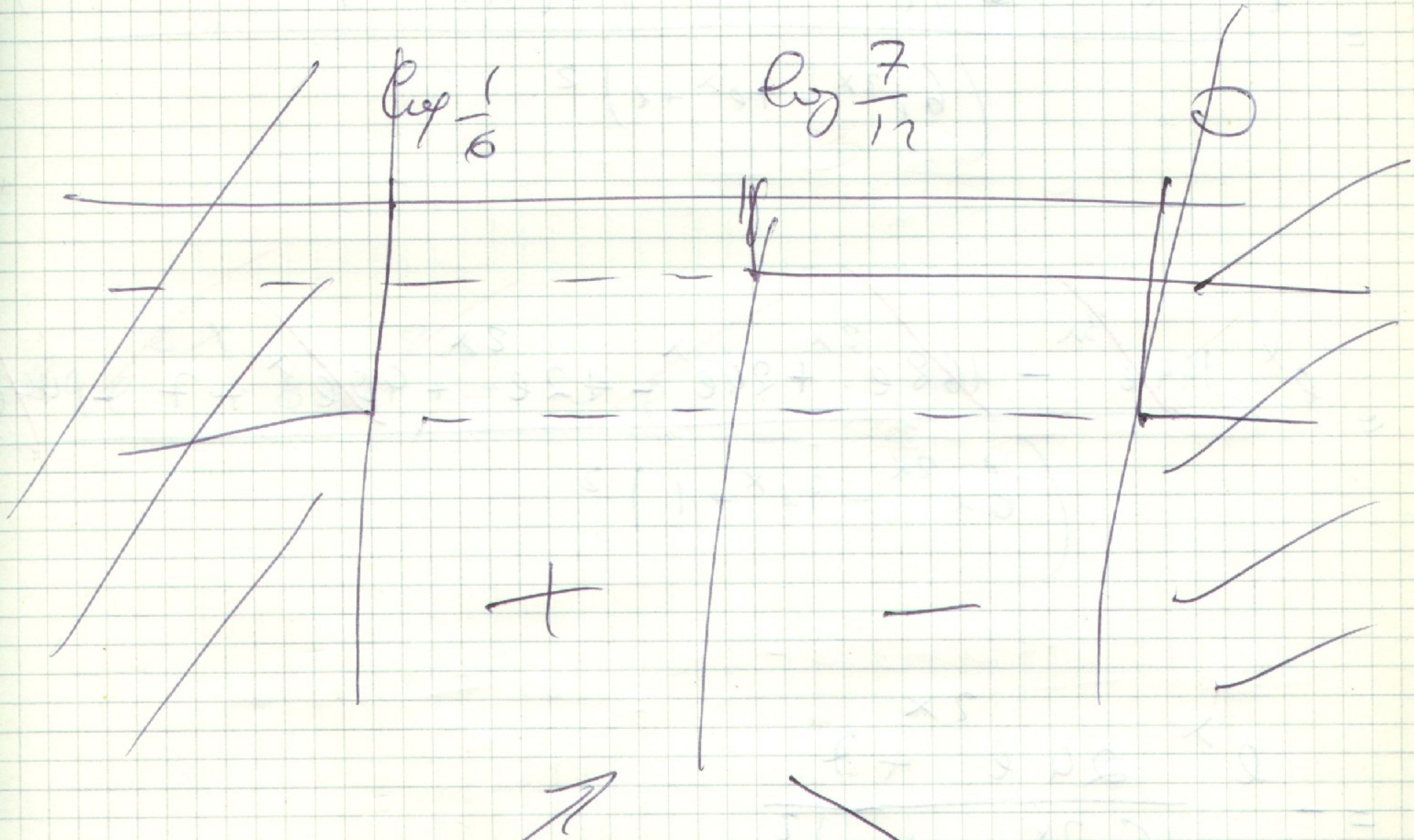
$$x=\log\frac{7}{12}$$

segno $f'(x)$



$$f'(x)>0 \implies \begin{cases} 12e^{x}-7>0 \\ 6e^{2x}-7e^{x}+1>0 \end{cases}$$

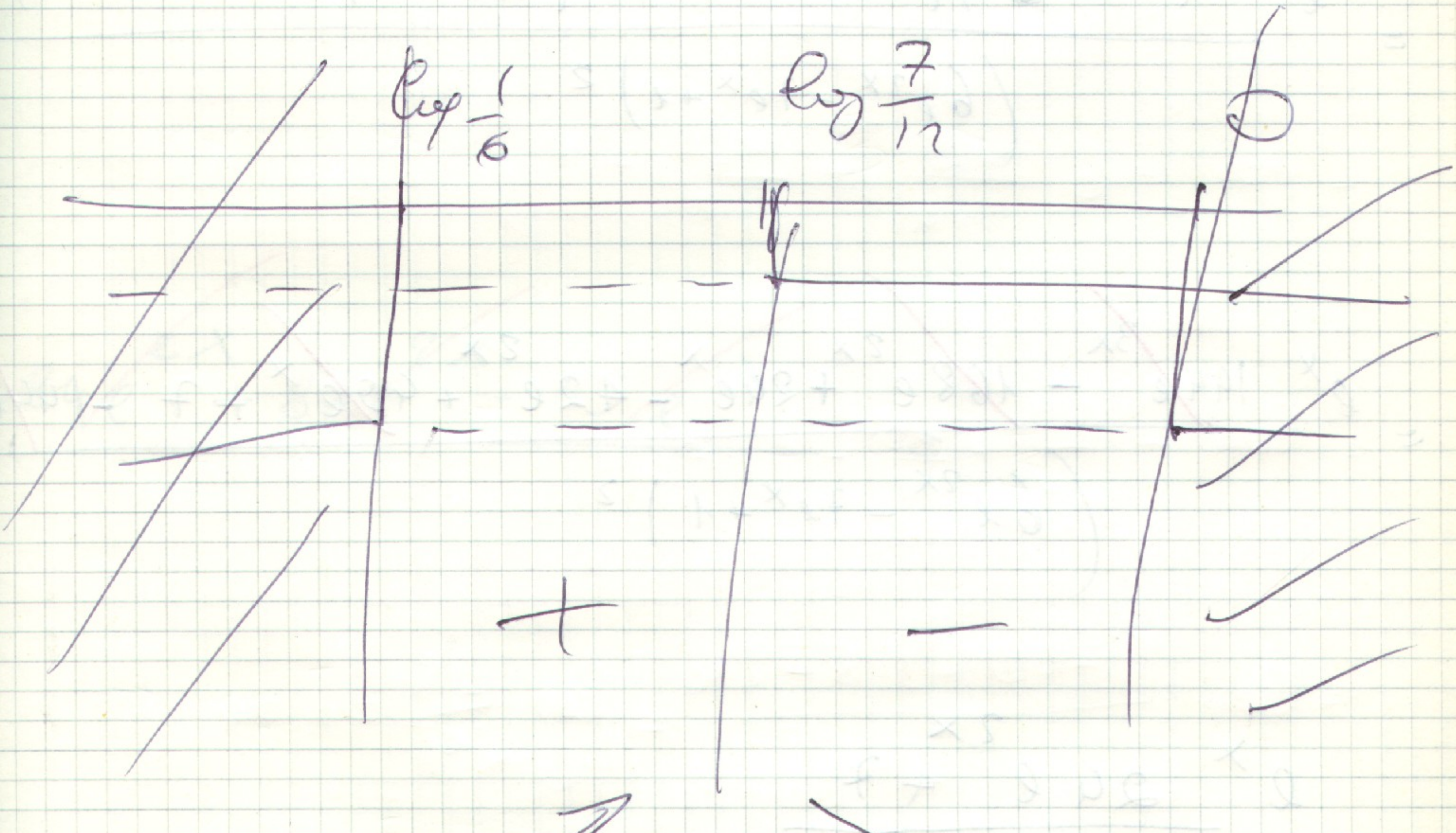
$$\begin{cases} x > \log \frac{7}{12} \\ x < \log \frac{1}{6} \quad ; \quad x > 0 \end{cases}$$



$x=\log\frac{7}{\ }$ è un punto di max

$6e \quad -7e \quad +1 >$

$$\begin{cases} x > \log \frac{7}{12} \\ x < \log \frac{1}{6} \; ; \; x > 0 \end{cases}$$



$x = \log \frac{7}{12}$ è un punto di max

$$f''(x) = D\ \frac{12e^{2x} - 7e^{x}}{6e^{2x} - 7e^{x} + 1} =$$

$$= \frac{(24e^{2x} - 7e^{x})(6e^{2x} - 7e^{x} + 1) - (12e^{2x} - 7e^{x})(12e^{2x} - 7e^{x})}{(6e^{2x} - 7e^{x} + 1)^{2}}$$

$$= e^{x}\ \frac{(24e^{x} + 7)(6e^{2x} + 7e^{x} + 1) - (12e^{x} - 7)(12e^{2x} - 7e^{x})}{(6e^{2x} - 7e^{x} + 1)^{2}}$$

$$= e^{x}\ \frac{\cancel{144e^{3x}} - \cancel{168e^{2x}} + 24e^{x} - 42e^{2x} + \cancel{49e^{x}} - 7 - \cancel{144e^{3}}}{(6e^{2x} - 7e^{x} + 1)^{2}}$$

$$= - e^{x}\ \frac{24e^{2x} + 7}{(6e^{2x} - 7e^{x} + 1)^{2}}$$

$f''(x) = 0 \Rightarrow 24e^{2x} + 7 = 0$ avec

$$f''(x) = D\,\frac{12e^{2x}-7e^{x}}{6e^{2x}-7e^{x}+1} =$$

$$= \frac{(24e^{2x}-7e^{x})(6e^{2x}-7e^{x}+1)-(12e^{2x}-7e^{x})(12e^{2x}-7e^{x})}{(6e^{2x}-7e^{x}+1)^{2}}$$

$$= e^{x}\,\frac{(24e^{2x}+7)(6e^{2x}+7e^{x}+1)+(12e^{x}-7)(12e^{2x}-7e^{x})}{(6e^{2x}-7e^{x}+1)^{2}}$$

$$= e^{x}\,\frac{\cancel{144e^{3x}}-\cancel{168e^{2x}}+24e^{x}-42e^{2x}+\cancel{49e^{x}}-7-\cancel{144e^{3x}}}{(6e^{2x}-7e^{x}+1)^{2}}$$

$$= -e^{x}\,\frac{24e^{2x}+7}{(6e^{2x}-7e^{x}+1)^{2}}$$

$f''(x)=0 \Rightarrow 24e^{2x}+7=0$ mai

non ci sono flessi

la $f''(x)$ è sempre negativa

la funzione è sempre convessa

$$\underline{+84\cancel{e}^{2x}+84\cancel{e}^{2x}-49\cancel{e}^{x}} =$$

$f\left(\log \frac{7}{12}\right) =$

$= \log\left(-6e^{2\log\frac{7}{12}} + 7e^{\log\frac{7}{12}} - 1\right) =$

$= \log\left(-6e^{\log\frac{49}{144}} + 7 \cdot \frac{7}{12} - 1\right)$

$= \log\left(-6\,\frac{49}{144} + \frac{49}{12} - 1\right)$

$= \log\left(-\frac{49}{24} + \frac{49}{12} - 1\right) =$

$= \log \frac{-49+98-24}{24} = \log\frac{25}{24} = 0{,}04$

$y = 2x + \log 6$

per $x = 0$ $\quad y = \log 6 = 1{,}79$

$f\left(\log \frac{7}{12}\right) =$

$= \log\left(-6e^{2\log \frac{7}{12}} + 7e^{\log \frac{7}{12}} + 1\right) =$

$= \log\left(-6e^{\log \frac{49}{144}} + 7 \cdot \frac{7}{12} - 1\right)$

$= \log\left(-6\,\frac{49}{144} + \frac{49}{12} - 1\right)$

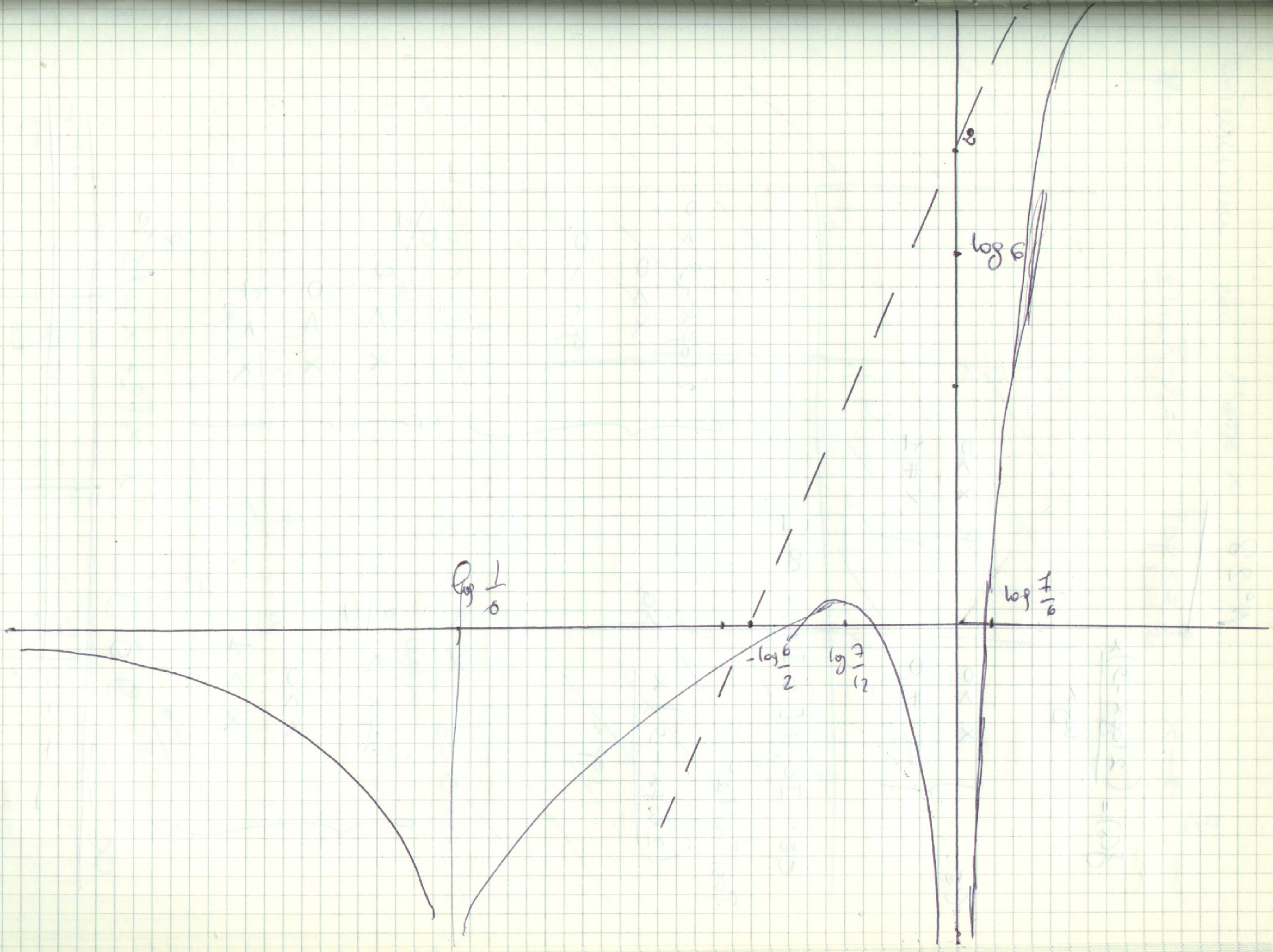
$= \log\left(-\frac{49}{24} + \frac{49}{12} - 1\right) =$

$= \log \frac{-49+98-24}{24} = \log \frac{25}{24} = 0{,}04$

$y = 2x + \log 6$

per $x = 0$ $\quad y = \log 6 = 1{,}79$

per $y = 0$ $\quad x = -\frac{\log 6}{2} = 0{,}895$


2
log 6
log 1/6
-log 6/2
log 7/12
log 7/6

$$f(x) = \frac{(2\log x - 9)x}{\log x}$$

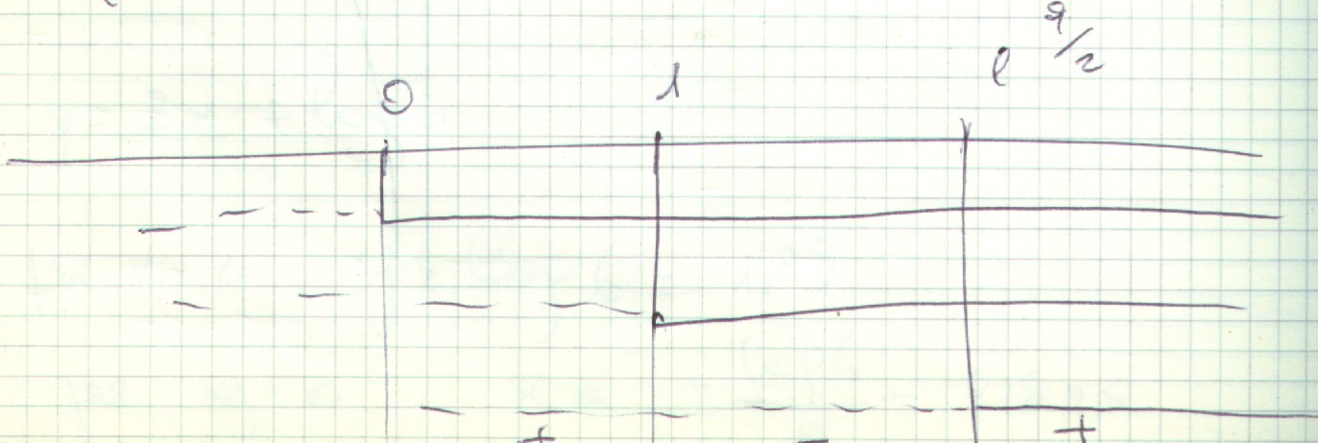
CE: $\begin{cases} x > 0 \\ \log x \neq 0 \end{cases}$ $\begin{cases} x > 0 \\ x \neq 1 \end{cases}$

CE: $]0,1[\cup]1,+\infty[$

Positività:

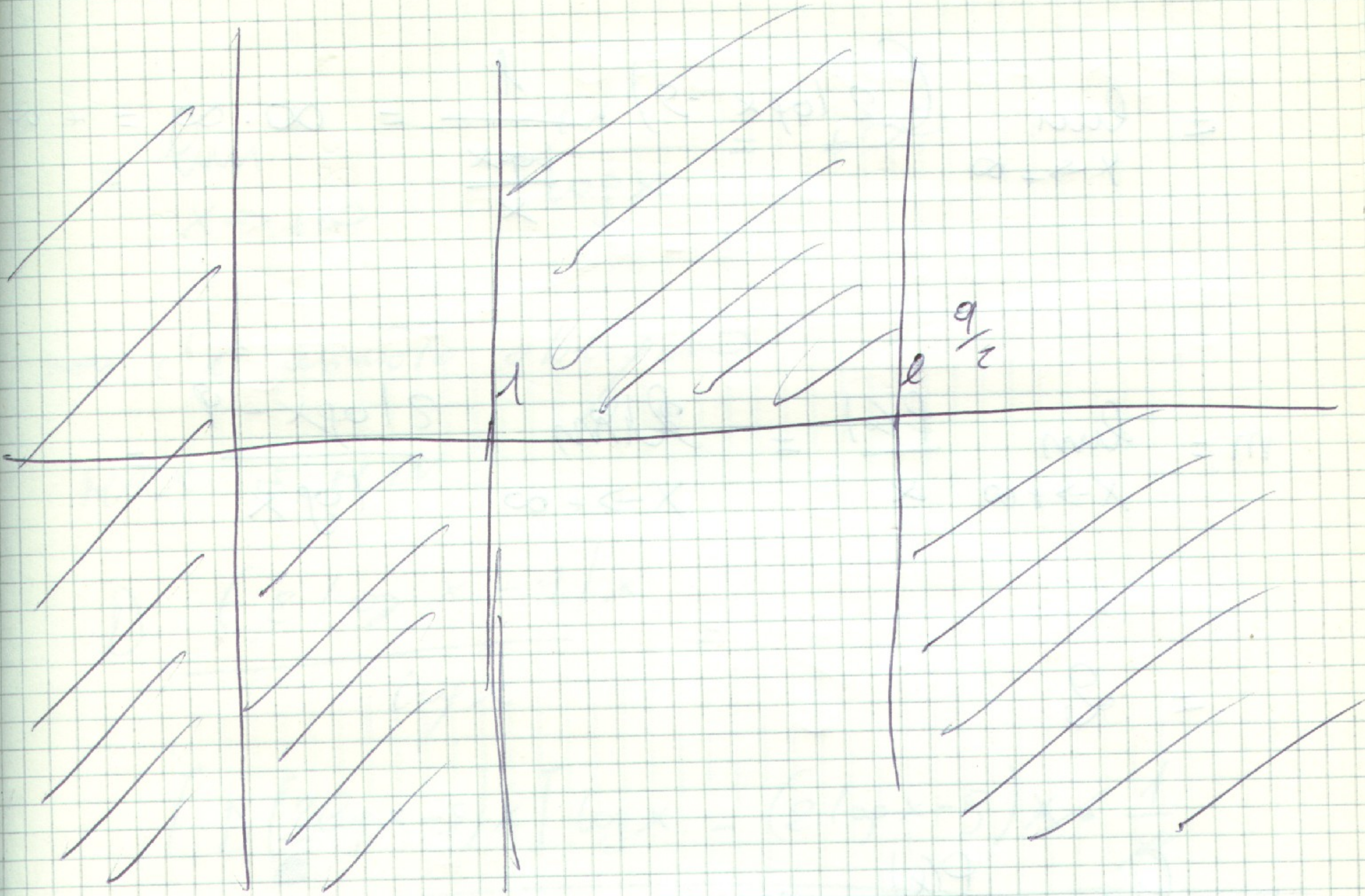
$\frac{(2\log x - 9)\,x}{\log x} > 0$ $\begin{cases} 2\log x - 9 > 0 \\ x > 0 \\ \log x > 0 \end{cases}$

$\begin{cases} \log x > \frac{9}{2} \\ x > 0 \\ x > 1 \end{cases}$ $\begin{cases} x > e^{\frac{9}{2}} \\ x > 0 \\ x > 1 \end{cases}$



Intersezione con l'asse x

$y = 0$ $\quad 2\log x - 9 = 0 \quad \log x = \frac{9}{2} \quad x = e^{\frac{9}{2}}$

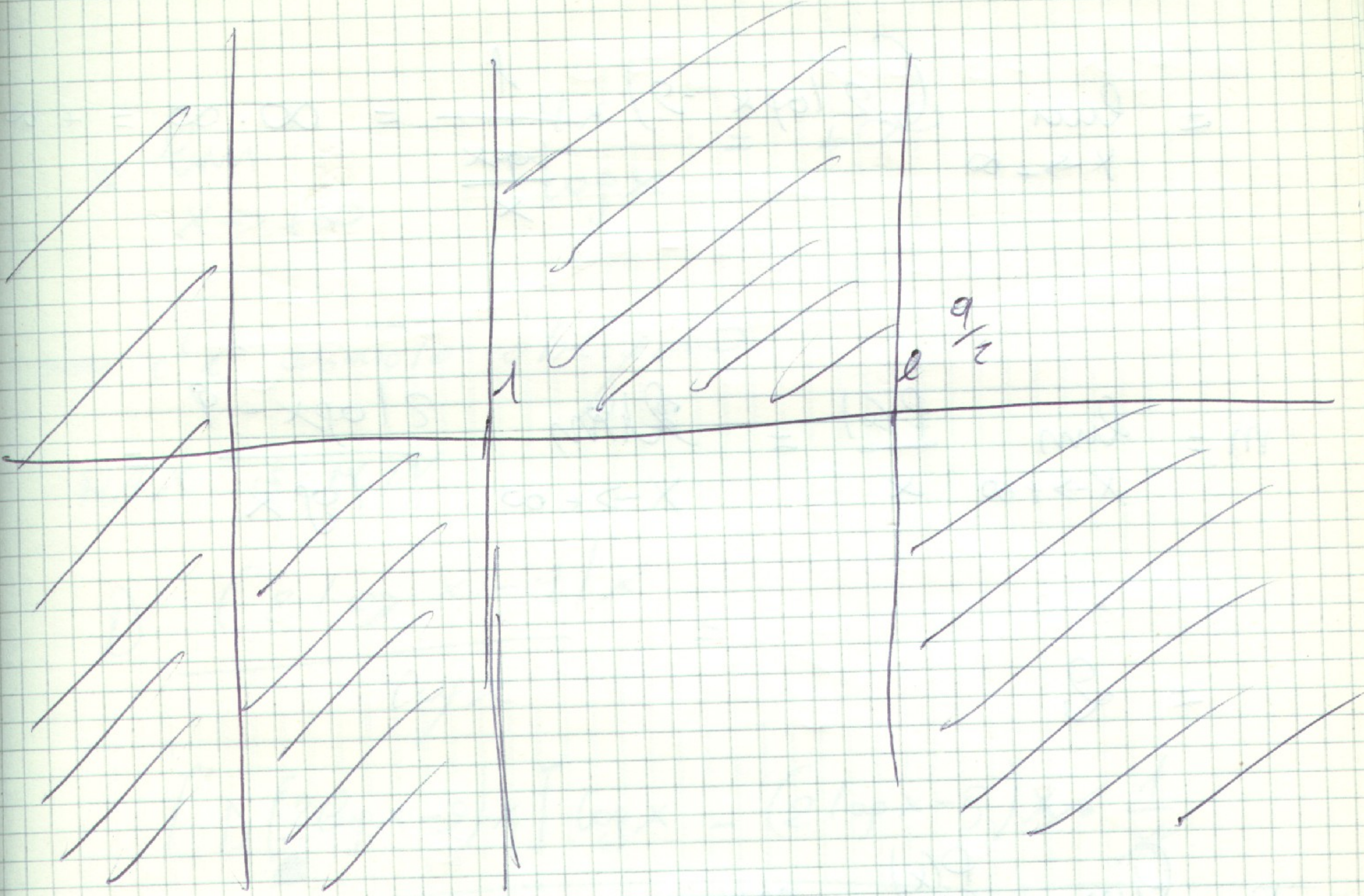


$$\lim_{x\to 0} \frac{(2\log x - 9)\,x}{\log x} = 2x - 9\frac{x}{\log x} = 0+0=0$$

$$\lim_{x\to 1^-} \frac{(2\log x - 9)\,x}{\log x} = \frac{-9}{0^-} = 9$$

$$\lim_{x\to e^{\frac{9}{2}}{}^-} \frac{(2\log x - 9)\,x}{\log x} = \frac{0^- e^{\frac{9}{2}}}{\frac{9}{2}} = -\infty$$

$y=0$ $\quad 2\log x - 9 = 0 \quad \log x = \frac{9}{2} \quad x = e^{/2}$



$$\lim_{x\to 0} \frac{(2\log x - 9)\,x}{\log x} = 2x - 9\frac{x}{\log x} = 0+0=0$$

$$\lim_{x\to 1^-} \frac{(2\log x - 9)\,x}{\log x} = \frac{-9}{0^-} = 9$$

$$\lim_{x\to e^{\frac{9}{2}-}} \frac{(2\log x - 9)\,x}{\log x} = \frac{0^- e^{9/2}}{\frac{9}{2}} = -\infty$$

$$\lim_{x\to e^{\frac{9}{2}+}} f(x) = +\infty$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} 2x - \frac{9x}{\log x}$$

$$= \lim_{x \to +\infty} (2 \log x - 9) \cdot \frac{1}{\frac{\log x}{x}} = \infty \cdot \infty = +\infty$$

$$m = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{2 \log x - 9}{\log x} =$$

$$= 2$$

$$n = \lim_{x \to +\infty} \frac{f(x)}{\cancel{x}} - mx =$$

$$= \lim_{x \to +\infty} \frac{(2 \log x - 9)x}{\log x} - 2x =$$

$$= \lim_{x \to +\infty} x \left(\frac{2 \log x - 9}{\log x} - 2 \right) =$$

$$= \lim_{x \to +\infty} x \, \frac{2 \log x - 9 - 2 \log x}{\log x} =$$

$X \to +\infty$

$$= \lim_{x \to +\infty} (2 \log x - 9) \cdot \frac{1}{\frac{\log x}{x}} = \infty \cdot \infty = +\infty$$

$$m = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{2 \log x - 9}{\log x} =$$

$$= 2$$

$$n = \lim_{x \to +\infty} \frac{f(x)}{\cancel{x}} - mx =$$

$$= \lim_{x \to +\infty} \frac{(2 \log x - 9)\, x}{\log x} - 2x =$$

$$= \lim_{x \to +\infty} x \left(\frac{2 \log x - 9}{\log x} - 2 \right) =$$

$$= \lim_{x \to +\infty} x \, \frac{2 \log x - 9 - 2 \log x}{\log x} =$$

$$= \lim_{x \to +\infty} x \frac{-9}{\log x} =$$

$$= \lim_{x \to +\infty} \frac{-9x}{\log x} = +\infty$$

non c'è asintoto obliquo

MAX E MIN

$$D \frac{(2\log x - 9)x}{\log x} =$$

$$= \frac{[D(2\log x - 9)x] \log x - (2\log x - 9)\cancel{x} \cdot \frac{1}{\cancel{x}}}{\log^2 x} =$$

$$= \frac{\left(\frac{2}{\cancel{x}} \cdot \cancel{x} + (2\log x - 9) \cdot 1\right) \log x - 2\log x + 9}{\log^2 x} =$$

$$= \frac{\cancel{2\log x} + 2\log^2 x - 9\log x - \cancel{2\log x} + 9}{\log^2 x}$$

$$= \frac{2\log^2 x - 9\log x + 9}{}$$

$$= \lim_{x \to +\infty} \frac{-9x}{\log x} = +\infty$$

non c'è asintoto obliquo

MAX E MIN

$$D\,\frac{(2\log x - 9)x}{\log x} =$$

$$= \frac{\left[D\left((2\log x - 9)x\right)\right]\log x - (2\log x - 9)\cancel{x}\cdot\frac{1}{\cancel{x}}}{\log^2 x} =$$

$$= \frac{\left(\frac{2}{\cancel{x}}\cdot\cancel{x} + (2\log x - 9)\cdot 1\right)\log x - 2\log x + 9}{\log^2 x} =$$

$$= \frac{\cancel{2\log x} + 2\log^2 x - 9\log x - \cancel{2\log x} + 9}{\log^2 x}$$

$$= \frac{2\log^2 x - 9\log x + 9}{\log^2 x}$$

$f'(x) = 0$

$\Downarrow$

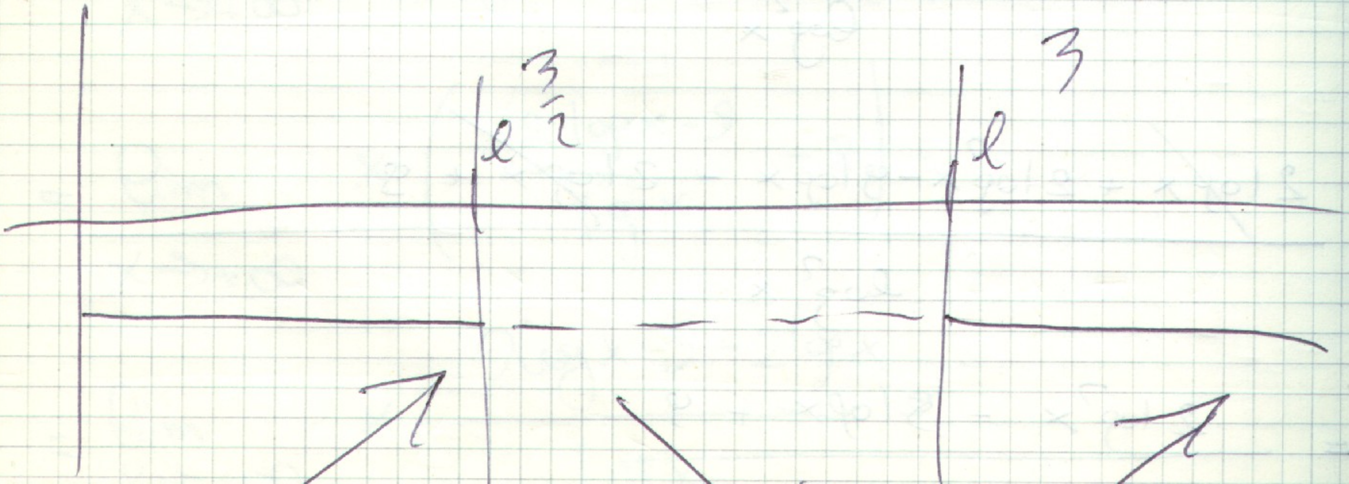
$2 \log^2 x - 9 \log x + 9 = 0$

$$\log x = \frac{9 \pm \sqrt{81 - 72}}{4} = \frac{9 \pm 3}{4}$$

$$\log x = \begin{cases} 3 \\ \frac{3}{2} \end{cases}$$

$x = e^{3}$ $\qquad$ $x = e^{\frac{3}{2}}$

$f'(x) > 0$ $\qquad$ $x < e^{\frac{3}{2}}$ $\qquad$ $x > e^{\frac{3}{2}}$



$x = e^{\frac{3}{2}}$ punto di MAX

$x = e^{3}$ punto di MIN

$$f(e^{\frac{3}{2}}) = \frac{(2\lg e^{\frac{3}{2}} - 9)e^{\frac{3}{2}}}{\log e^{\frac{3}{2}}} =$$

$$= \frac{(2 \cdot \frac{3}{2} - 9)e^{\frac{3}{2}}}{\frac{3}{2}} =$$

$$= \frac{-6e^{\frac{3}{2}}}{\frac{3}{2}} = -\overset{2}{\cancel{6}} \cdot \frac{2}{\cancel{3}} e^{\frac{3}{2}} =$$

$$= -4e^{\frac{3}{2}}$$

$f($

$$f(e^3) = \frac{(2\log e^3 - 9)e^3}{\log e^3} =$$

$$= \frac{(2 \cdot 3 - 9)e^3}{3} =$$

$$= \frac{-3e^3}{3} = -e^3$$

$$f''(x) = D\,\frac{2\log^2 x - 9\log x + 9}{\log^2 x} =$$

$$= \frac{\left(4\log x \cdot \frac{1}{x} - \frac{9}{x}\right)\log^2 x - (2\log^2 x - 9\log x + 9)}{\log^{4\,3} x}$$

$$= \frac{\cancel{4\log^2 x} - 9\log x - \cancel{4\log^2 x} + 18\log x - 18}{x\log^3 x}$$

$$= \frac{9 \log x - 18}{x \log^3 x}$$

$f''(x) = 0 \Longrightarrow \log x = \frac{18}{9}$; $\log x = 2$

$x = e^2$

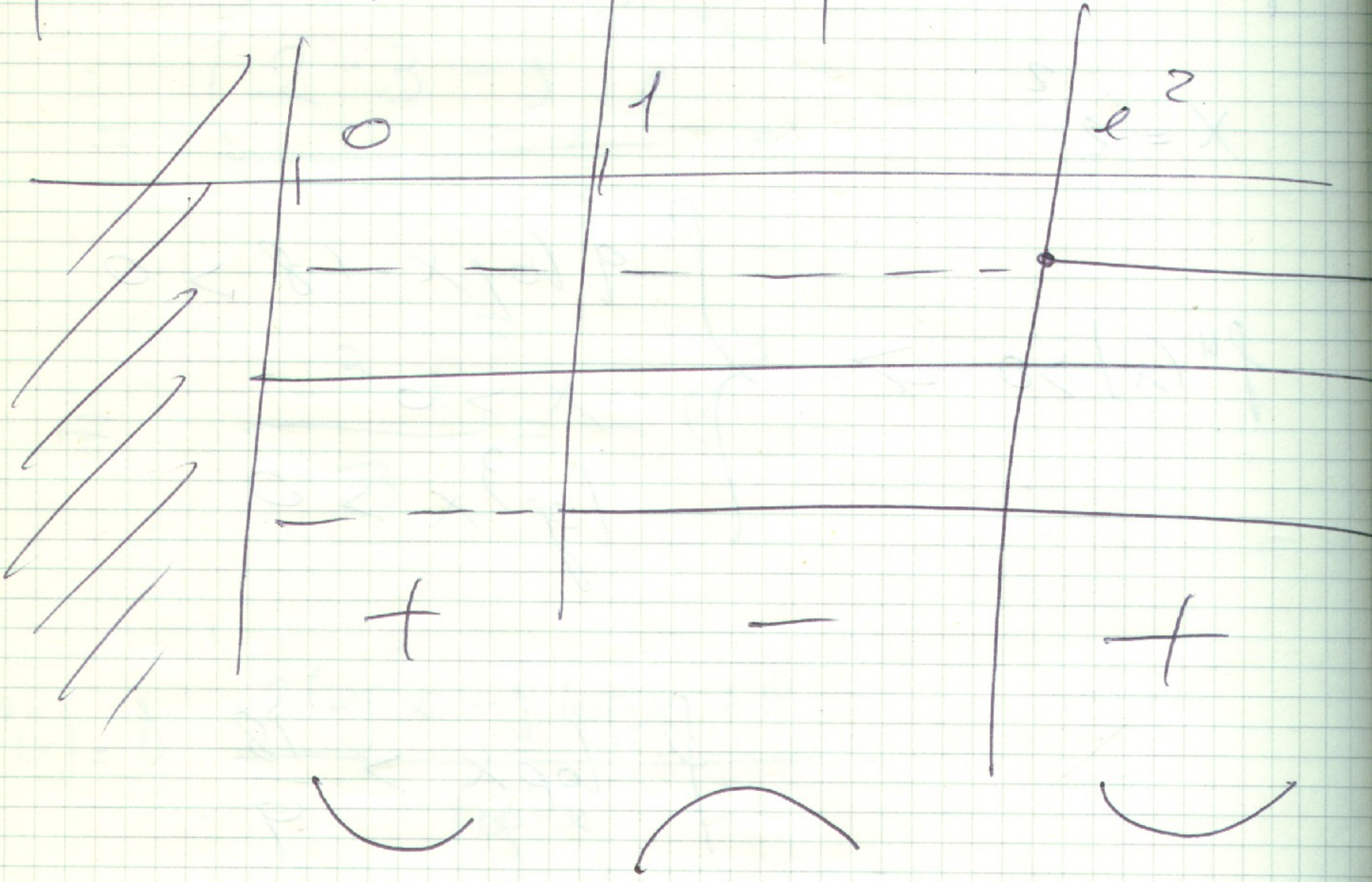
$$f''(x) > 0 \Longrightarrow \begin{cases} 9 \log x - 18 > 0 \\ x > 0 \\ \log^3 x > 0 \end{cases}$$

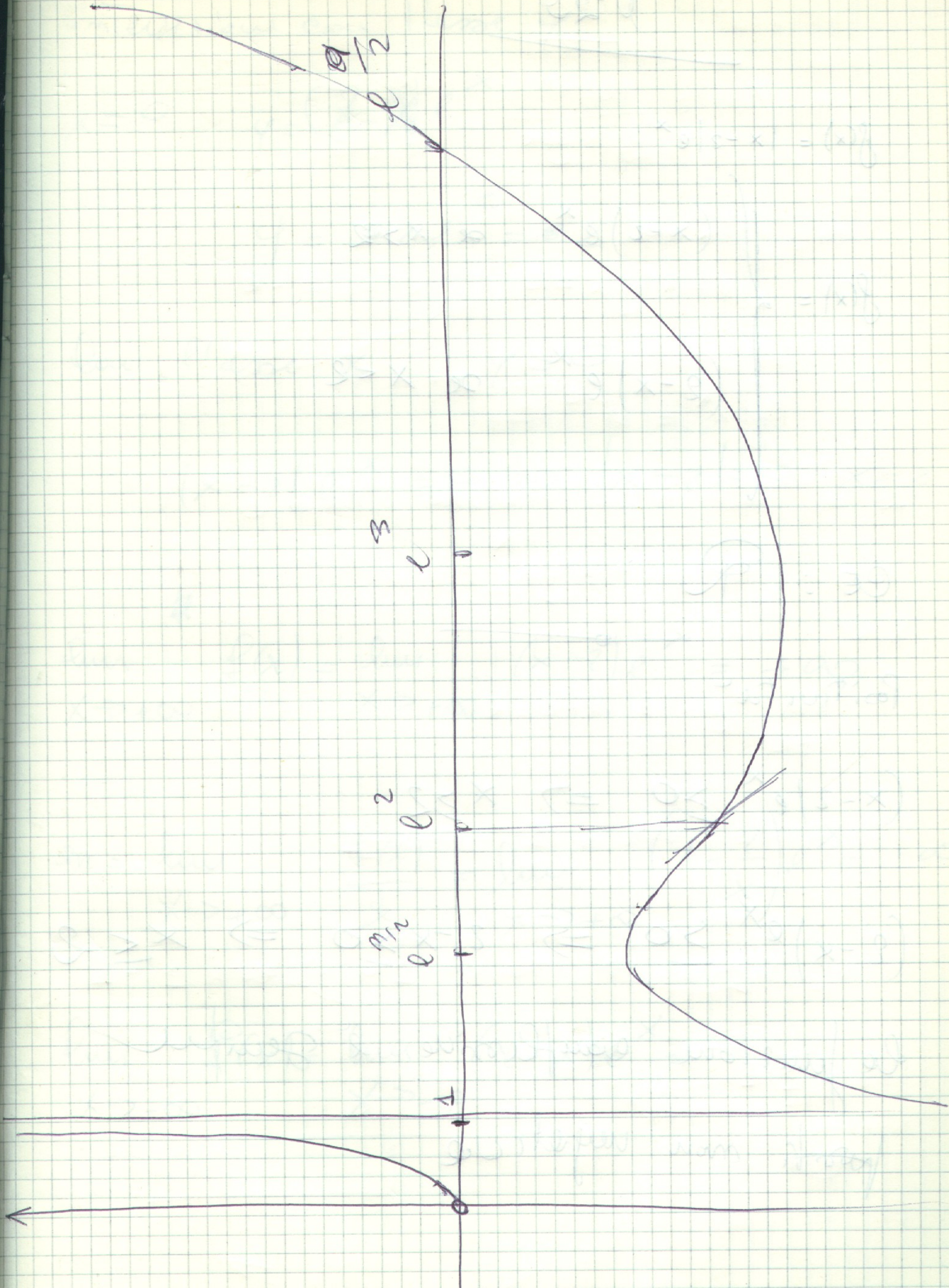
$$\begin{cases} \log x > \frac{18}{9} \\ x > 0 \\ \log x > 0 \end{cases}$$

$$\frac{\cancel{2\log x} \cdot \frac{1}{x}}{\quad} =$$

$-$

$$\begin{cases} \log x > 2 \\ x > 0 \\ x > 1 \end{cases} \qquad \begin{cases} x > e^2 \\ x > 0 \\ x > 1 \end{cases}$$



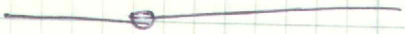
0
1
$e^{3/2}$
e^2
e^3
$e^{9/2}$

N°29

$f(x) = |x-2|e^x$

$$f(x) = \begin{cases} (x-2)e^x & \text{se } x>2 \\ (2-x)e^x & \text{se } x<2 \end{cases}$$

CE : $\mathbb{R}$



Positività

$(x-2)e^x \geq 0 \Rightarrow x \geq 2$

$(2-x)e^x \geq 0 \Rightarrow 2-x \geq 0 \Rightarrow x \leq 2$

la funzione complessiva è sempre ~~posit~~ non negativa

Intersezione con l'asse y

$x=0 \qquad y=(2-\cancel{x})e^{x}$

$$y = 2e^{0} = 2$$

Con l'asse x $\qquad y=0$

$$|x-2|e^{x}=0 \Rightarrow |x-2|=0 \quad x=2$$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} (x-2)e^{x} = +\infty$$

$$m = \lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \left(1-\frac{2}{x}\right)e^{x} = +\infty$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} (2-x)e^{x} =$$

$$f(x) = \frac{\log x}{e + x\log x}$$

C.E.: $\begin{cases} x > 0 \\ e + x\log x \neq 0 \end{cases}$ $\begin{cases} x > 0 \\ x\log x \neq -e \end{cases}$

~~$\log x^x \neq$ [illegible]~~

la eq. non si può risolvere. Studiamo

$$g(x) = e + x\log x$$

$x > 0$

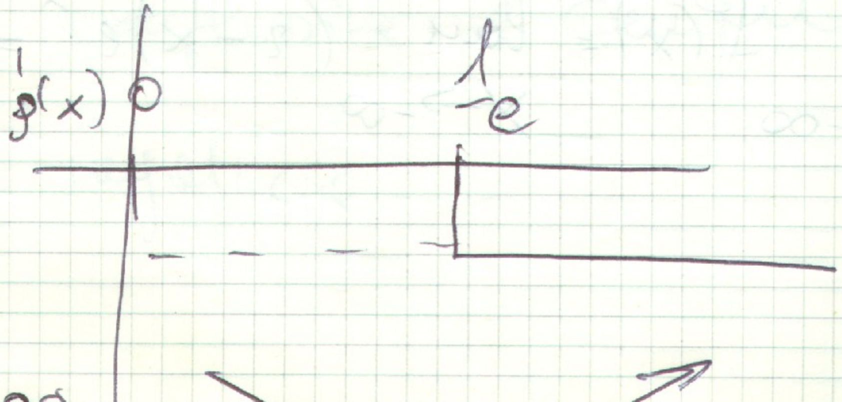
$$\lim_{x \to 0^+} g(x) = e + 0 = e$$

$$g'(x) = \log x + \frac{\cancel{x}}{\cancel{x}} = \log x + 1$$

$$g'(x) \geq 0 \Rightarrow \log x + 1 \geq 0 \Rightarrow \log x \geq -1$$

$$x \geq e^{-1}$$

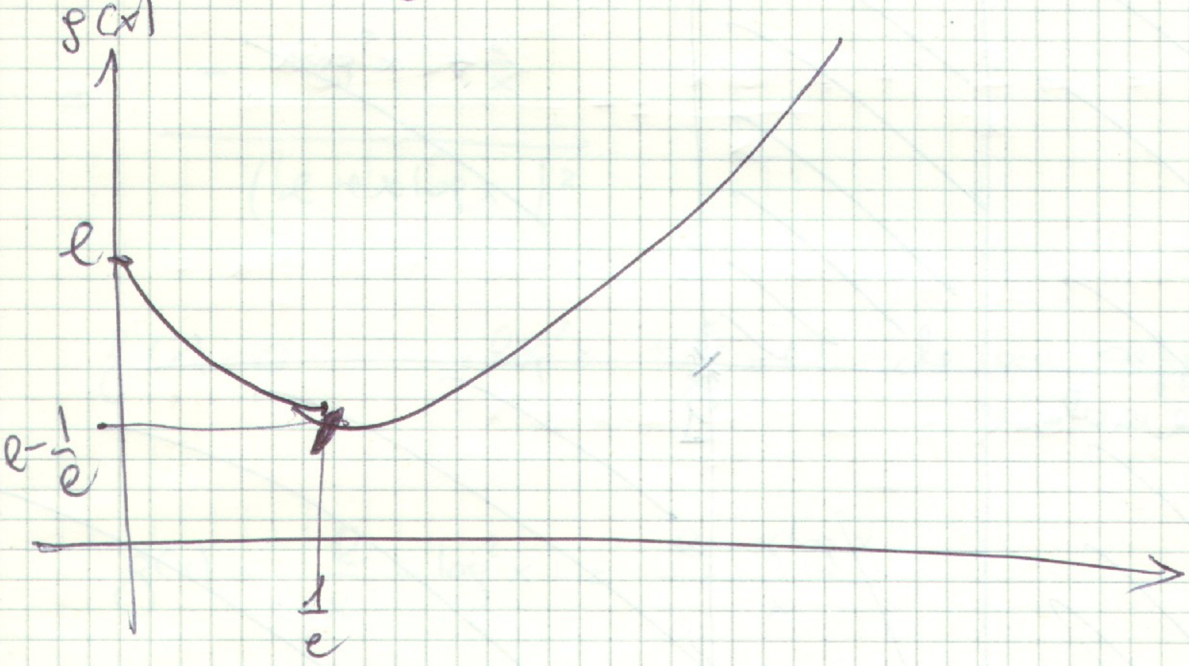
~~g(x~~



$g(x)$ ha un minimo in $x = \frac{1}{e}$

$$g\left(\frac{1}{e}\right) = e + \frac{1}{e}\log\frac{1}{e} = e + \frac{1}{e}\log e^{-1} =$$

$$= e - \frac{1}{e} > 0$$



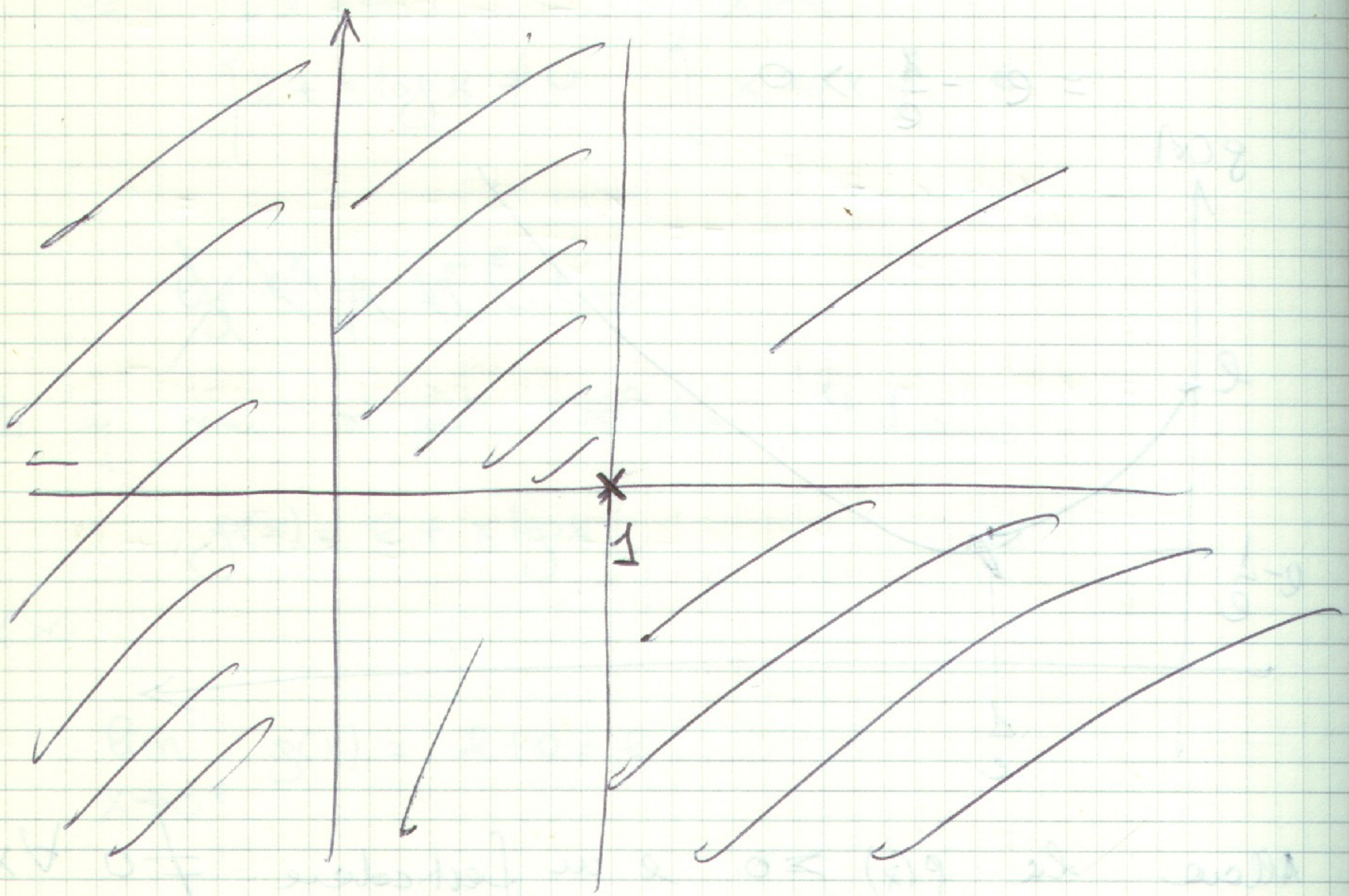
Allora la $g(x) \geq 0$ e in particolare $\neq 0 \ \forall x > 0$

$$e - x\log x \neq 0 \quad \forall x > 0$$

$$c \in \,]0, +\infty[$$

Positività

$$\begin{cases} \log x > 0 \\ e + x\log x > 0 \end{cases} \qquad \begin{cases} x > 1 \\ \forall x > 0 \end{cases}$$



Intersezione con l'asse x per x = 1

$$\lim_{x \to 0^+} \frac{\log x}{e + x\log x} = \frac{-\infty}{e} = -\infty$$

$$\lim_{x \to +\infty} \frac{\log x}{x\log x + e} = \lim_{x \to +\infty} \frac{\cancel{\log x}}{\cancel{\log x}\left(x + \frac{e}{\log x}\right)} = \frac{1}{\infty + 0} = 0$$

MAX E MIN

$$D\,\frac{\log x}{e+x\log x}=\frac{\frac{1}{x}(e+x\log x)-\log x\left(\log x+\frac{x}{x}\right)}{(e+x\log x)^2}=$$

$$=\frac{\frac{e}{x}+\cancel{\log x}-\log^2 x+\cancel{\log x}}{(e+x\log x)^2}$$

$$=\frac{-\log^2 x+\frac{e}{x}}{(e+x\log x)^2}$$

$f'(x)=0 \qquad -\log^2 x+\frac{e}{x}=0$ che non sappiamo risolvere

$$g(x)=\frac{e}{x}-\log^2 x=\frac{e-x\log^2 x}{x}$$

$$\cancel{\lim_{x\to 0} g(x)}=\lim_{x\to 0} x\log^2 x=0\cdot(+\infty)$$

$$=\lim_{x\to 0}\frac{\log^2 x}{\frac{1}{x}}=\lim_{x\to 0}\frac{2(\log x)\cdot\cancel{\frac{1}{x}}}{-\frac{1}{\cancel{x^2}}}=$$

$$=\lim_{x\to 0^+} -2x\log x=0$$

$$\lim\quad g(x)=\frac{e}{\ }=+\infty$$

$$D\,\frac{\log x}{e + x\log x} = \frac{\frac{\;}{x}(e + x\log x) - \log x\left(\log x + \frac{\;}{x}\right)}{(e + x\log x)^2} =$$

$$= \frac{\frac{e}{x} + \cancel{\log x} - \log^2 x + \cancel{\log x}}{(e + x\log x)^2}$$

$$= \frac{-\log^2 x + \frac{e}{x}}{(e + x\log x)^2}$$

$f'(x) = 0 \qquad -\log^2 x + \frac{e}{x} = 0$ che non sappiamo risolvere

$$g(x) = \frac{e}{x} - \log^2 x = \frac{e - x\log^2 x}{x}$$

$$\cancel{\lim_{x\to 0} g(x) =} \quad \lim_{x\to 0} x\log^2 x = 0\cdot(+\infty)$$

$$= \lim_{x\to 0} \frac{\log^2 x}{\frac{1}{x}} = \lim_{x\to 0} \frac{2(\log x)\cdot\cancel{\frac{1}{x}}}{-\frac{1}{\cancel{x^2}}} =$$

$$= \lim_{x\to 0^+} -2\,x\log x = 0$$

$$\lim_{x\to 0^+} g(x) = \frac{e}{0} = +\infty$$

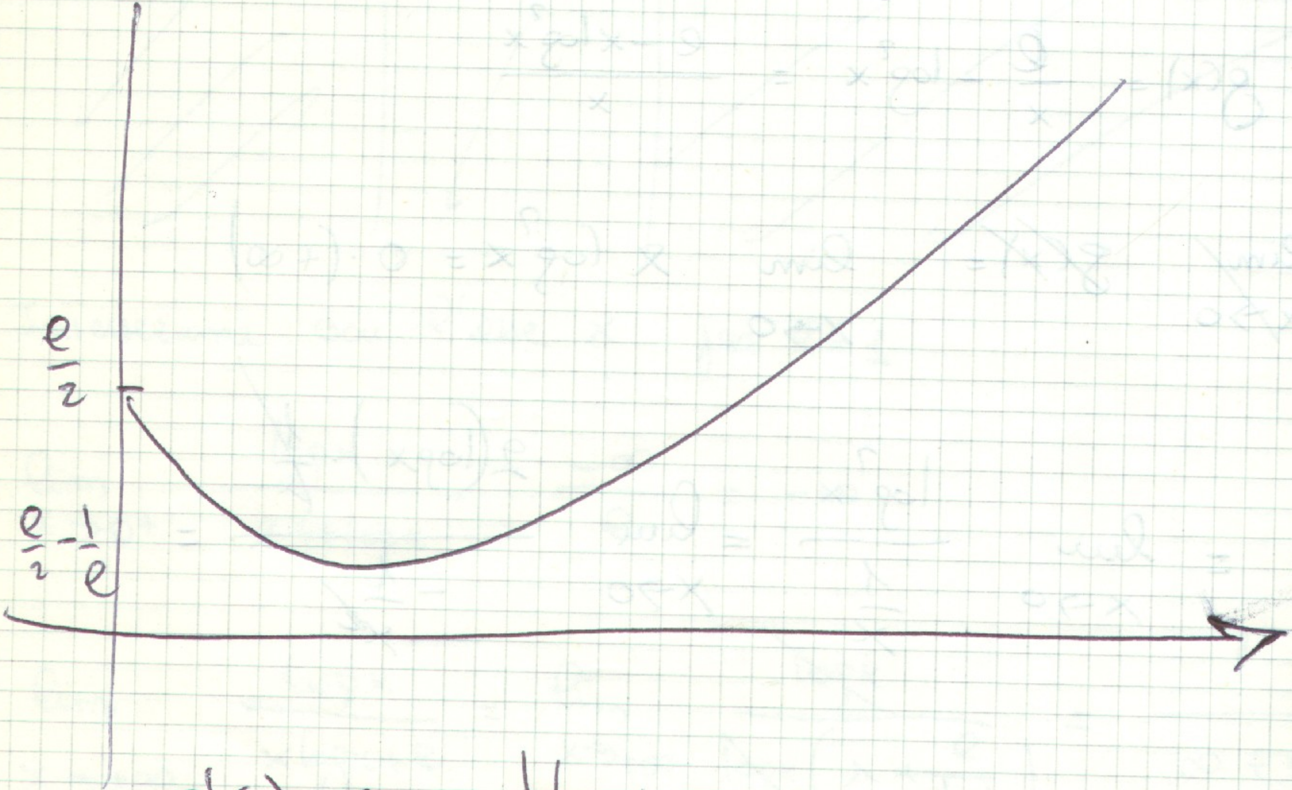
$$\lim_{x\to+\infty} g(x) = -\lim_{x\to+\infty}\left(\frac{e}{x} - \log^2 x\right| =$$

$$= 0 - \infty = -\infty$$

$$g'(x) = -\frac{e}{x^2} - \frac{2\log x}{x} = -\frac{e+2x\log x}{x^2}$$

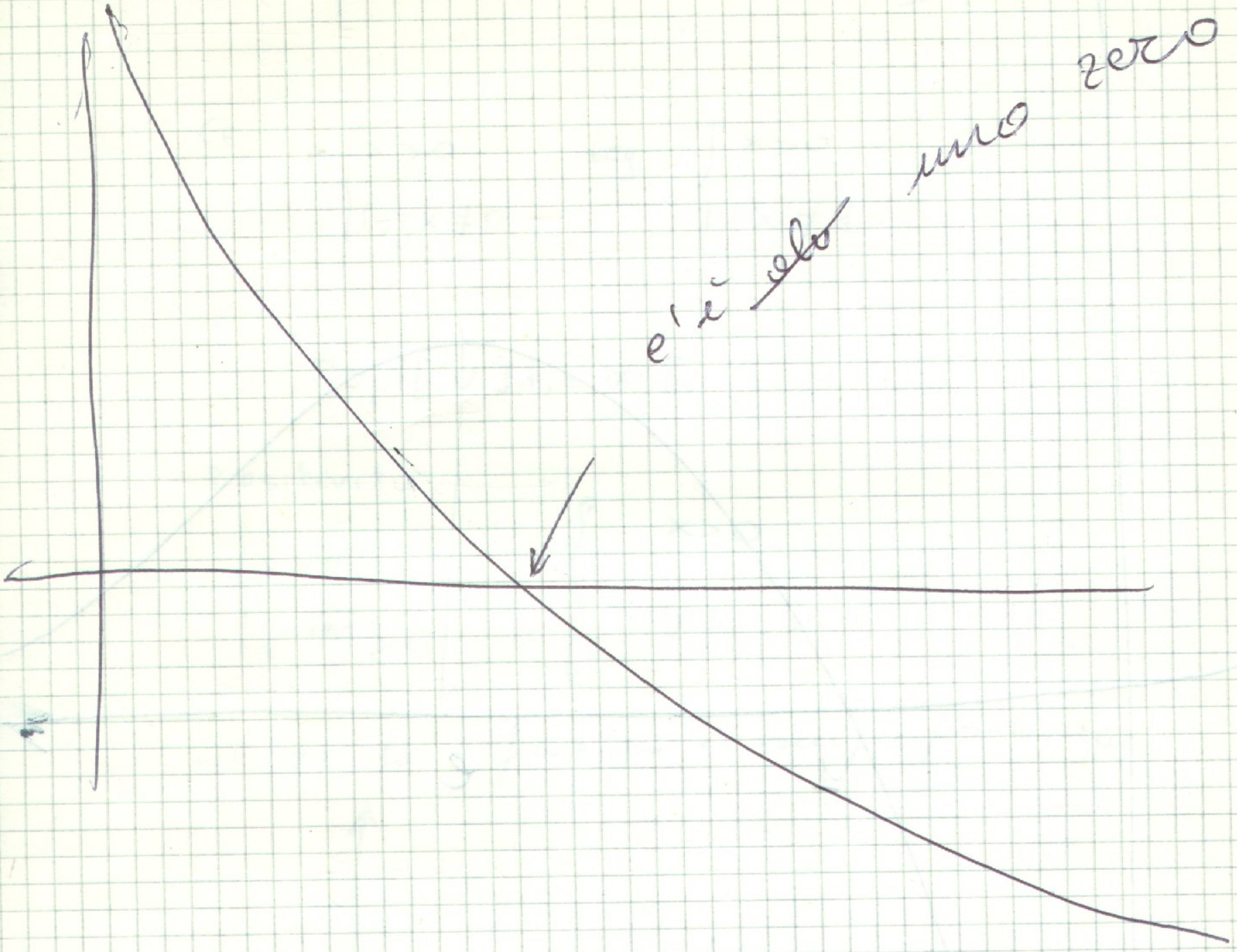
sfruttando i risultati di $\underline{e + x\log x} =$

$$e+2x\log x = 2\left(\frac{e}{2} + x\log x\right)$$



$$g'(x) < 0 \quad \forall x > 0$$

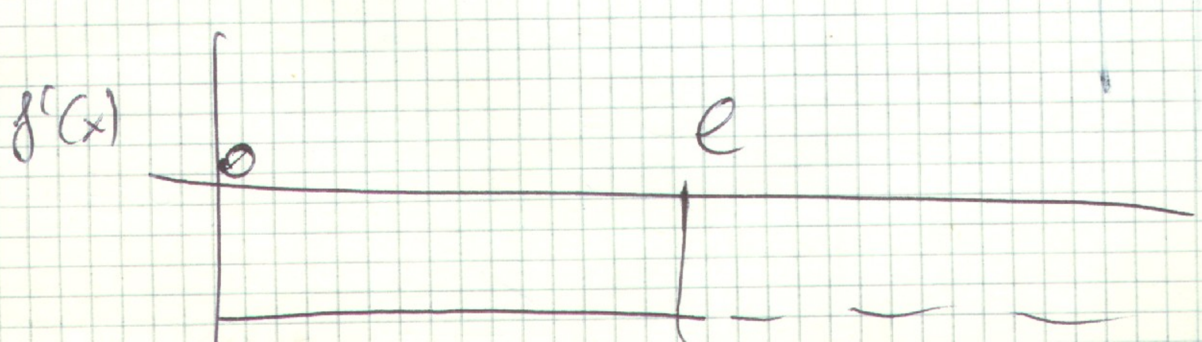
f(x) e' sempre decrescente

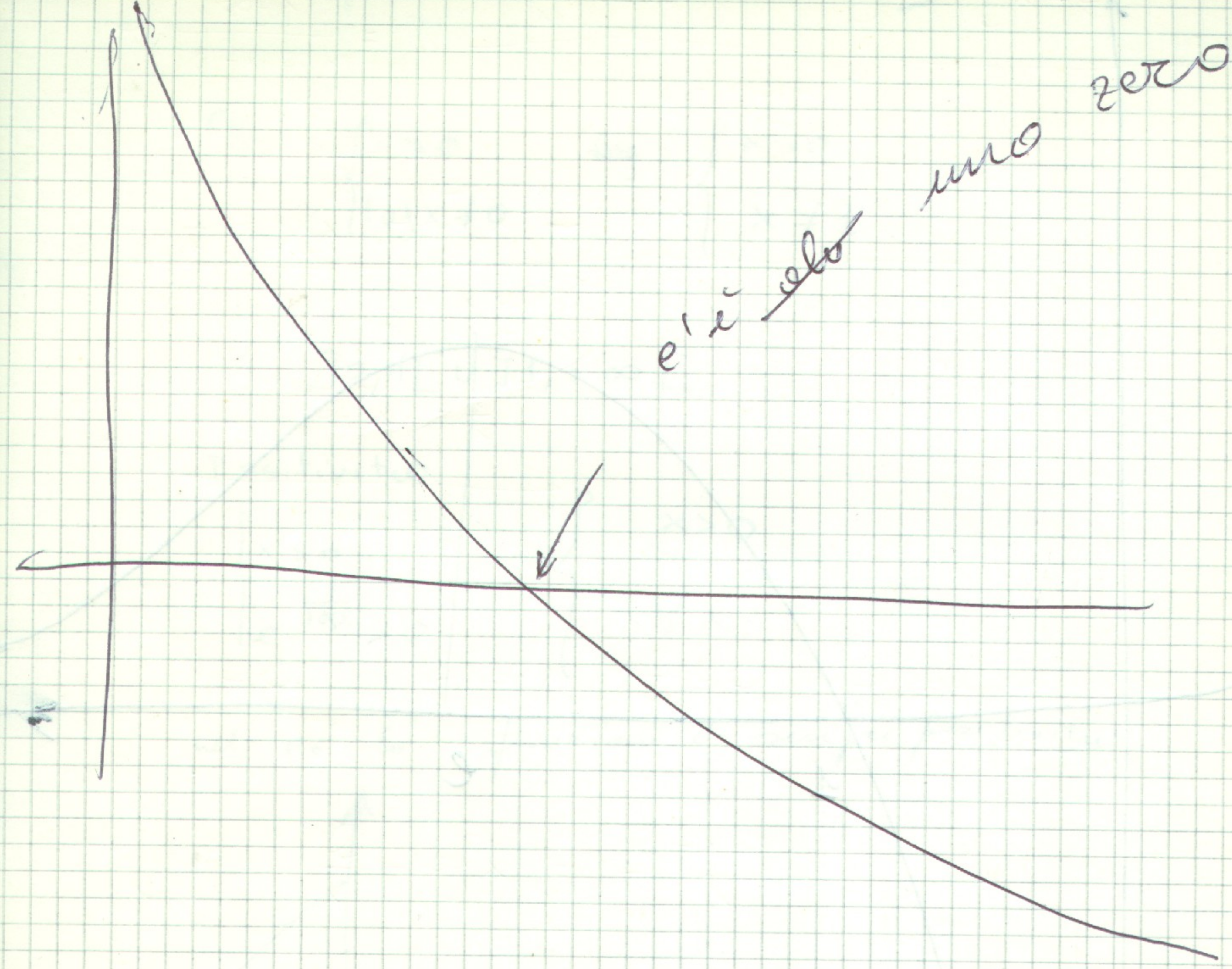


quindi f'(x) si annulla in un punto

$$f'(e) = -\log e + \frac{e}{e} = -1 + 1 = 0$$

$\underline{x = e}$

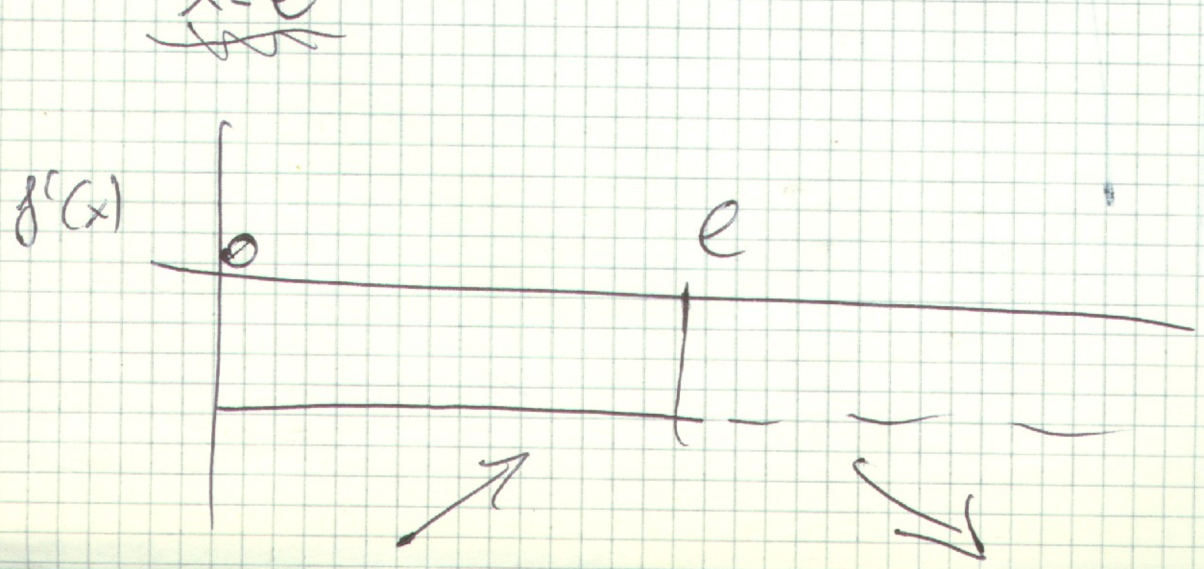


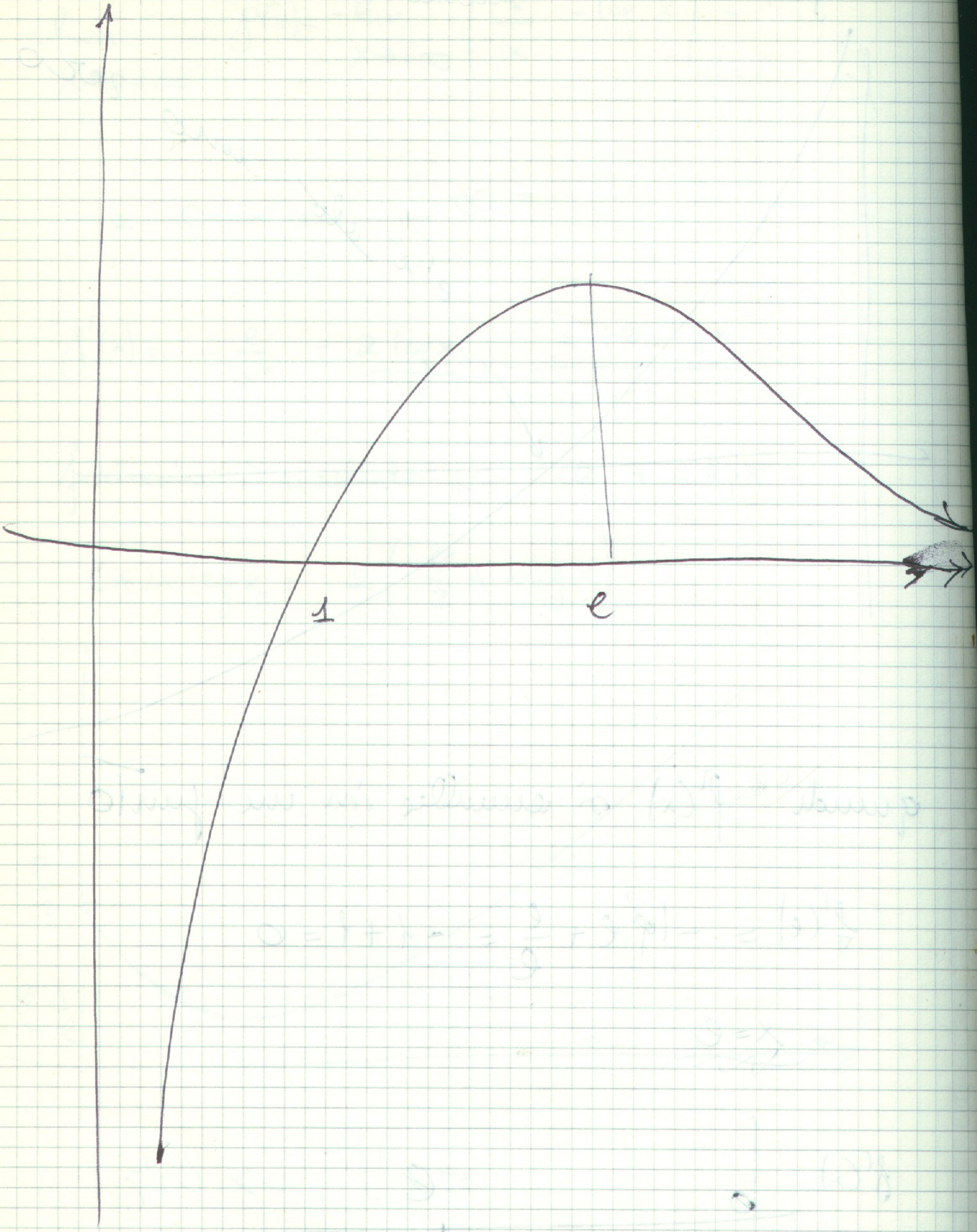


quindi $f'(x)$ si annulla in un punto

$$f'(e) = -\log e + \frac{e}{e} = -1 + 1 = 0$$

$x = e$




1
e

30

$$f(x) = x e^{\frac{1}{\log x}}$$

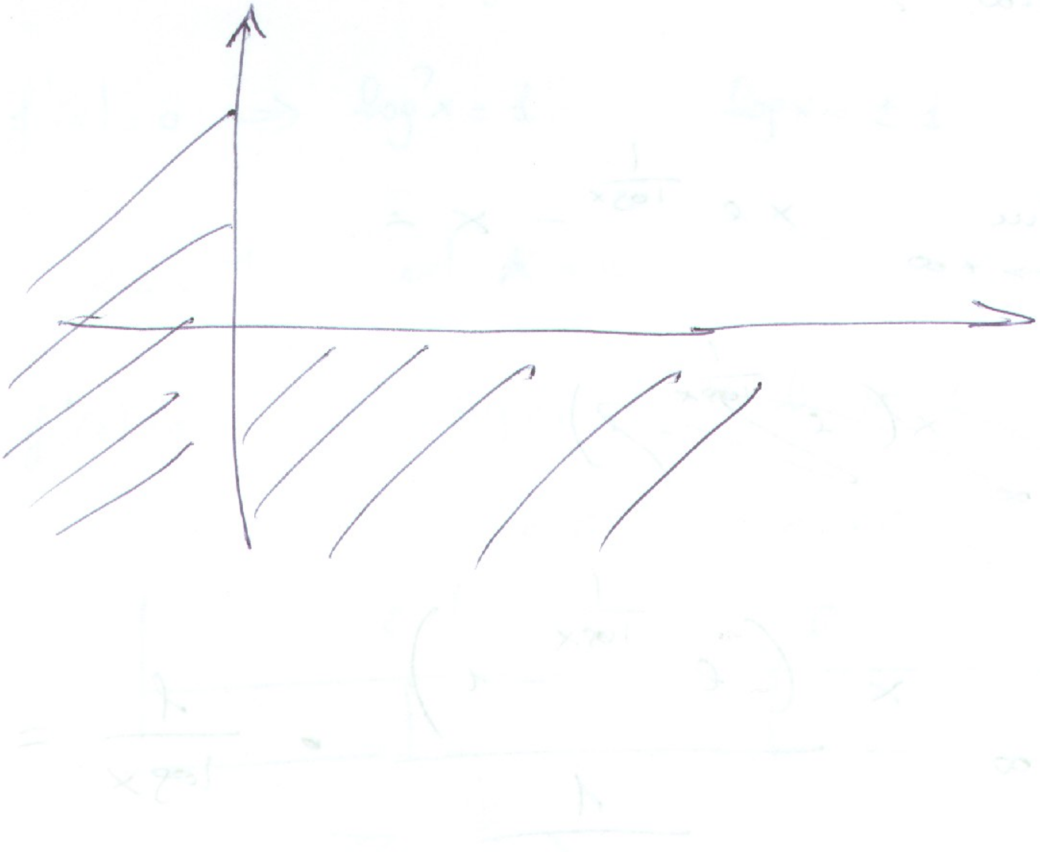
CE: $\begin{cases} x > 0 \\ \log x \neq 0 \end{cases}$ ~~log~~ $\begin{cases} x > 0 \\ x \neq 1 \end{cases}$

$]0,1[\cup]1,+\infty[$

Positività

$\begin{cases} x > 0 \\ e^{\frac{1}{\log x}} > 0 \end{cases}$ $\begin{cases} x > 0 \\ \forall x \end{cases}$

nel CE la funzione è sempre positività



$$\lim_{x \to 0^+} x e^{\frac{1}{\log x}} = 0 \, e^{-\infty} = 0$$

$$\lim_{x \to 1^-} x e^{\frac{1}{\log x}} = 1 \cdot 0 = 0$$

$$\frac{e^y - 1}{y}$$

$$\lim_{x \to 1^+} x e^{\frac{1}{\log x}} = +\infty$$

$$\lim_{x \to \infty} x e^{\frac{1}{\log x}} = +\infty$$

$$m = \lim_{x \to +\infty} \frac{\cancel{x} e^{\frac{1}{\log x}}}{\cancel{x}} = 1$$

$$n = \lim_{x \to +\infty} x e^{\frac{1}{\log x}} - x =$$

$$= \lim_{x \to +\infty} x \left(e^{\frac{1}{\log x}} - 1 \right)$$

$$= \lim_{x \to +\infty} x \frac{\left(e^{\frac{1}{\log x}} - 1 \right)}{\frac{1}{\log x}} \cdot \frac{1}{\log x} =$$

$$=$$

$$= \lim_{x \to +\infty} \frac{x}{\log x} \cdot \frac{e^{\frac{1}{\log x}} - 1}{\frac{1}{\log x}} =$$

$$\begin{array}{r} 15\times \\ 4 \\ \hline 60\times \\ 4 \\ \hline 240 \end{array}$$

$= \cancel{0}\ +\infty$ l'asintoto obliquo non c'e'

MAX e MIN

$$D\, x e^{\frac{1}{\log x}} = e^{\frac{1}{\log x}} + x e^{\frac{1}{\log x}} \cdot \frac{\left(-\frac{1}{x}\right)}{\log^2 x} =$$

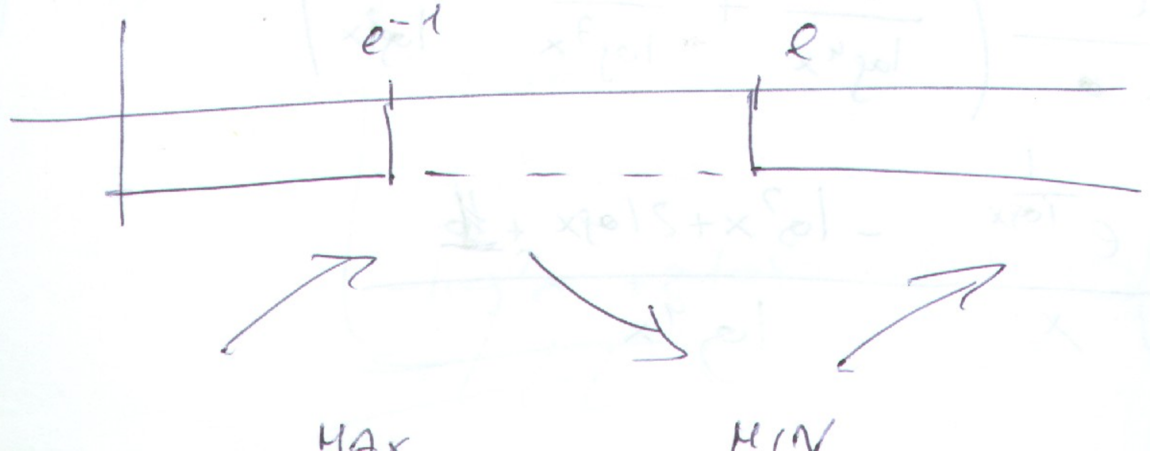
$$= e^{\frac{1}{\log x}} - \frac{e^{\frac{1}{\log x}}}{\log^2 x} = e^{\frac{1}{\log x}} \frac{\log^2 x - 1}{\log^2 x}$$

$f'(x) = 0 \Rightarrow \log^2 x = 1 \qquad \log x = \pm 1$

$x = e^{-1}$ ed $x = e$

$f'(x) > 0 \qquad \log x < -1 \qquad \log x > 1$

$x < e^{-1} \qquad x > e$



$$f''(x) = D\left[e^{\frac{1}{\log x}}\left(1 - \frac{1}{\log^2 x}\right)\right] =$$

$$= e^{\frac{1}{\log x}}\left(-\frac{\frac{1}{x}}{\log^2 x}\right)\left(1-\frac{1}{\log^2 x}\right) +$$

$$+ e^{\frac{1}{\log x}}\left(0 + \frac{2\log x}{\log^4 x}\cdot\frac{1}{x}\right) =$$

$$= e^{\frac{1}{\log x}}\left[-\frac{1}{x\log^2 x} + \frac{1}{x\log^4 x} + \frac{2}{x\log^3 x}\right] =$$

$$= \frac{e^{\frac{1}{\log x}}}{x\ \cancel{x}}\left(\frac{1}{\log^4 x} + \frac{2}{\log^3 x} - \frac{1}{\log^2 x}\right)$$

$$= \frac{e^{\frac{1}{\log x}}}{x}\ \frac{-\log^2 x + 2\log x + 1}{\log^4 x}$$

$f''(x) = 0$

$-\log^2 x + 2\log x + 1 = 0$

$y = \log x$

$-y^2 + 2y + 1 = 0$

$y^2 - 2y - 1 = 0$

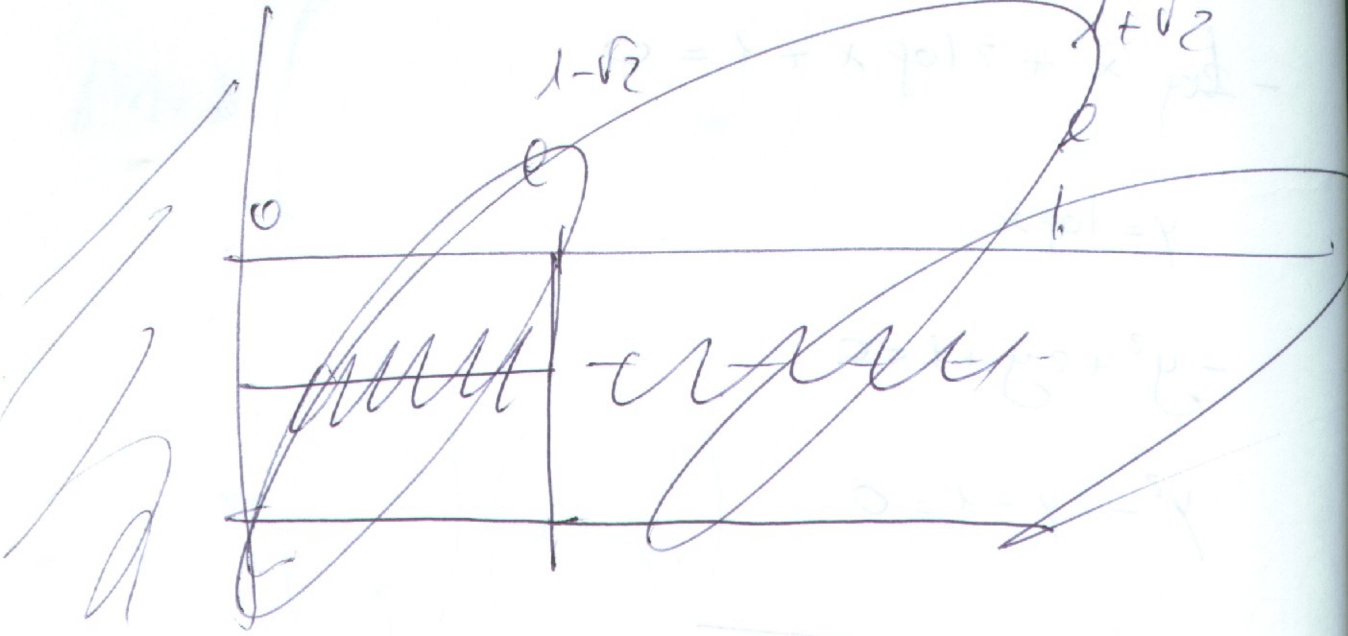
$$y = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$$

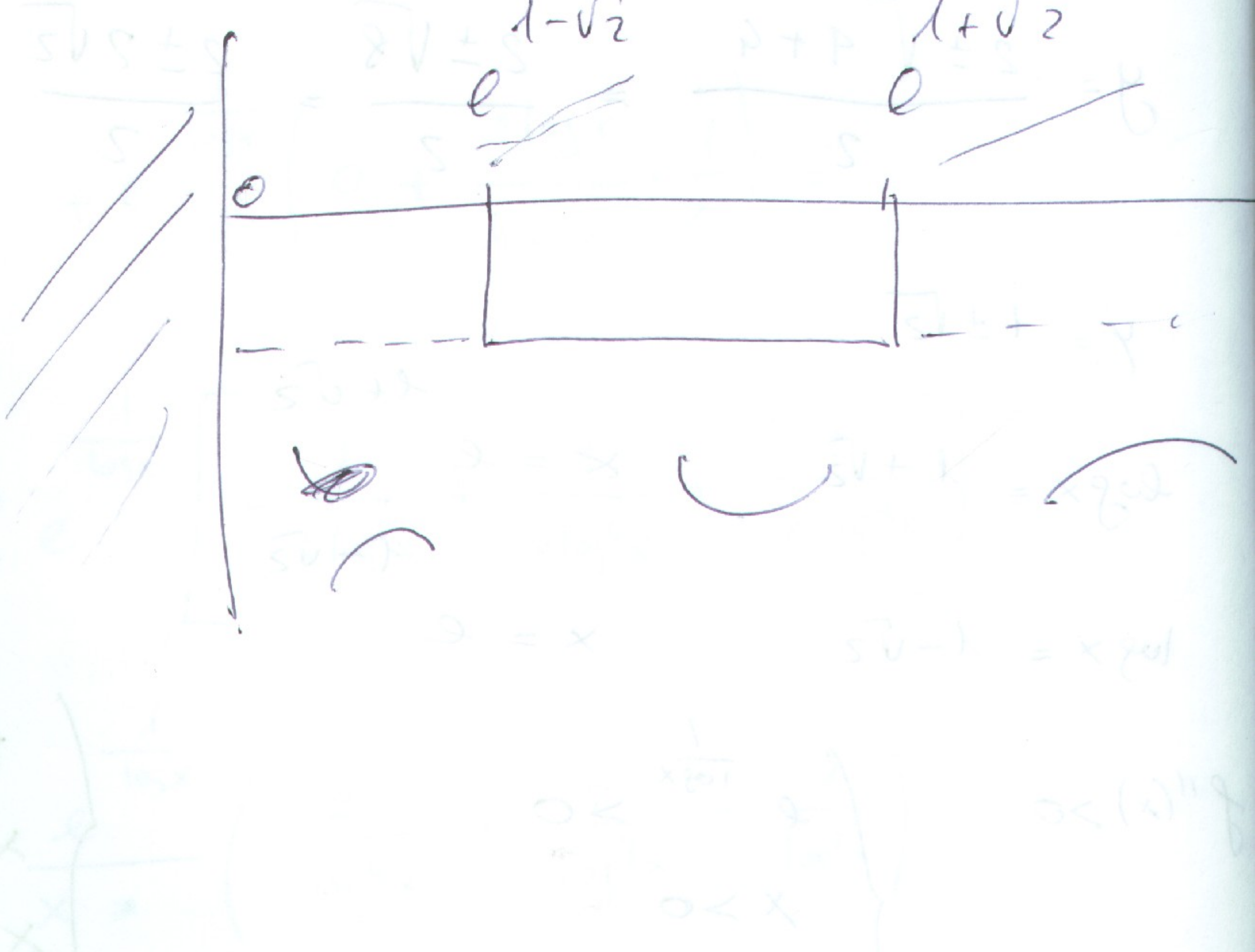
$y = 1 \pm \sqrt{2}$

$\log x = 1 + \sqrt{2} \qquad x = e^{1+\sqrt{2}}$

$\log x = 1 - \sqrt{2} \qquad x = e^{1-\sqrt{2}}$

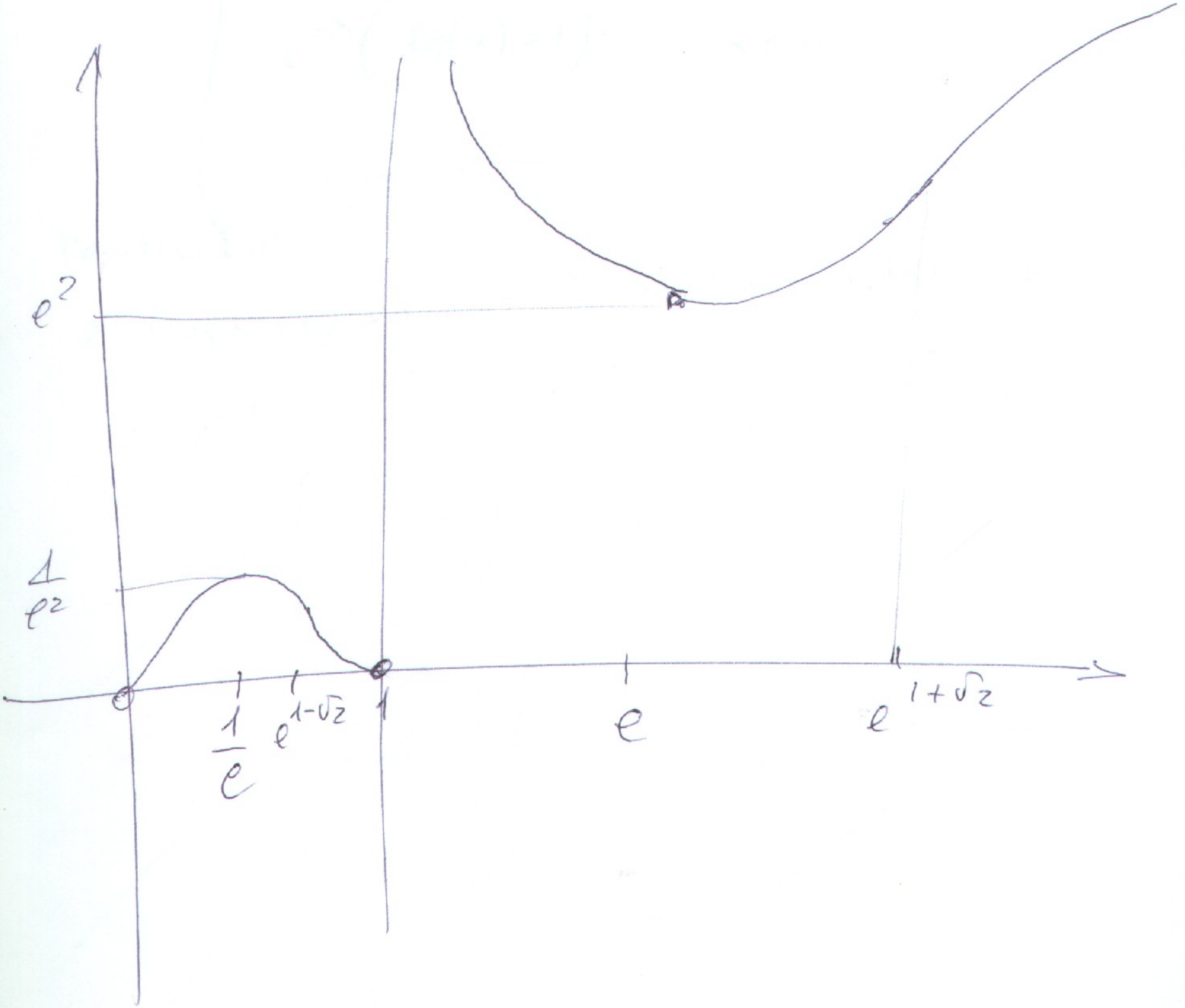
$$f''(x) > 0 \qquad \begin{cases} e^{\frac{1}{\log x}} > 0 \\ x > 0 \\ \log^4 x > 0 \\ -\log^2 x + 2\log x + 1 > 0 \end{cases} \qquad \begin{cases} \forall x \\ x > 0 \\ x > 0 \\ e^{1-\sqrt{2}} < x < e^{1+\sqrt{2}} \end{cases}$$


1-√2
1+√2
e
e
0
1


1-√2
1+√2
e
e
0

$$f\left(\frac{1}{e}\right) = \frac{1}{e} \cdot e^{\frac{1}{\log e^{-1}}} = \frac{1}{e} \cdot e^{-1} = \frac{1}{e^2}$$

$$f(e) = e \cdot e^{\frac{1}{\log e}} = e \cdot e = e^2$$



$f(x) = e^{-x}\left(\log|x| + \frac{x}{|x|}\right)$

31

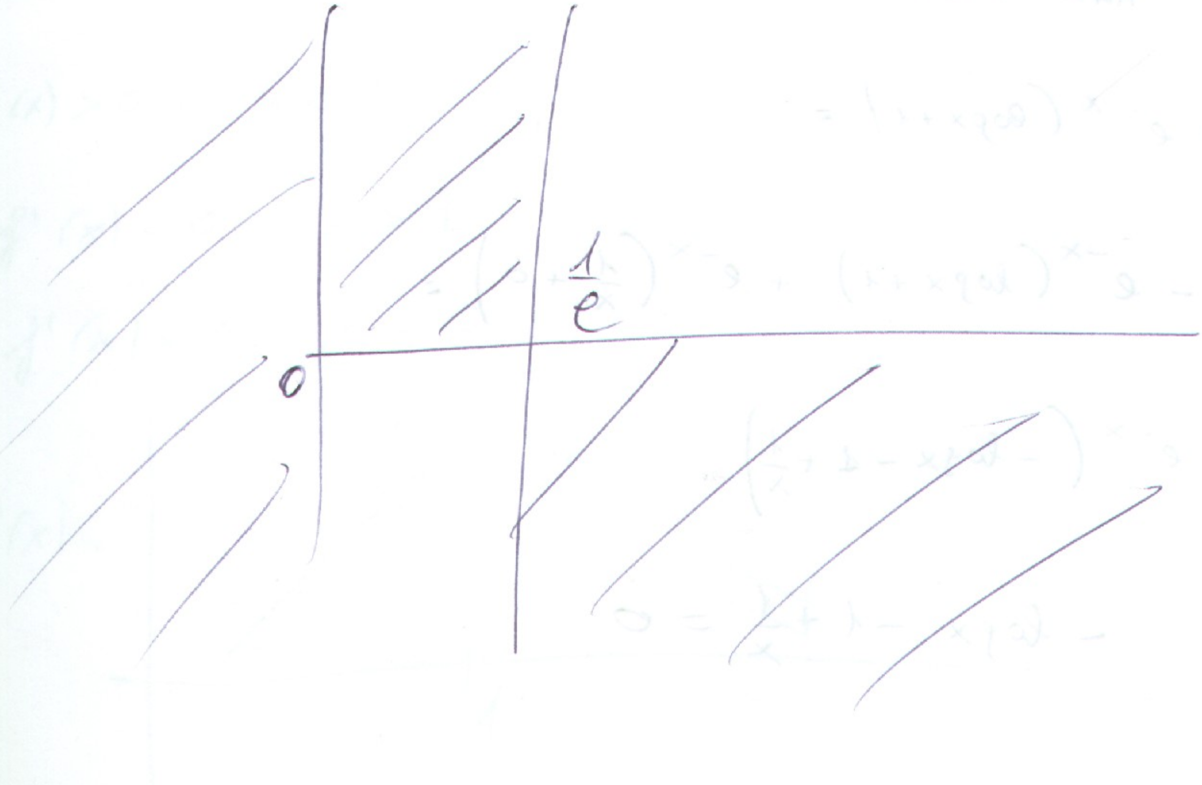
CE: $|x| \neq 0$ $x \neq 0$ $R - \{0\}$

$$f(x) = \begin{cases} e^{-x}(\log x + 1) & x > 0 \\ e^{-x}(\log(-x) - 1) & x < 0 \end{cases}$$

Positività

$e^{-x}(\log x + 1) > 0 \Rightarrow \log x + 1 > 0 \Rightarrow \log x > -1$

$x > e^{-1}$



$$\lim_{x \to 0^+} e^{-x}(\log x + 1) =$$

$$= 1(-\infty + 1) = -\infty$$

$$\lim_{x \to +\infty} e^{-x}(\log x + 1) =$$

$$= \lim_{x \to +\infty} \frac{\log x + 1}{e^x} = \lim_{x \to +\infty} \frac{\frac{1}{x}}{e^x} =$$

$$= 0$$

MAX E MIN

$$D\, e^{-x}(\log x + 1) =$$

$$= -e^{-x}(\log x + 1) + e^{-x}\left(\frac{1}{x} + 0\right) =$$

$$= e^{-x}\left(-\log x - 1 + \frac{1}{x}\right)$$

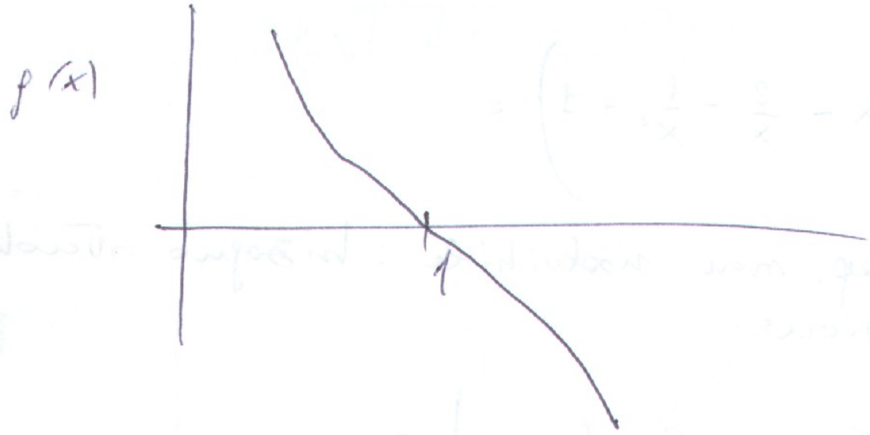
$$-\log x - 1 + \frac{1}{x} = 0$$

$g(x) = -\log x + \frac{1}{x} - 1$

$g'(x) = -\frac{1}{x} + \frac{1}{x^2} = -\frac{x+1}{x^2}$

per $x>0$ $g'(x) > 0$ $g(x)$ è una funzione decrescente

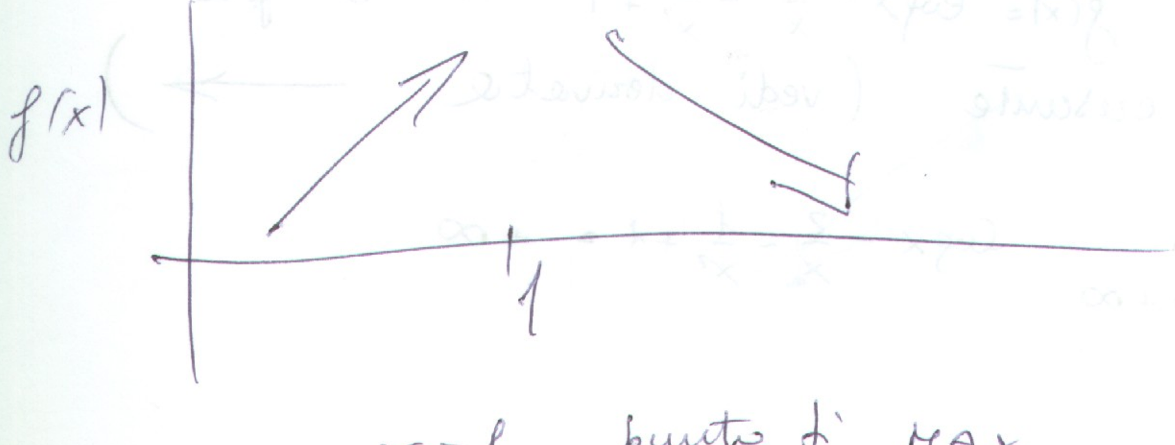
$g(1) = -0 + 1 - 1 = 0$



$f'(x) > 0$ $x < 1$

$f'(x) < 0$ $x > 1$

$f'(x) = 0$ $x = 0$



$x = 1$ punto di MAX

$$f''(x) = D\, e^{-x}\left(-\log x - 1 + \frac{1}{x}\right) =$$

$$= -e^{-x}\left(-\log x - 1 + \frac{1}{x}\right) + e^{-x}\left(-\frac{1}{x} - \frac{1}{x^2}\right) =$$

$$= e^{-x}\left(\log x + 1 - \frac{1}{x} - \frac{1}{x} - \frac{1}{x^2}\right)$$

$$= e^{-x}\left(\underline{\log x - \frac{2}{x} - \frac{1}{x^2}} + 1\right) =$$

Eq. $f''(x) = 0$ eq. non risolvibile : bisogno studiare
funzione

$=$ ~~e^{-x}~~

$$\lim_{x \to 0} \left(\log x - \frac{2}{x} - \frac{1}{x^2} + 1\right) =$$

$$= D.\, \lim_{x \to 0^+} \frac{1}{x^2}\left(x^2 \log x - 2x - 1 + x^2\right) = -\frac{1}{0} = -\infty$$

la $f(x) = \log x - \frac{2}{x} - \frac{1}{x^2} + 1$ è sempre crescente (vedi derivata $\longrightarrow$)

$$\lim_{x \to +\infty} \log x - \frac{2}{x} - \frac{1}{x^2} + 1 = +\infty$$

$g(x) = \lg x - \frac{2}{x} - \frac{1}{x^2} + 1$

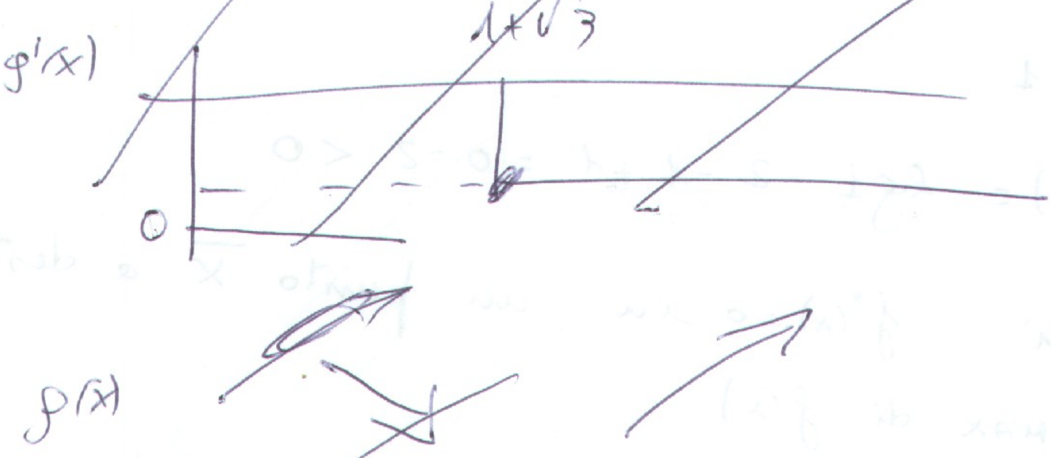
$g'(x) = \frac{1}{x} + \frac{2}{x^2} + \frac{2}{x^3} = \frac{1}{x}\left(1 + \frac{2}{x} + \frac{2}{x^2}\right) =$

$= \frac{1}{x} \quad \frac{x^2 + 2x + 2}{x^2}$ $\quad \Delta = 4 + 8 < 0$

per $x > 0$ la $g'(x)$ è sempre positiva

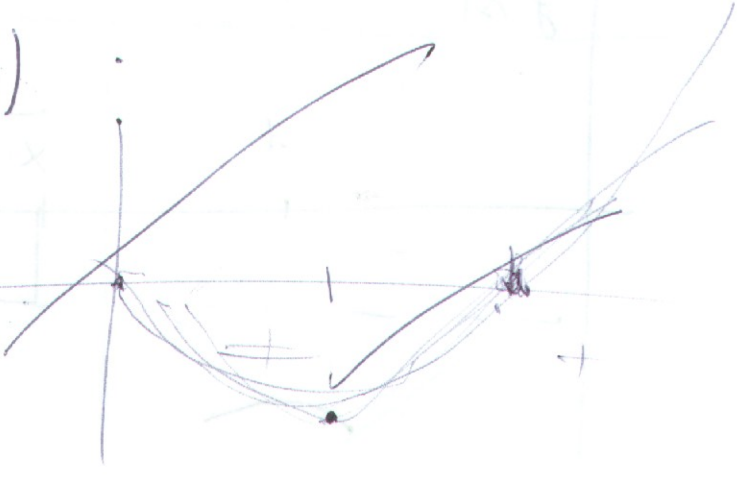
~~$x = 1 \pm \sqrt{1+2} = 1 \pm \sqrt{3}$~~

$x = 1 + \sqrt{3}$

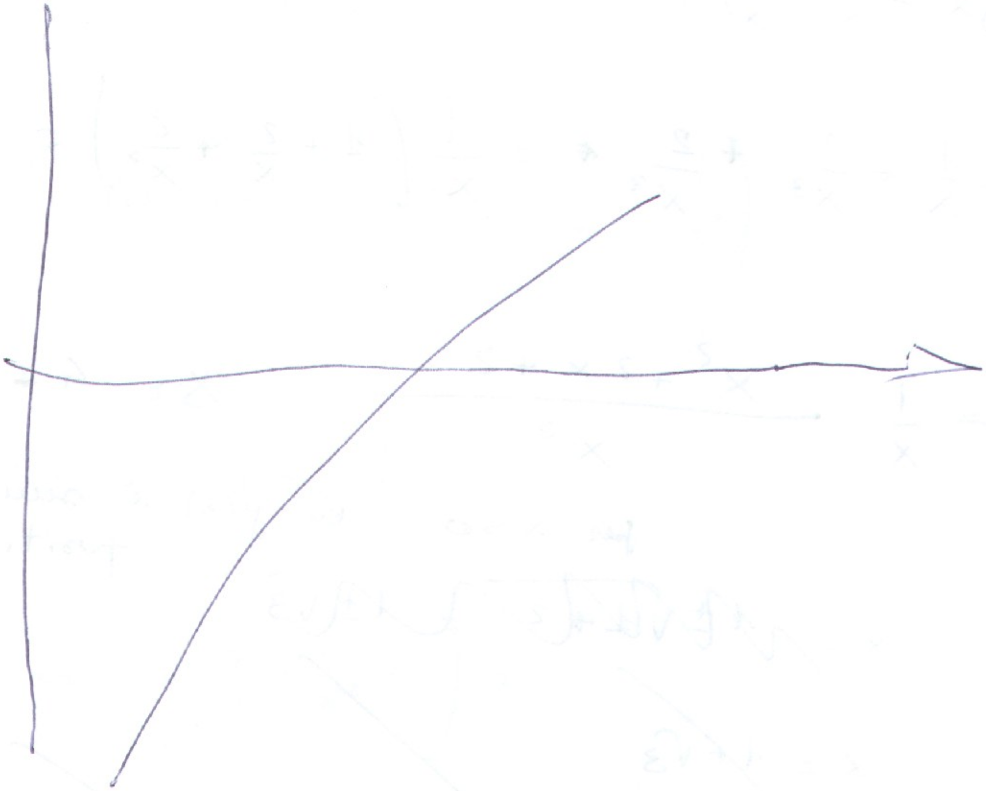


bio

$f''(1+\sqrt{3})$



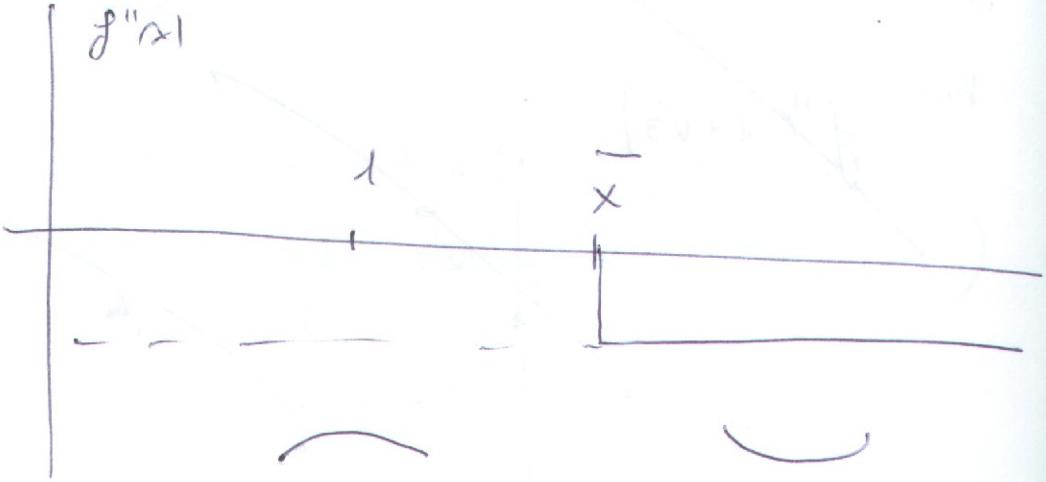
quindi la $g(x)$ attraversa l'asse delle x solo in un punto



in $x = 1$

$$g(1) = \log 1 - 2 - \cancel{1} + \cancel{1} = 0 - 2 < 0$$

quindi $f''(x) = 0$ in un punto $\overline{x}$ a destra del MAX di $f(x)$



Studiamo ora la

$$e^{-x}\left(\log(-x)-1\right) \quad \text{per } x<0$$

CE: $x \neq 0$ $\quad]-\infty,0[$

Positività

$$f(x) \geq 0 \qquad \begin{cases} e^{-x} > 0 \\ \log(-x)-1>0 \end{cases} \qquad \begin{cases} \forall x \\ \log(-x) > \cancel{0}\ 1 \end{cases}$$

$$\begin{cases} \forall x \\ -x > e \end{cases} \qquad \begin{cases} \forall x \\ x < -e \end{cases}$$

$$\lim_{x \to 0^-} e^{-x}\left(\log(-x) - 1\right) = 1(-\infty - 1)$$

$$= -\infty$$

$$\lim_{x \to -\infty} e^{-x}\left(\log(-x) - 1\right) = e^{+\infty}\left(\log(+\infty) - 1\right) =$$

$$= +\infty$$

~~MAX E MIN~~

$$m = \lim_{x \to -\infty} \frac{e^{-x}}{x}\left(\log(-x) - 1\right) =$$

$$= \lim_{x \to -\infty} \frac{e^{-x}}{x} \lim_{x \to -\infty} \left[\log(-x) - 1\right] =$$

$$= \lim_{x \to -\infty} - e^{-x} \cdot \lim_{x \to -\infty} \left[\log(-x) - 1\right] =$$

$$= -\infty$$

non c'è asintoto obliquo.

MAX E MIN

$$Df(x) = -e^{-x}\left[\log(-x)-1\right] + e^{-x}\left[-\frac{1}{x}(-1)\right] =$$

$$= -e^{-x}(\log(-x)-1) + \frac{e^{-x}}{x}$$

$$= e^{-x}\left(\frac{1}{x} - \log(-x) + 1\right)$$

$f'(x)=0 \qquad \frac{1}{x} - \log(-x) + 1 = 0$

eq. non risolvibile

$u(x,y,z) = 3x^2yz + xy^2z$

$\frac{\partial u}{\partial x} = 6xyz + y^2z$

$\frac{\partial^2 u}{\partial x^2} = 6yz$

$\frac{\partial u}{\partial x\partial y}$

②

$\frac{\partial u}{\partial x\partial y} = 6xz$

$\frac{\partial u}{\partial y} = 3x^2z$

$\frac{\partial u}{\partial y\partial x} = 6xz$

$\frac{\partial u}{\partial x} = f_x$

Studiamo la

$$g(x) = \frac{1}{x} + 1 - \log(-x)$$

$$g(-1) = -1 + 1 - 0 = 0$$

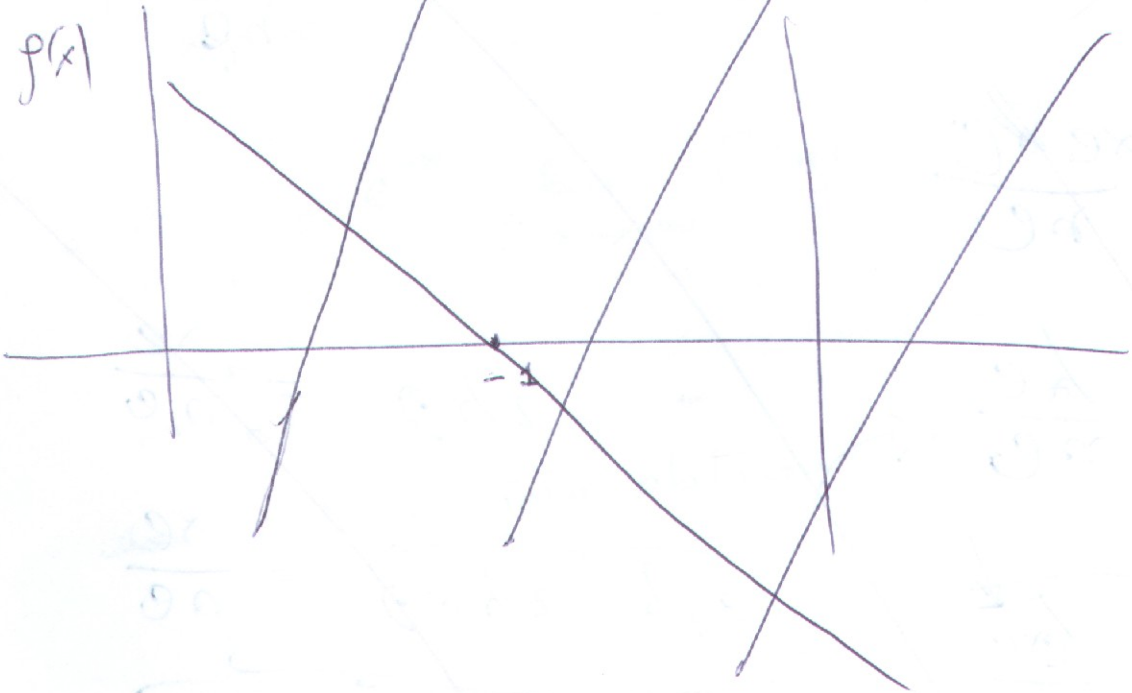
$$g'(x) = -\frac{1}{x^2} - \left(-\frac{1}{x}\right)(-1) = -\frac{1}{x^2} + \frac{1}{x} = -\frac{x+1}{x^2}$$

$g'(x) = 0$ per $x = 1$

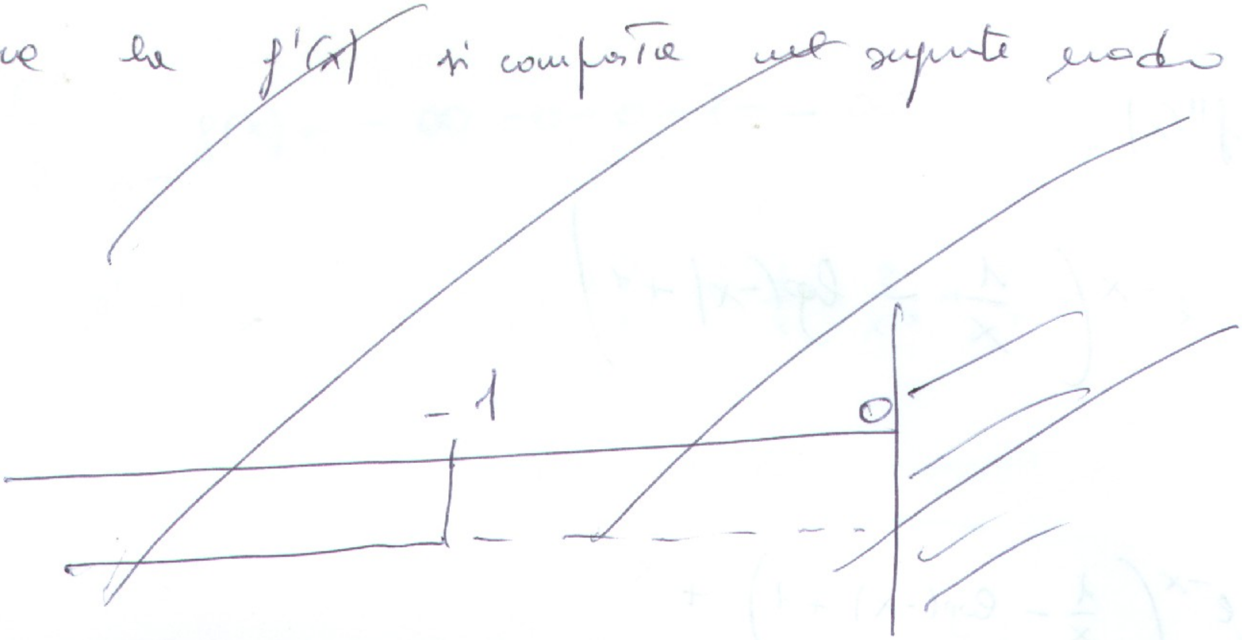
quindi $g'(x) > 0$ per $x + 1 < 0$ $\quad x < -1$

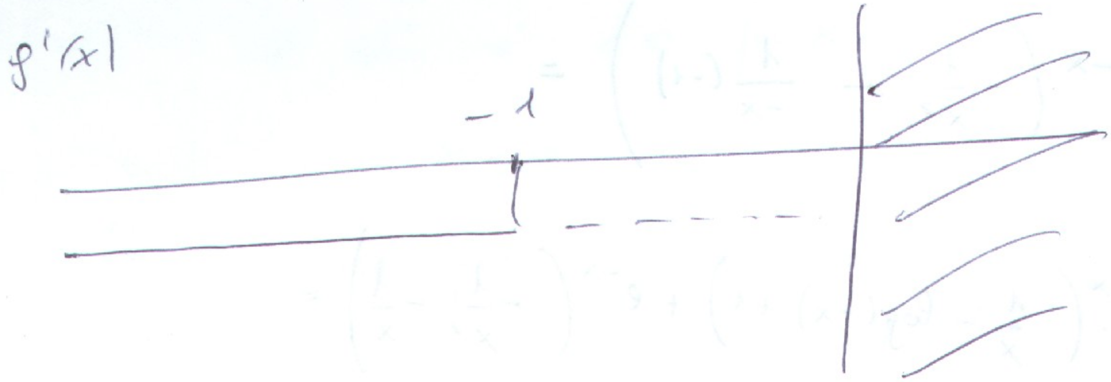
quindi ~~per~~ $x < 0$ la $g'(x)$ è sempre negativa

la $g(x)$ è sempre decrescente

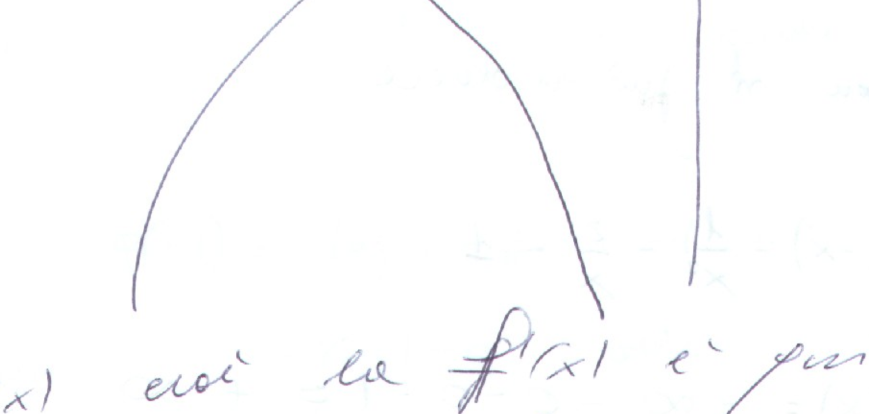


allora la $f'(x)$ si comporta nel seguente modo





la $f(x)$ cresce prima di -1 e decresce dopo



la $f(x)$ cioè la $f'(x)$ è quindi sempre negativa cioè la $f(x)$ è sempre

$f''(x)$

$$D\ e^{-x}\left(\frac{1}{x}-\log(-x)+1\right)$$

$$=-e^{-x}\left(\frac{1}{x}-\log(-x)+1\right)+$$

$$+\ e^{-x}\left(-\frac{1}{x^2}-\frac{1}{-x}(-1)\right)=$$

$$=-e^{-x}\left(\frac{1}{x}-\log(-x)+1\right)+e^{-x}\left(-\frac{1}{x^2}-\frac{1}{x}\right)=$$

$$=e^{-x}\left(\log(-x)-\frac{1}{x}-1-\frac{1}{x^2}-\frac{1}{x}\right)=$$

$$=e^{-x}\left(\log(-x)-\frac{1}{x^2}-\frac{2}{x}-1\right)$$

$f''(x)=0$ non si può risolvere

$$g(x)=\log(-x)-\frac{1}{x^2}-\frac{2}{x}-1$$

$$\lim_{x\to-\infty} g(x)=+\infty-0-0-1=+\infty$$

$$\lim_{x \to 0^-} g(x) = -\infty - 0 - 0 - 1 = -\infty$$

$$g'(x) = -\frac{1}{x}(-1) + \frac{2}{x^3} + \frac{2}{x^2} = \frac{1}{x} + \frac{2}{x^3} + \frac{2}{x^2} =$$

$$= \frac{x^2 + 2x + 2}{x^3} .$$

il numeratore ha $\Delta < 0$ ed è > 0 $\forall x$

il den. è < 0 per $x < 0$

quindi per $x < 0$ $g'(x) < 0$

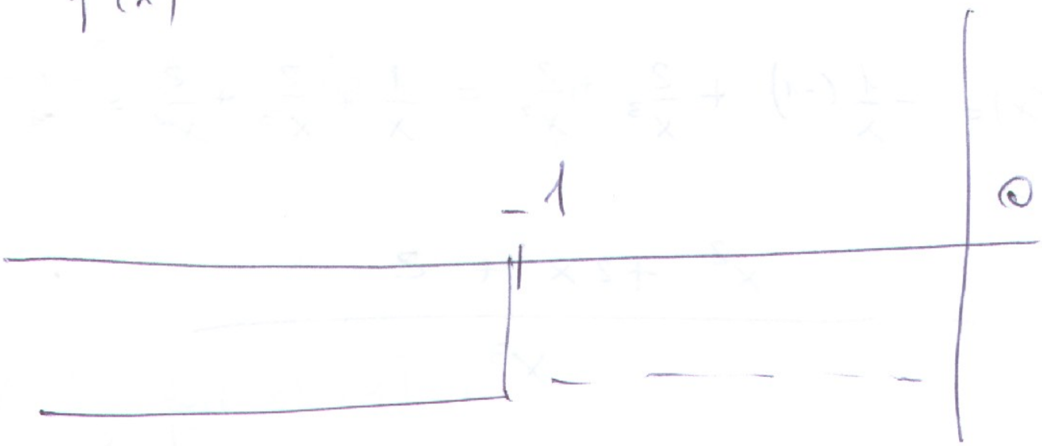
la $g(x)$ è sempre decrescente.

quindi la $f''(x)$ è uguale a zero solo in un punto $\bar{x}$

f''[illegible] $g(-1) = \log 1 - \frac{1}{\infty} + 2 - 1 = 0$

quindi $f''(-1) = 0$ allora $x = 1$ è un punto di flesso

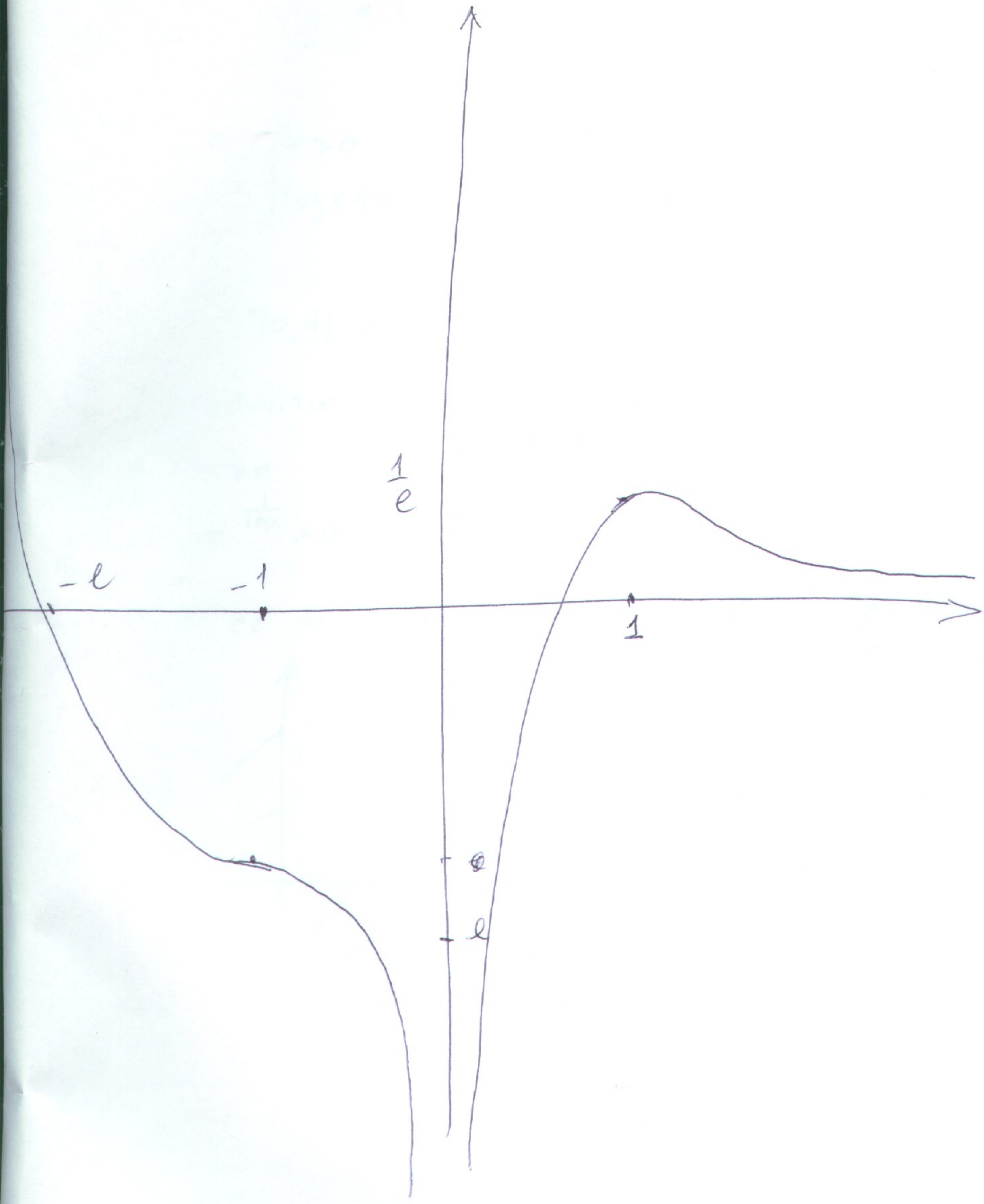
$f''(x)$



$f(x)$

$f(-1) = e^1 (\log(1) - 1) = -e$

$f(1) = :$


1/e
-e
-1
1
e